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Solve the Transportation Problem of Trapezoidal Fuzzy Numbers using Robust's Ranking Technique

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A B S T R A C T

Fuzzy set theory has been applied in many fields such as management, engineering, theory of matrices and so on. In this paper, some elementary operations on proposed Trapezoidal Fuzzy numbers (TrFNs) are defined and also have been defined some operations on Trapezoidal Fuzzy matrices (TrFMs). Using Robust's ranking technique method to solve the Fuzzy Transportation problem of Trapezoidal Fuzzy numbers.

Keywords: Trapezoidal Fuzzy number, Robust's Ranking Technique, Fuzzy Transportation problem

1. Introduction

Fuzzy sets have been introduced by Lofti.A. Zadeh. Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. Hisdal discussed the interval-valued fuzzy sets if higher type. Interval arithmetic was first suggested by Dwyer in 1951, by means of Zadeh's extension principle, the usual Arithmetic operations on real numbers can be extended to the ones defined on Fuzzy numbers. Dubosis and Prade have defined any of the fuzzy numbers. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single – valued numbers. Jhon studied an appraisal of theory and applications on type-2 fuzzy sets.

Trapezoidal Fuzzy number's (TrFNs) are frequently used in application, due to the presence of uncertainty in many mathematical formulations in different branches of science and technology. Presenting a new ranking function and arithmetic operations on type-2 generalized Trapezoidal fuzzy numbers by Stephen Dinagar and Anbalagan.

Fuzzy matrices were introduced for the first time by Thomason who discussed the convergence of power of fuzzy matrix. Several authors had presented a number of results on the convergence of power sequences of fuzzy matrices. Fuzzy matrices play an important role in scientific development. Two new operations and some applications of fuzzy matrices are given in Shymal.A.K. and Pal.M. .

Ragab et.al presented some properties of the min-max composition of fuzzy matrices. Kim presented some important results on determinant of square Fuzzy matrices and contributed with many research works. The adjoint of square Fuzzy matrix was defined by Thomson and Kim, Jaisankar and Mani proposed the Hessen berg of Trapezoidal fuzzy number matrices.

In this paper a new method is presented for solving the fuzzy transportation problem using Robust's ranking technique for the representative values of the fuzzy number. Using the proposed ranking method is discussed with illustration. It is very simple and easy to understand the fuzzy optimal solution of fuzzy transportation problems occurring in the real life situations.

1.1 Basic Preliminaries

Fuzzy number

A fuzzy set $\tilde{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

$(i)\tilde{A}$ is normal (ii) $\tilde{A}$ is convex (iii) The support of $\widetilde{A}$ is closed and bounded then $\widetilde{A}$ is called fuzzy number.

Trapezoidal Fuzzy Number

A fuzzy number $\tilde{A}^{TzL}=(a_1, a_2, a_3, a_4)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}^{TzL}}(x) = \begin{cases} 0 & ; \quad x \leq a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & ; \quad a_1 \leq x \leq a_2 \\ 1 & ; \quad a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & ; \quad a_3 \leq x \leq a_4 \\ 0 & ; \quad x > a_4 \end{cases}$$

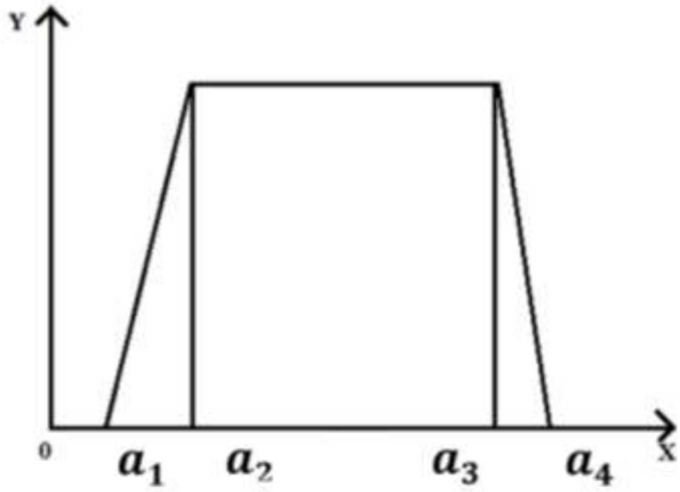

Fig:1 Trapezoidal Fuzzy number

Robust's Ranking Technique

Yager (1981) Let $\tilde{A}$ be a convex fuzzy number the Robust's ranking index is defined by

$R(\tilde{A}) = 1/2 \int_0^1 [\tilde{A}_\lambda^L, \tilde{A}_\lambda^U] \; = 1/2 \int_0^1 [(l' + (a - l')\lambda) + (r' - (r' - b)\lambda] d\lambda.$

In this paper, we give index $R(\tilde{A})$ is the representative value of the fuzzy number $\widetilde{A}$ and index $R(\tilde{C}_{ij})$ is the representative value of the fuzzy cost $\tilde{C}_{ij}$.

Fuzzy Transportation Problem

The mathematical formulation of the FTP is of the following form (Table 1):

$minimize\ \tilde{\Psi} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} X_{ij}$

Subject to $\sum_{j=1}^{n} X_{ij} \leq \alpha_i,\ i = 1,2, \dots, m$

$\sum_{i=1}^{m} X_{ij} \geq \beta_i \quad j = 1,2, \dots, n\ \ and\ X_{ij} \geq 0$ for all i and j,

Table 1: Fuzzy Transportation Table

	1	**2**	$\cdots$	**N**	α_i
1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\cdots$	$\tilde{c}_{1n}$	α_1
2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\cdots$	$\tilde{c}_{2n}$	α_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
β_j	β_1	β_2	$\cdots$	β_n	$\sum_{i=1}^{m} \alpha_i = \sum_{j=1}^{n} \beta_j$

where $\tilde{c}_{ij}$ is the fuzzy cost of transportation one unit of the goods ith source to the jth destination.

X_{ij} is the quantity transportation from ith source to the jth destination.

α_i is the total availability of the goods at ith source. β_i is the total requirements of the goods at jth destination.

$\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} \tilde{X}_{ij}$ is the total fuzzy transportation cost.

If $\sum_{i=1}^{m} \alpha_i = \sum_{j=1}^{n} \beta_j$, then FTP is said to be balanced.

If $\sum_{i=1}^{m} \alpha_i \neq \sum_{j=1}^{n} \beta_j$, then FTP is said to be unbalanced

1.2 Operation on Trapezoidal Fuzzy Numbers (TrFNs)

Let $\widetilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ and $\widetilde{B}^{TzL} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers (TrFNs) then we defined,

Addition

$\widetilde{A}^{TzL} + \widetilde{B}^{TzL} = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$

Subtraction

$\widetilde{A}^{TzL} - \widetilde{B}^{TzL} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

Multiplication

$\widetilde{A}^{TzL} \times \widetilde{B}^{TzL} = (a_1\Re(B), a_2\Re(B), a_3\Re(B), a_4\Re(B))$

where $\Re(\widetilde{B}^{TzL}) = \left(\dfrac{b_1+b_2+b_3+b_4}{4}\right)$ or

$\Re(\widetilde{b}^{TzL}) = \left(\dfrac{b_1+b_2+b_3+b_4}{4}\right)$

Division

$\widetilde{A}^{TzL} / \widetilde{B}^{TzL} = \left(\dfrac{a_1}{\Re(\widetilde{B}^{TzL})}, \dfrac{a_2}{\Re(\widetilde{B}^{TzL})}, \dfrac{a_3}{\Re(\widetilde{B}^{TzL})}, \dfrac{a_4}{\Re(\widetilde{B}^{Tzl})}\right)$

where $\Re(\widetilde{B}^{TzL}) = \left(\dfrac{b_1+b_2+b_3+b_4}{4}\right)$ or

$\Re(\widetilde{b}^{TzL}) = \left(\dfrac{b_1+b_2+b_3+b_4}{4}\right)$

Scalar multiplication

$$K\,\widetilde{A}^{TzL} = \begin{cases} \left(ka_1, ka_2, ka_3, ka_4\right) & if\ k \geq 0 \\ \left(ka_4, ka_3, ka_2, ka_1\right) & if k < 0 \end{cases}$$

1.3 Operation on Trapezoidal Fuzzy Matrices (TrFMs)

Let A = ($\widetilde{a}_{ij}^{TzL}$) and B =($\widetilde{b}_{ij}^{TzL}$) be two trapezoidal fuzzy matrices (TrFMs) of same order. Then, we have the following

i. **Addition**

A+B = $\left(\widetilde{a}_{ij}^{TzL} + \widetilde{b}_{ij}^{TzL}\right)$

ii. **Subtraction**

A-B = $\left(\widetilde{a}_{ij}^{TzL} - \widetilde{b}_{ij}^{TzL}\right)$

iii. For A = $\left(\widetilde{a}_{ij}^{TzL}\right)_{m\times n}$ and B = $\left(\widetilde{b}_{ij}^{TzL}\right)_{n\times k}$ then AB = $\left(\widetilde{c}_{ij}^{TzL}\right)_{m\times k}$

where $\widetilde{c}_{ij}^{TzL} = \sum_{p=1}^{n} \widetilde{a}_{ip}^{TzL} . \widetilde{b}_{pj}^{TzL}$, i=1,2,…,m and j=1,2,…,k.

iv. A^T or A^1 = $\left(\widetilde{a}_{ji}^{TzL}\right)$

v. $KA = \left(K\,\widetilde{a}_{ij}^{TzL}\right)$ where K is scalar.

1.3.1 Examples

1) If $A=\begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix}$ and

B= $\begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

Then A+B= $\left(\tilde{a}_{ij}^{TzL} + \tilde{b}_{ij}^{TzL}\right)$

$A+B=\begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix}+\begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

$=\begin{bmatrix}(1,5,7,11) & (5,9,13,17)\\ (4,8,10,18) & (3,6,13,18)\end{bmatrix}$

2) If $A=\begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix}$ and

B= $\begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

Then A − B= $\left(\tilde{a}_{ij}^{TzL} - \tilde{b}_{ij}^{TzL}\right)$

$A-B=\begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix}-\begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

$=\begin{bmatrix}(-8,-2,0,2) & (-7,-3,1,5)\\ (-11,-1,1,3) & (-4,1,8,11)\end{bmatrix}$

3) If $A=\begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix}$ and

B= $\begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

Then A.B= $(\tilde{a}_{ij}^{TzL}\tilde{b}_{ij}^{TzL})$

$A.B=\begin{bmatrix}(-1,2,3,4) & (2,4,6,8)\\ (1,4,5,6) & (4,5,9,10)\end{bmatrix}\begin{bmatrix}(2,3,4,7) & (3,5,7,9)\\ (3,4,5,12) & (-1,1,4,8)\end{bmatrix}$

$$=\begin{bmatrix}(-1,2,3,4)(4)+(2,4,6,8)(6) & (-1,2,3,4)(6)+(2,4,6,8)(3)\\ (1,4,5,6)(4)+(4,5,9,10)(6) & (1,4,5,6)(6)+(4,5,9,10)(3)\end{bmatrix}$$

$A.B=\begin{bmatrix}(8,32,48,64) & (0,24,36,48)\\ (28,46,74,84) & (18,39,57,66)\end{bmatrix}$

2. Robust ranking technique

Numerical example

Consider fuzzy transportation problem with three sources that is $\boldsymbol{S_1, S_2, S_3}$ and three destinations$\boldsymbol{D_1, D_2, D_3}$. The cost of transporting one unit of the goods from $\boldsymbol{i}$th source to the $\boldsymbol{j}$th destination given whose elements are trapezoidal fuzzy numbers are shown in Table 2. Find out the minimum cost total fuzzy transportation.

From Table 2, the cost of transporting one unit of the goods from $\boldsymbol{i}$th source to the $\boldsymbol{j}$th destination are TrFNs. We use Robust's ranking technique for calculating the membership function of the TrFNs.

Table 2. The Fuzzy Transportation Table

SOURCE	D_1	D_2	D_3	Supply (α_i)
S_1	(3,5,7,14)	(2,4,8,13)	(3,5,9,15)	35
S_2	(2,5,8,10)	(3,6,9,12)	(4,7,10,16)	40
S_3	(3,6,8,13)	(4,8,10,15)	(5,9,13,15)	50
Demand(β_j)	45	55	25	

Using Robust's Ranking Technique to convert the transportation table.

$\boldsymbol{R(\bar{A}) = 1/2\int_0^1 [A_\lambda^{-L}, A_\lambda^{-U}]d\lambda}$

$= \mathbf{1/2}\int_{0}^{1}[((\boldsymbol{l'} + \boldsymbol{a} - \boldsymbol{l''})\boldsymbol{\lambda}) + (\boldsymbol{r'} - (\boldsymbol{r'} - \boldsymbol{b})\boldsymbol{\lambda})]\boldsymbol{d\lambda},$ where $(\boldsymbol{l'}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{r'}) = (\overline{\boldsymbol{A}})$.

Let us consider above table where the element (3,5,7,14) is TrFN.

The $\boldsymbol{\lambda}$-cut of the TrFN (3,5,7,14) is $[\boldsymbol{C}_{\boldsymbol{11\lambda}}^{-L}, \boldsymbol{C}_{\boldsymbol{11\lambda}}^{-U}] = [\mathbf{3} + \mathbf{2}\boldsymbol{\lambda}, \mathbf{14} - \mathbf{7}\boldsymbol{\lambda}]$.

Therefore we obtain.

$$\boldsymbol{R}(\boldsymbol{C}_{\mathbf{11}}) = \boldsymbol{R}(\mathbf{3}, \mathbf{5}, \mathbf{7}, \mathbf{14}) = \frac{\mathbf{1}}{\mathbf{2}}\int_{\mathbf{0}}^{\mathbf{1}}[(\mathbf{3} + (\mathbf{5} - \mathbf{3})\boldsymbol{\lambda}) + (\mathbf{14} - (\mathbf{14} - \mathbf{7})\boldsymbol{\lambda})]\boldsymbol{d\lambda}$$

$= \frac{1}{2}\int_0^1 (17 - 5\lambda)d\lambda = \frac{1}{2}\left[17\lambda - \frac{5\lambda^2}{2}\right]_0^1 = \frac{1}{2}\left[\left(17 - \frac{5}{2}\right) - 0\right]$

$R(C_{11}) = 7.25$

$R(C_{12}) = R(2,4,8,13) = \frac{1}{2}\int_0^1 [(2 + (4 - 2)\lambda) + (13 - (13 - 8)\lambda)]d\lambda$

$= \frac{1}{2}\int_0^1 (15 - 3\lambda)d\lambda = \frac{1}{2}\left[15\lambda - \frac{3\lambda^2}{2}\right]_0^1 = \frac{1}{2}[\left(57 - \frac{3}{2}\right) - 0]$ $R(C_{12}) = 6.75$

$R(C_{13}) = R(3,5,9,15) = \frac{1}{2}\int_0^1 [(3 + 2\lambda) + (15 - 6\lambda)]d\lambda$

$= \frac{1}{2}\int_0^1 (18 - 4\lambda)d\lambda = \frac{1}{2}\left[18\lambda - \frac{4\lambda^2}{2}\right]_0^1 = \frac{1}{2}[\left(18 - \frac{4}{2}\right) - 0]$ $R(C_{13}) = 8$

$R(C_{21}) = R(2,5,8,10) = \frac{1}{2}\int_0^1 [(2 + 3\lambda) + (10 - 2\lambda)]d\lambda$

$= \frac{1}{2}\int_0^1 (12 + \lambda)d\lambda = \frac{1}{2}\left[12\lambda + \frac{\lambda^2}{2}\right]_0^1 = \frac{1}{2}[\left(12 + \frac{1}{2}\right) - 0]$ $R(C_{21}) = 6.25$

$R(C_{22}) = R(3,6,9,12) = \frac{1}{2}\int_0^1 [(03 + 3\lambda) + (12 - 3\lambda)]d\lambda$

$= \frac{1}{2}\int_0^1 15\, d\lambda = \frac{1}{2}[15\lambda]_0^1 = \frac{1}{2}\left[\left(15(1)\right) - 0\right]$

$R(C_{22}) = 7.5$

$R(C_{23}) = R(4,7,10,16) = \frac{1}{2}\int_0^1 [(4 + 3\lambda) + (16 - 6\lambda)]d\lambda$

$= \frac{1}{2}\int_0^1 (20 - 3\lambda)d\lambda = \frac{1}{2}\left[20\lambda - \frac{3\lambda^2}{2}\right]_0^1 = \frac{1}{2}[\left(20 - \frac{3}{2}\right) - 0]$ $R(C_{23}) = 9.25$

$R(C_{31}) = R(3,6,8,13) = \frac{1}{2}\int_0^1 [(3 + 3\lambda) + (13 - 5\lambda)]d\lambda$

$= \frac{1}{2}\int_0^1 (16 - 2\lambda)d\lambda = \frac{1}{2}\left[16\lambda - \frac{2\lambda^2}{2}\right]_0^1 = \frac{1}{2}[(16 - 1) - 0]$ $R(C_{31}) = 7.5$

$R(C_{32}) = R(4,8,10,15) = \frac{1}{2}\int_0^1 [(4 + 4\lambda) + (15 - 5\lambda)]d\lambda$

$= \frac{1}{2}\int_0^1 (19 - \lambda)d\lambda = \frac{1}{2}\left[19\lambda - \frac{\lambda^2}{2}\right]_0^1 = \frac{1}{2}\left[\left(19 - \frac{1}{2}\right) - 0\right]$

$R(C_{32}) = 9.25$

$R(C_{33}) = R(5,9,13,15) = \frac{1}{2}\int_0^1 [(5 + 4\lambda) + (15 - 2\lambda)]d\lambda$

$= \frac{1}{2}\int_0^1 (20 + 2\lambda)d\lambda = \frac{1}{2}\left[20\lambda + \frac{2\lambda^2}{2}\right]_0^1 = \frac{1}{2}[(20 + 1) - 0]$ $R(C_{33}) = 10.5$

Table 3: Fuzzy transportation table after ranking

Source	D_1	D_2	D_3	Supply
S_1	7.25	6.75	8	35
S_2	6.25	7.5	9.25	40
S_3	7.5	9.25	10.5	50
Demand	45	55	25	125

3. IBFS of Fuzzy Transportation Problem

Step 1: Using Robust's Ranking Technique to convert the transportation table, we get Table 3.

Step 2: Since $\sum_{i=1}^{3}\boldsymbol{\alpha_i} = \sum_{j=1}^{3}\boldsymbol{\beta_j} = \mathbf{125}$, the FTP is balanced. From the above table is balanced.

Step 3: Use VAM method to find the IBFS.

Hence optimal assignment basic variable is

$\boldsymbol{X_{12}} = \mathbf{10}, \boldsymbol{X_{13}} = \mathbf{25}$, $\boldsymbol{X_{22}} = \mathbf{40}, \boldsymbol{X_{31}} = \mathbf{45}, \boldsymbol{X_{32}} = \mathbf{5}$ and minimum fuzzy transportation cost is

Transportation cost = $(\mathbf{10} \times \mathbf{6.75}) + (\mathbf{25} \times \mathbf{8}) + (\mathbf{40} \times \mathbf{7.5}) + (\mathbf{45} \times \mathbf{7.5}) + (\mathbf{5} \times \mathbf{9.25})$

$= \mathbf{67.5 + 200 + 300 + 337.5 + 462.5}$

$\mathbf{T.cost = 951.25}$.

Table 4: IBFS Fuzzy Transportation Table

Source	D_1	D_2	D_3	Supply
S_1	7.25	10 6.75	25 8	35
S_2	6.25	40 7.5	9.25	40
S_3	45 7.5	5 9.25	10.5	50
Demand	45	55	25	125

4. Optimal Solution of Fuzzy Transportation Problem

Using MODI Method in Table 4, we get

$\boldsymbol{C_{12} = U_1 + V_2 = 6.75 \Rightarrow U_1 = 6.75}$

$\boldsymbol{C_{22} = U_2 + V_2 = 7.5 \Rightarrow U_2 = 7.5}$

$\boldsymbol{C_{32} = U_3 + V_2 = 9.25 \Rightarrow U_3 = 9.25}$

$\boldsymbol{C_{13} = U_1 + V_3 = 8 \Rightarrow 6.75 + V_3 = 6.75}$

$\boldsymbol{\Rightarrow V_3 = 8 - 6.75 \Rightarrow V_3 = 1.25}$

$\boldsymbol{C_{31} = U_3 + V_1 = 7.5 \Rightarrow 9.25 + V_1 = 7.5}$

$\boldsymbol{\Rightarrow V_1 = 7.5 - 9.25 \Rightarrow V_1 = -1.75.}$

Finally get MODI method table, we get Table 5

Here all $\boldsymbol{C_{ij} - Z_{ij} \leq 0}$, then the table is optimal.

Transportation cost = $(\mathbf{10} \times \mathbf{6.75}) + (\mathbf{25} \times \mathbf{8}) + (\mathbf{40} \times \mathbf{7.5}) + (\mathbf{45} \times \mathbf{7.5}) + (\mathbf{5} \times \mathbf{9.25})$

$= \mathbf{67.5 + 200 + 300 + 337.5 + 462.5}$

$\mathrm{T.cost} = 951.25.$

Table 5: Optimal Fuzzy Transportation Table

5 7.25 -2.25	6.75 10	8 25	$U_1 = 6.75$
5.75 6.25 -0.5	7.5 40	8.75 9.25 -0.5	$U_2 = 7.5$
7.5 45	9.25 5	10.5 9.25 -0.5	$U_3 = 9.25$
$V_1 = -1.75$	$V_2 = 0$	$V_3 = 1.25$	

5. Conclusion

In this paper, concentrated the notion of the Trapezoidal fuzzy matrices and operations of trapezoidal fuzzy matrices are defined. Few illustrations based on operations of trapezoidal fuzzy matrices have also been justified. To solve the Transportation Problem using the Robust's ranking technique with illustration. To solve the Assignment Problem, Sequencing Problem and so on is using the Robust's ranking technique of Trapezoidal fuzzy numbers will discuss in future.

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