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Heuristic Approach and the Mathematical Fluency of Grade 10 Students

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A B S T R A C T

Students in the present struggle and mathematically challenge how word problems should be solved correctly and effectively. One of the ways to resolve issues like this is the proper application of techniques and methodologies bound in the mathematics education field. This led to the study that attempted to determine the effect of the heuristic approach on the mathematical fluency of Grade 10 students in a public high school in San Pablo City. Eighty students participated, of which forty students were taught using the traditional approach, and the other forty were exposed to the heuristic approaches. Inferential statistics also showed a significant difference in mathematical fluency based on the mean pre-test and post-test of both experimental and control groups, implying improved performance after instruction. Comparison of the level of mathematical fluency based on test results after traditional instruction and the integration of the heuristic approaches showed significant differences in favor of the experimental group. Thus, using heuristic approaches has improved students' fluency levels better than the traditional approach. This suggested that the heuristic approach had a measurable and positive impact on the student's mathematical performance. Additionally, the significant increase in scores suggests that students became more capable of applying learned concepts to solve mathematical problems.

Keywords: heuristic approach, mathematical fluency, traditional instruction

Introduction

Given the recent tech advancements and the increase in the need for critical thinking skills, the learning environment has drastically changed to equip students with the necessary tools for success in an ever-dynamic world. Pivotal to this transformation is the desire to develop mathematical competence, an invaluable skill set that is indispensable for coping with the complexities of diverse academic and professional fields (Barham, 2020). Despite all technological advancements, students in the present are struggling and mathematically challenged as to how a problem should be solved correctly and effectively. One of the ways to resolve issues like this is the proper application of techniques and methodologies bound in the field of education, specifically in mathematics (Lee, 2017).

The heuristic teaching approach implies that it differs from more traditional forms of mathematics instruction (Pathak, 2023). Instead of restricting students to algorithms and predetermined solutions, it allows them to build an inquiry-based approach to mathematics. The intent is to enable the students to find solutions by themselves, making them feel happy that they have taken up a problem as their own (Andrini, 2016). The heuristic approach laid its foundation at the high school level, and therefore, high school becomes a bridge that connects basic concepts with higher-order mathematics topics (Tambunan, 2018).

Rosyada and Retnawati (2021) hinted that educator are not free from controversy and challenges when shifting towards a heuristic approach in mathematics education. Others believe it might depart the structured learning environment in which students are used, likely affecting their ability to understand complex mathematical concepts. On the other hand, supporters of the heuristic method argue that it allows children to understand mathematics more broadly and apply knowledge in different situations (Kaitera & Harmoinen, 2022). This present study would like to add its voice to this never-ending debate by looking at the impact of the heuristic approach on the mathematics fluency of high school students.

Being proficient in math is highly crucial for an individual. It helps individuals effectively grasp and apply arithmetic topics (Hirschmüller & Egloff, 2018). In the past, math instruction was frequently focused on learning and applying formulas (Danesi, 2016). However, nowadays, many people find out that there is another kind of teaching, the so-called heuristic approach, which is also very important. This form of teaching involves a careful thought process, problem-solving skills, and fundamental knowledge of how math works (Gurat, 2018). This is especially important at the high school level when learners are at a crucial moment of their learning journey.

Foster (2017) mentioned that mathematical fluency is essential because the student's ability to understand and use mathematical concepts properly depends on this ability. Considering high school, a vital landmark phase in a student's study life, the importance of mathematical fluency can only be further accentuated. However, mathematics education has traditionally been known to be the kind of lecture where one is compelled to memorize and follow rote methods. On the other hand, however, there are emerging considerations regarding the heuristic approach as an alternative pedagogical approach that leads to critical thinking, problem-solving, and a better understanding of mathematical principles (Gurat, 2018).

In addition, mathematical fluency among high school students is not merely an abstraction of accuracy but a multidimensional competency. It includes the ability of a student to have a deeper understanding of mathematical concepts, apply them logically, and communicate them with ease and confidence (Tikhomirova, 2017). The road to mathematical fluency is paved through the smooth fusion of primary, critical thinking, and problem-solving skills, ensuring that a student blends through a wide range of mathematical challenges that arise in the classroom and day-to-day life (Best, 2019). Being a high school student, the juncture is highly critical for students to consolidate their mathematical knowledge, and this makes mathematical fluency a valuable asset not only for advanced learning but also for the development of necessary analytical skills needed for further academic and professional activities (Gafoor & Kurukkan, 2015).

In other words, mathematical fluency serves as the basis for mathematical learning, providing students with the necessary tools and confidence to navigate the various hurdles that arise during their academic journey and beyond. It is the starting point for advanced thinking, problem-solving, and a greater appreciation for the beauty and application of mathematics in various settings (Best, 2019).

This study, "Heuristic Approaches and the Mathematical Fluency of Grade 10 Students," reveals the significance of heuristic ways of accomplishing tasks and their influence on Grade 10 students' mathematical fluency. Moreover, this thesis intends to narrow the local aspects of educational discourse (Wang, 2023) by shedding some light on the role of intuitive approaches in the development of mathematical fluency of Grade 10 students through scrutiny of this dynamic; the research would try to give a substantial contribution to the current dialogue pertaining innovative pedagogical practices, aiming to enhance the educational experience and effectiveness for students at the helm of the academic establishments.

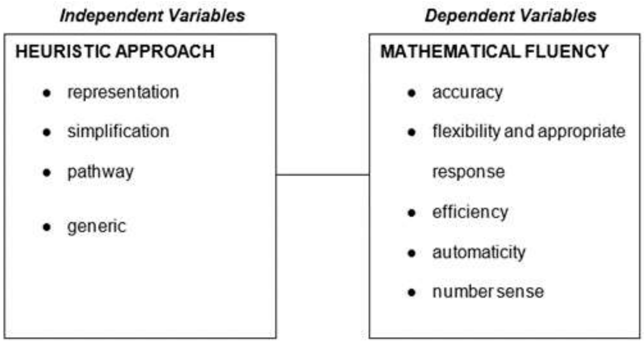

Figure 1. Research Framework

Research Problem

Specifically, this study aimed to answer the following questions:

1. What is the level of mathematical fluency of Grade-10 students in both groups based on the mean pretest and posttest, in terms of:

a. accuracy;

b. flexibility and appropriate response;

c. efficiency;

d. automaticity; and

e. number sense?

2. Is there a significant difference between the pretest and posttest mean score of mathematical fluency of Grade-10 students in the control group?

3. Is there a significant difference between the pretest and posttest mean score of mathematical fluency of Grade-10 students who used the heuristic approaches?

4. Is there a significant difference between the mean pre-test of mathematical fluency of Grade-10 students in the control group and those who used the heuristic approach?

5. Is there a significant difference between the mean post-test of mathematical fluency of Grade-10 students in the control group and those who used the heuristic approach?

Materials and Methods

This study utilized a quasi-experimental pretest and post-test design to analyze and decipher the causal processes between heuristic strategies and the mathematical fluency of Grade 10 learners. A quasi-experimental pretest-post-test research design is a type of research methodology that possesses similarities with experimental designs but lacks full control over variables due to its quasi-experimental nature (Creswell, 2014).

Employed by a stratified random sampling strategy, the respondents of the study were 80 Grade 10 students at San Pablo City Integrated High School. 40 students were included in the control group and the other 40 students were in the experimental group. Stratified random sampling has numerous applications and benefits, such as studying population demographics and life expectancy (Hayes, 2024).

Four lesson exemplars were utilized in a span of four weeks. These exemplars introduced the students to different heuristic approaches with practices through a learning activity sheet. The approaches aimed to improve the mathematical fluency of students. To assess mathematical fluency of students, a pretest and post test in the form of learning activity sheets are prepared.

The initial phase involved the independent application of descriptive statistics to the pretest and post-test scores. Measures such as means, standard deviations, and ranges concisely summarize each phase's central tendencies and variations. This facilitates a clear understanding of the initial mathematical fluency landscape and the subsequent changes observed.

Inferential statistics, specifically paired-sample t-tests and independent t-tests, determined the statistical significance of observed differences between the pretest and post-test scores. This analytical approach allows for a precise assessment of whether the changes are statistically meaningful, providing robust evidence of the impact of heuristic approaches on mathematical fluency.

Inferential statistics was applied to determine the magnitude of the difference between the two groups (Fritz et al., 2012). This gave insights into the practical relevance of the heuristic intervention. Effect size calculations contributed to a more nuanced understanding of the impact on mathematical fluency dimensions, enhancing the interpretative depth of the findings.

Fusing descriptive statistics, inferential statistics, and effect size calculations enabled a comprehensive interpretation of the pretest and post-test data. Findings are not solely confined to statistical significance but are contextualized within the specific changes observed in mathematical fluency dimensions. This integrated approach ensures a thorough and refined understanding of the effectiveness of heuristic approaches in shaping Grade 10 students' mathematical fluency.

Results and Discussions

The table below, the level of mathematical fluency of the students in both groups in terms of accuracy is described from inadequate to proficient. There were 37 or 92.5% in the experimental group and 40 or 100% in the control group were deficient in the pre-test. The results indicated that students of both groups had gaps in understanding key concepts, they struggled to solve problems accurately. These gaps include lack of prior knowledge and applications of getting the value of quartile and deficiency in understanding the steps in solving problems involving quartile. They also tended to commit errors in calculations using the four fundamental operations. Lastly, the students made mistakes on choosing the value that corresponds to the computed position.

On the other hand, 24 or 60% in experimental group were proficient and 25 or 62.5% in the control group are approaching proficiency. This means that both groups achieved higher levels of accuracy. After being exposed to heuristic approaches, most students in the experimental group were able to analyze problems critically and apply appropriate problem-solving strategies and tended to make no errors in the result. More so, the control group also achieved a higher level of accuracy with the use of lecture-discussion teaching strategy.

Table 1. Level of mathematical fluency in terms of accuracy

Scores	Pre-test				Post test				Verbal Interpretation
	Exp		Control		Exp		Control		
	F	%	f	%	F	%	F	%	
3.50-4.00	0	0	0	0	24	60	10	25	Proficient
2.50-3.49	0	0	0	0	12	30	25	62.5	Developing
1.50-2.49	3	7.5	0	0	4	10	5	12.5	Emerging
1.00-1.49	37	92.5	40	100	0	0	0	0	Inadequate
Total	40	100	40	100	40	100	40	100	

Legend: 3.50-4.0 Proficient; 2.50-3.49 Approaching Proficiency; 1.50-2.49 Processing; 1.0-1.49 Deficient

Table 2 depicts the level of mathematical fluency of Grade 10 students in terms of flexibility and appropriate response. Based on the table above for the pre-test, 40 or 100% in the experimental group and 36 or 90% in the control group generally described as inadequate which means that students are deficient. This indicates that students had low to mid-level ability as to flexibility and appropriate response.

Conversely, there are no students who attained inadequate after the discussion of the lesson as shown in the result of the post test. Numerically speaking, 17 or 42.5% belonged to proficient and approaching proficiency for experimental group while 21 or 52.5% of the total number of students are approaching proficiency in the control group. This implies that students' abilities in the areas of flexibility and the appropriate response in the solving of math problems can be affected by several factors, among which are their cognitive skills, problem-solving strategies, and mathematical knowledge (Isyrofinnisak et al., 2020). High flexibility in problem-solving denotes a student's capacity to tackle a problem from different angles, employ diverse strategies, and adjust their approach as required.

Table 2. Level of mathematical fluency in terms of flexibility and appropriate response

Scores	Pre-test				Post test				Verbal Interpretation
	Exp		Control		Exp		Control		
	F	%	f	%	F	%	F	%	
3.50-4.00	0	0	0	0	17	42.5	5	12.5	Proficient
2.50-3.49	0	0	0	0	17	42.5	21	52.5	Developing
1.50-2.49	0	0	4	10	6	15	14	35	Emerging
1.00-1.49	40	100	36	90	0	0	0	0	Inadequate
Total	40	100	40	100	40	100	40	100	

Legend: 3.50-4.0 Proficient; 2.50-3.49 Approaching Proficiency; 1.50-2.49 Processing; 1.0-1.49 Deficient

The next table displays the efficiency level of Grade 10 students. There are 40 or 100% in the experimental group and 39 or 97.5% in the control group who are deficient in the pre-test considering efficiency as a mathematical fluency. Majority of students in the groups are inadequate and emerging. Same as for accuracy and flexibility and appropriate response, there are no students who are developing and proficient. Students at these levels are not able to solve problems using shortcuts and strategies to make solving easier. This is due to the lack of mathematical concepts, ineffective problem-solving strategies, or the problems in the organization and execution of the problem-solving steps efficiently (Scheibling-Sève et al., 2020)

Table 3. Level of mathematical fluency in terms of efficiency

Scores	Pre-test				Post test				Verbal Interpretation
	Exp		Control		Exp		Control		
	F	%	f	%	F	%	F	%	
3.50-4.00	0	0	0	0	10	25	3	7.5	Proficient
2.50-3.49	0	0	0	0	24	60	21	52.5	Developing
1.50-2.49	0	0	1	2.5	5	12.5	13	32.5	Emerging
1.00-1.49	40	100.0	39	97.5	1	2.5	3	7.5	Inadequate
Total	40	100	40	100	40	100	40	100	

Legend: 3.50-4.0 Proficient; 2.50-3.49 Approaching Proficiency; 1.50-2.49 Processing; 1.0-1.49 Deficient

After exposing the students to heuristic approaches, larger numbers in the experimental group are developing or 24 (60%) of the total number of students. This indicates that students applied the heuristic approach properly and efficiently solve the problems to arrive. Furthermore, as Eisenmann et al. (2015) explains, high efficiency in solving problems entails the use of efficient strategies, for example, the division of complex problems into manageable steps, the identification of the key information, and the application of the relevant mathematical techniques effectively. There are several students who are proficient and emerging. Moreover, there is only one student who is inadequate in the experimental group.

On the other hand, as for the control group, the majority of students were also developing or approaching proficiency. There were 21 or 52.5% of the total number of students who are developing. Some students are emerging and proficient which means that they were proficient and processing. There were only 3 or 7.5% of the total number of students who are inadequate. This suggests that students who were exposed to traditional ways of learning the lesson are able to solve the problems correctly with a given conventional method of solving problems.

The table below presents the mathematical fluency of Grade 10 students in terms of automaticity. There were 35 or 87.5 in the experimental group and 38 or 95% were inadequate. It shows that the majority of students in both groups are described as deficient to processing.

Table 4. Level of mathematical fluency in terms of automaticity

Scores	Pre-test				Post test				Verbal Interpretation
	Exp		Control		Exp		Control		
	F	%	f	%	F	%	F	%	
3.50-4.00	0	0	0	0	29	72.5	22	55	Proficient
2.50-3.49	0	0	0	0	7	17.5	15	37.5	Developing
1.50-2.49	5	12.5	2	5	4	10	2	5	Emerging
1.00-1.49	35	87.5	38	95	0	0	1	2.5	Inadequate
Total	40	100	40	100	40	100	40	100	

Legend: 3.50-4.0 Proficient; 2.50-3.49 Approaching Proficiency; 1.50-2.49 Processing; 1.0-1.49 Deficient

In automaticity part of the pre-test, it allows the students to record the time they spent in solving the problems. They were asked to write the time started and time finished. Most students recorded more than 10 minutes to finish a problem that landed in adequate level. It indicates students are slow and shows an effortful approach to solving math problems. These involve frequent pauses, errors, or the need to mentally calculate basic operations like addition, subtraction, multiplication, or division instead of recalling them automatically (Santos, 2020). It is also observed that there is no student in the proficient and developing levels before the applying heuristic approaches.

On one hand, after exposure to heuristic approaches and traditional ways of teaching the lesson as shown in the result of the post test, there were 29 or 72.5 % in the experimental group and 22 or 55% in the control group are proficient. This means majority of students in both groups is described as proficient. On this note, students portray high automaticity, which is a sign of a student's capacity to solve math problems quickly, correctly and with very little mental effort. Students can do basic calculations quickly, remember mathematical facts easily, and use procedural knowledge fast and accurately (Eisenmann et al, 2015).

Table 5. Level of mathematical fluency in terms of number sense

Scores	Pre-test				Post test				Verbal Interpretation
	Exp		Control		Exp		Control		
	F	%	f	%	F	%	F	%	
3.50-4.00	0	0	0	0	9	22.5	3	7.5	Proficient
2.50-3.49	6	15	4	10	24	60	13	32.5	Developing
1.50-2.49	21	52.5	19	47.5	7	17.5	19	47.5	Emerging
1.00-1.49	13	32.5	17	42.5	0	0	5	12.5	Inadequate
Total	40	100	40	100	40	100	40	100	

Legend: 3.50-4.0 Proficient; 2.50-3.49 Approaching Proficiency; 1.50-2.49 Processing; 1.0-1.49 Deficient

In contrast, after exposure to heuristic approaches and traditional ways of teaching the lesson, the table above shows that 24 or 60% in the experimental group were developing while 19 or 47.5% were emerging which means that most of them were approaching proficiency and processing. There were some students who are proficient and inadequate. Therefore, students can quickly and accurately identify arithmetic operations, recognize and apply number patterns, estimate quantities effectively, make connections between mathematical ideas and enumerate mathematical knowledge. Students with high number sense indicates a student's strong understanding and proficiency in numerical concepts, operations, and relationships (Moscoso et al., 2020)

The following table shows that there is a noticeable difference in the mathematical fluency of the students in the control group based on pre-test and posttest mean scores. This is seen in all components of mathematical fluency namely, accuracy, flexibility and appropriate response, efficiency, automaticity and number sense. This means that the students have developed their level of mathematical fluency after the lesson is taught. Though the traditional method of teaching is applied to the control group, noticeable differences are observed in the mean pre-test and post test results.

Table 6. Difference between the pre-test and posttest mean scores in mathematical fluency of the control group

Mathematical Fluency	Pre-test		Post Test		t	df	Sig. (2-tailed)
	Mean	SD	Mean	SD			
Accuracy	0.90	0.37	3.15	0.51	-20.877	39	0.000
Flexibility and Appropriate Response	0.86	0.43	2.81	0.54	-16.641	39	0.000
Efficiency	0.76	0.33	2.59	0.64	-14.470	39	0.000
Automaticity	0.88	0.58	3.40	0.57	-16.392	39	0.000
Number sense	1.70	0.64	2.34	0.78	-3.926	39	0.000

Legend: Sig (2-tailed) ≤ .05 (Significant); Sig (2-tailed) ≥ .05 (Not significant)

Table 7. Difference between the pre-test and posttest mean scores in mathematical fluency of the experimental group

Mathematical Fluency	Pre-test		Post Test		t	df	Sig. (2-tailed)
	Mean	SD	Mean	SD			
Accuracy	0.92	0.21	3.44	0.58	-25.956	39	0.000
Flexibility and Appropriate Response	0.83	0.22	3.20	0.68	-19.604	39	0.000
Efficiency	0.80	0.22	3.00	0.68	-18.672	39	0.000
Automaticity	0.93	0.41	3.59	0.61	-20.226	39	0.000
Number sense	1.81	0.73	3.03	0.64	-8.181	39	0.000

Legend: Sig (2-tailed) ≤ .05 (Significant); Sig (2-tailed) ≥ .05 (Not significant)

Significant differences were noted in the pre-test and posttest mean scores of the mathematical fluency of the students in the experimental group. This is true for all mathematical fluency, all in favor of the mean post test scores, suggesting that the discussion of the lessons with the integration of heuristic approaches was able to improve the mathematical fluency of the students.

Table 8. Difference between the pre-test mean scores in mathematical fluency of the experimental and control group

Mathematical Fluency	Experimental		Control		t	df	Sig. (2-tailed)
	Mean	SD	Mean	SD			
Accuracy	0.92	0.21	0.90	0.37	0.300	78	0.765
Flexibility and Appropriate Response	0.83	0.22	0.86	0.43	-0.394	78	0.695
Efficiency	0.80	0.22	0.76	0.33	0.644	78	0.521
Automaticity	0.93	0.41	0.88	0.58	0.487	78	0.627
Number sense	1.81	0.73	1.70	0.64	0.683	78	0.497

Legend: Sig (2-tailed) ≤ .05 (Significant); Sig (2-tailed) ≥ .05 (Not significant)

This means that the level of mathematical fluency of the students based on the pretest scores are significantly related. This implicates that the level of mathematics fluency of both the groups were relatively comparable in terms of their mathematical fluency levels. The results imply that there are no statistically significant differences between the experimental and control groups' pre-test mean scores that aligns with findings from studies such as Fyfe and Rittle-Johnson (2018), which emphasize the importance of prior knowledge and feedback in influencing mathematical fluency. Prior knowledge and feedback might have contributed to the similarity in pre-test scores between the groups.

In summary, the data from Table 8, coupled with insights from related literature and studies, indicate that the Experimental and Control Groups had comparable levels of mathematical fluency before the intervention, suggesting a level playing field for assessing the impact of the intervention on post-test scores.

The table above below that the mathematical fluency of the students in the experimental and control group have significant differences based on pre-test mean scores in terms of accuracy, flexibility and appropriate response, efficiency and number sense. This means that students in the experimental group have achieved a higher level of mathematical fluency than those under the control group.

Table 9. Difference between the posttest mean scores in mathematical fluency of the experimental and control group

Mathematical Fluency	Experimental		Control		t	df	Sig. (2-tailed)
	Mean	SD	Mean	SD			
Accuracy	3.44	0.58	3.15	0.51	2.410	78	0.018
Flexibility and Appropriate Response	3.20	0.68	2.81	0.54	2.889	78	0.005
Efficiency	3.00	0.68	2.59	0.64	2.768	78	0.007
Automaticity	3.59	0.61	3.40	0.57	1.409	78	0.163
Number sense	3.03	0.64	2.34	0.78	4.341	78	0.000

Legend: Sig (2-tailed) ≤ .05 (Significant); Sig (2-tailed) ≥ .05 (Not significant)

In terms of accuracy, the application of the heuristic approaches to the experimental group helps students to achieve more correct answers in computing for quartiles 1, 2 or 3 than the control group. This is because students were able to visualize how the quartiles were determined through diagrams, listing and making patterns. In terms of flexibility and appropriate response, the experimental group was exposed to various strategies in line with the use of heuristic approaches. Thus, helping them in getting higher scores compared to control group. For efficiency, students were able to have freedom in choosing shortcuts that respond to solve problems efficiently. The experimental group was able to get higher scores than the control group as they were able to use suitable shorter ways to attain the correct answer. All these descriptions align with the studies by Lemaire and Siegler (2014) and Fyfe et al. (2018), which makes an impact on the effectiveness of heuristic approaches in improving and developing mathematical fluency, problem-solving flexibility, and efficiency.

As for number sense, the experimental group significantly outperforms the control group. The students were asked to list key concepts and knowledge necessary for the solutions in solving quartiles. The experimental group was able to reach higher scores than the control group because students recalled and acquired those concepts and knowledge from previous mathematical learnings. This finding is in line with studies by Gijlers et al. (2018) and Engle et al. (2021), emphasizing the role of heuristic approaches in developing and transferring number sense skills, leading to more meaningful and relevant problem-solving experiences.

However, there is no significant difference in their mathematical fluency in terms of automaticity. This means that the level of mathematical fluency of the students based on the post test scores are only similar in terms of automaticity. Since automaticity measures the speed of students in answering the test, it depicts that both groups accomplished the problems with almost the same time spent. The length of time they spent in answering the test are mostly the same but the quality of work when it comes to other mathematical fluency differs. This research concludes that the effect of heuristic techniques on the automatic processing is in line with the findings of Schneider et al. (2017) and Rittle-Johnson et al. (2020) who found that although they promote automatic processing to some degree, they do not always lead to a significant improvement in automaticity when compared to other fluency aspects.

Conclusions

The study revealed that exposing heuristic approaches, demonstrated superior performance in mathematical fluency dimensions related to accuracy, flexibility, efficiency, and number sense compared to the control group. While heuristic approaches may not always lead to significant improvements in automaticity, they contribute significantly to overall mathematical fluency and problem-solving skills.

Recommendations

Teachers may be willing to accept and consider the varied ways of student's answering mathematical problems as long as they provide the right answers. Since significant differences are found in the pretest and posttest mean score in the control group, teachers may still utilize lecture-discussion method of traditional teaching in solving word problems in mathematics. More so, as some students showed low performance in problem - solving, teachers may use this to look for the gaps and plan a program and intervention to be given to students so that they will be able to improve their mathematical fluency and since significant differences are found in the pretest and posttest mean score in the experimental group, teachers may integrate heuristic approaches into the curriculum can effectively improve students' mathematical fluency and can help enhance overall student performance. Also, educators may consider adopting heuristic approaches to promote deeper learning and flexibility in mathematical problem-solving and future researchers may consider further exploring the long-term effects of heuristic teaching on various aspects of mathematical proficiency.

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