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Flexural and Split Tensile Strengths Evaluation of Asbestos Fibre Reinforced Concrete [AFRC] Based on Scheffe's (5,2) Model

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ABSTRACT

Controlling the cost of the conventional reinforcement with latest research innovations or techniques is one of the best ways of reducing the cost of building/ construction projects for economic benefit. Recent researches have shown that one of the ways of achieving this feat is through the techniques of incorporating fibres to partially or wholly replace the conventional reinforcement bar. The use of Asbestos Fibre (AF) as a replacement for conventional reinforcement is one of those new techniques that can be employed under controlled environment. This research study therefore is aimed at using Scheffe's Second Degree Model for five component mixture to optimize the Flexural Strength and Split Tensile Strength of Asbestos Fibre Reinforced Concrete (AFRC). Using Scheffe's Simplex method, the Flexural Strength and Split Tensile Strength of AFRC were determined for different mix proportions. Control experiments were also carried out and the flexural and split tensile strengths determined. The test statistics using the Student's t-test validated the results. Maximum design strengths recorded for the flexural test at 14 and 28 days were 6.67MPa and 8.18MPa respectively, while those recorded for the splitting tensile test were 4.72MPa and 3.08MPa respectively. AFRC controllable design strength values are capable of sustaining major construction projects such as construction of walkways, pavement slabs, building, airports, bridges etc, still maintaining both economic and safety advantages

Keywords: Flexural Strength, Split Tensile Strength, AFRC, Scheffe's (5,2) Optimization Model, Mixture Design

1.INTRODUCTION

Concrete, according to Oyenuga (2008) is a composite inert material comprising of a binder course (cement), mineral filter or aggregates and water. Concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building materials throughout the world (Syal and Goel, 2007). Concrete, according to Neville (1990), plays an important part in all building structures owing to its numerous advantages that ranges from low built in fire resistance, high compressive strength to low maintenance. Although concrete is one of the most widely used construction material, it has got its own drawbacks. According to Shetty (2006), plain concrete possesses a very low tensile strength, limited ductility, low shear strength and little resistance to cracking. As all stakeholders in the construction industries are focusing on sustainable technology that can be safe and economical, attempts have been made to improve the concrete properties with relatively new construction material developed through extensive research and development work. This has led to the reinforcement of the tension zone of the concrete with conventional steel bars. Due to the expensive nature of the conventional reinforcement, further researches have shown that incorporation of fibres into the concrete would act as crack arrester and would substantially improve its static as well as dynamic properties. Therefore, a type of concrete known as Fibre reinforced concrete (FRC) has been on focus. FRC is a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete as well as uniformly dispersed. Fibres are usually used in concrete to control cracking due to plastic shrinkage and to drying shrinkage. They also reduce the permeability of concrete and thus reduce bleeding of water. Some types of fibres produce greater impact, abrasion, and shatter resistance in concrete. Combining fibres with concrete can produce a range of materials which possess enhanced tensile strength, compressive strength, elasticity, toughness, and durability etc. Asbestos Fibre Reinforced Concrete (AFRC) is concrete mixture where the conventionally steel reinforcement in concrete production is partially or wholly replaced with Asbestos Fibre (AF). AF is obtained by extraction of asbestos-containing rock which is crushed and milled to produce a thread like fibrous material known as asbestos. Before now, Asbestos was considered as good fibre reinforcement since it was inexpensive, readily available, and easily blended into the mix. Figure 1 shows a typical diagram of an asbestos concrete. Special properties of AFRC under investigation in this present work are the flexural strength and the split tensile strength. Flexural strength is the ability of the material to withstand bending forces applied perpendicular to its longitudinal axis. It is defined as the maximum bending stress that can be applied to the material before it yields. Furthermore, splitting tensile strength test on concrete cylinder is a method to determine the tensile strength of concrete. It is generally carried out to obtain the tensile strength of concrete, and the stress field in the tests is actually a biaxial stress field with compressive stress three times greater than the tensile stress. The split tensile strength test is an indirect method of testing tensile strength of concrete and is generally greater than direct tensile strength and lower than flexural strength (modulus of rupture).

In order to optimize the present AFRC concrete mixture, it is good for us to explain the concept of optimization. The objective of mix design, according to Shacklock (1974), is to determine the most appropriate proportions in which to use the constituent materials to meet the needs of construction work. Optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as strength, workability and durability etc. On the account of the widely varying properties of the constituent materials, the conditions that prevail at the site of work, the exposure condition, and the conditions that are demanded for a particular work for which the mix is designed, the design of concrete mix according to (Shetty, 2006) has not being a simple task. By definition, concrete mix design according to Jackson and Dhir (1996) remains the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. From the above definition, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. In the context of the above guidelines, the empirical mix design methods and procedures proposed by Hughes (1971), ACI- 211(1994) and DOE (1988) appears to be a little bit complex and time consuming as they involve a lot of trial mixes and complex statistical calculations before the desired strength of the concrete can be reached. Thus, optimization of the concrete mixture design proves to be the fastest method, best option, most convenient and the most efficient way of selecting concrete mix ratios /proportions for better efficiency and better performance of concrete when compared with usual empirical methods .Typical examples of optimization model is Scheffe's Model. It could be in the form of Scheffe's Second Degree Model or Scheffe's Third Degree Model. In this present study, Scheffe's Second Degree Model for five components mixtures (namely Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Asbestos Fibre are presented.

In this recent work, the use of Scheffe's Second Degree Polynomial Model in the optimization of the Flexural Strength and Split Tensile Strength of AFRC is examined. Although, there have been little works done on the general asbestos and optimization applications, none has been able to address the subject matter in full. For example, Chaudhary and others (2017) performed experimental analysis on asbestos fibre reinforcement concrete composite. Samir and others (2014) investigated the use of asbestos –free fibre –cement waste as a partial substitute of Portland cement in mortar. On optimization, recent works have shown that many researchers have used Scheffe's method to carry out one form of optimization work or the other. For example, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were based on the use of Scheffe' model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Mbadike and Osadebe (2013) applied Scheffe's (4,2) model to optimize the compressive strength of Laterite Concrete. Egamana and Sule (2017) carried out an optimization work on the compressive strength of periwinkle shell aggregate concrete Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3) where comparison was made between second degree model and third degree model. Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. Again, Nwachukwu and others (2022g) applied Scheffe's Third Degree Model to optimize the compressive strength of SFRC. In what is termed as introduction of six component mixture and its Scheffe's formulation ,Nwachukwu and others (2022h) developed and use Scheffe's (6,2) Model to optimize the compressive strength of Hybrid- Polypropylene – Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2022 i) applied Scheffe's (6,2) model to optimize the Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2022j) applied Scheffe's (6,2) model in the Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC) .Nwachukwu and others (2022k) applied the use of Scheffe's Second Degree Polynomial Model to optimize the compressive strength of Mussel Shell Fibre Reinforced Concrete (MSFRC). Nwachukwu and others (2022 l) carried out an optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model. Nwachukwu and others (2023a) applied Scheffe's Third Degree Regression Model to optimize the compressive strength of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023b) applied Scheffe's (6,3) Model in the Optimization Of Compressive Strength of Concete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2023c) applied Scheffe's (6,2) model to optimize the Flexural Strength And Split Tensile Strength Of Hybrid Polypropylene Steel Fibre Reinforced Concrete (HPSFRC). Finally, Nwachukwu and others (2023d) made use of Scheffe's Second Degree Model In The Optimization Of Compressive Strength Of Asbestos Fibre Reinforced Concrete (AFRC). Nwachukwu and others (2023e) used optimization techniques in the Flexural Strength And Split Tensile Strength determination of Hybrid Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023f) applied Scheffe's Optimization model in the evaluation of Flexural Strength And Split Tensile Strength Of Plastic Fibre Reinforced Concrete (PLFRC). Nwachukwu and Opara (2023) in their paper presented at the Conference Proceedings of the Nigeria Society of Engineers, demonstrated the use of Snail Shells Ash (SSA) in the partial replacement of cement using Scheffe's (5,2) optimization model. Nwachukwu and others (2024a) applied the use of Scheffe's (6,2) model to evaluate the optimum flexural and split tensile strengths of Periwinkle Shells Ash (PSA)-

Mussel Shells Ash (MSA)- Cement Concrete (PMCC). Nwachukwu and others (2024b) applied the use of Scheffe's (6,2) model to evaluate the optimum compressive strength of Periwinkle Shells Ash (PSA)- Snail Shells Ash (SSA)- Cement Concrete (PSCC). Nwachukwu and others (2024c) applied Scheffe's (5,2) model to evaluate the compressive strength of Plastic Fibre Reinforced Concrete [PLFRC]**.** Nwachukwu and others (2024d) applied the use of Scheffe's Third Degree Model to optimize the compressive strength of HPNFRC. Nwachukwu and others (2024e) applied the use of Scheffe's Third Degree Regression Model to optimize the compressive strength of MSFRC. Nwachukwu and others (2024f) applied the use of Scheffe's Second Degree Model to optimize the flexural strength and split tensile strength of NFRC. Again, Nwachukwu and others (2024g) applied the use of Scheffe's Second Degree Model to optimize the flexural strength and split tensile strength of PFRC. Finally, Nwachukwu and others (2024h) applied the use of Scheffe's Second Degree Model to optimize the flexural strength and split tensile strength of PFRC. From the works reviewed so far, there is enough evidence that the subject matter has not been fully addressed as it can be envisaged that no work has been done on the use of Scheffe's Second Degree Model to optimize the flexural strength and split tensile strength of AFRC. Thus, there is urgent need for this present research work.

Fig 1: A Typical Example Of Asbestos Concrete

2. METHODOLOGY

2.1 MATERIALS FOR AFRC- FSTS MIXTURES

The constituent materials for laboratory examination are Water/Cement ratio, Cement, Fine and Coarse Aggregates and Asbestos Fibre. Potable water is obtained from clean water source and was applied in accordance with ASTM C1602/C1602M-22 (2022).. The cement is Dangote cement, a brand of Ordinary Portland Cement obtained from local distributors, which conforms to British Standard Institution BS 12 (1978). The fine aggregate, with sizes that ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite (as a coarse aggregate) of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. Both fine and coarse aggregates were procured and prepared in accordance with ASTM C33/C33M-18 (2018). The Asbestos Fibre used in this work is used under controlled environment as raw asbestos, generally is a very hazardous material. Generally, Asbestos is obtained by extraction of asbestos-containing rock which is crushed and milled to produce a thread like fibrous material known as asbestos. The asbestos contains thousands of fibres which can be further divided into microscopic fibrils that give rise to Asbestos Fibres (AF).

2.2. THEORITICAL BACKGROUND ON AFRC SCHEFFE'S (5, 2) OPTIMIZATION MODEL

In general, a simplex lattice in the Scheffe's optimization model is a structural representation of lines joining the atoms of a mixture, where the atoms are the constituent components of the mixture. For instance, when considering the AFRC mixture, the relevant constituent elements are the water, cement, fine aggregate, coarse aggregate and the asbestos fibre. With respect to Scheffe's theory, mixture components are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \ldots + X_q = 1 \; ; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad \textbf{(1)}$$

where $X_i \geq 0$ and $i = 1, 2, 3 \ldots q$, and q = the number of mixtures

2.2.1. AFRC SCHEFFE'S (5,2) SIMPLEX LATTICE DESIGN

The Scheffe's (q, m) simplex lattice design is characterized by the symmetric arrangements of points within the experimental region and a well-chosen regression equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \ldots, 1; \quad i = 1, 2, \ldots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which is 2 in this present study .For example a (3, 2) lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^{4}C_2 = 6$ points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are: $(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(0, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right)$. To evaluate the number of coefficients/ or terms/ or design points required for a given lattice , the following general formula is applied:

$$k = \frac{(q+m-1)!}{(q-1)!\,.\,m!} \quad \text{Or} \quad {}^{q+m-1}C_m \qquad \textbf{2(a-b)}$$

Where k = number of coefficients/ terms / point, q = number of components/mixtures = 5 in this present study m = number of deqree of polynomial = 2 in this present work. Using either of Eqn. (2), $k_{(5,2)}$ = 15 Consequently, the possible design points for Scheffe's (5,2) lattice can be as follows:

A_1 (1,0,0,0,0,); A_2 (0,1,0,0,0,); A_3 (0,0,1,0,0,); A_4 (0,0,0,1,0,), A_5 (0,0,0,0,1,);A_{12} (0.5, 0.5, 0, 0, 0); A_{13} (0.5, 0, 0.5, 0, 0); A_{14} (0.5, 0, 0, 0.5, 0); A_{15} (0.5, 0, 0, 0, 0.5); A_{23} (0, 0.5, 0.5, 0,0); A_{24} (0, 0.5, 0, 0.5, 0); A_{25} (0, 0.5, 0, 0, 0.5); A_{34} (0, 0, 0.5, 0.5, 0); A_{35} (0, 0, 0.5, 0, 0.5) and A_{45} (0, 0, 0, 0.5, 0.5) **(3)**

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable X_1, X_2, X_3, X_4 … X_q is given in form of: $P = b_0 + \sum bi\, xi + \sum bij xj + \sum bi\, jxjxk + + \sum bi^{j}{}_2 + \ldots i_n x i_2 x i_n$ **(4)**

where ($1 \leq i \leq q$, $1 \leq i \leq j \leq k \leq q$, $1 \leq i_1 \leq i_2 \leq \ldots \leq i_n \leq q$ respectively) , b = constant coefficients and P is the response (the response is a polynomial function of pseudo component of the mix) which represents the property under study, which ,in this case is the Flexural Strength (P^F) or Split Tensile Strength (P^S) as the case may be.

This research work is based on the (5, 2) simplex. The actual form of Eqn. (4) has already been developed by Nwachukwu and others (2017) and will be applied subsequently.

2.2.2. PSEUDO AND ACTUAL COMPONENTS.

In Scheffe's mixture design, the relationship between the pseudo components and the actual components is given as:

$$Z = A * X \qquad \textbf{(5)}$$

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging the equation, we have : $X = A^{-1} * Z$ **(6)**

2.2.3. POLYNOMIAL EQUATION FOR AFRC SCHEFFE'S (5, 2) SIMPLEX LATTICE

The polynomial equation by Scheffe (1958), describing the response is given in Eqn.(4). But, for Scheffe's (5,2) simplex lattice, the polynomial equation for five component mixtures has been derived from Eqn.(4) by Nwachukwu and others (2017) and the simplified version is given as follows:

$$P = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 +$$
$$\beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{45}X_4X_5 \qquad \textbf{(7)}$$

2.2.4 .SIMPLIFIED COEFFICIENTS OF THE AFRC SCHEFFE'S (5, 2) POLYNOMIAL EQUATION

From the work of Nwachukwu and others (2022h), the simplified equations for the coefficients of the Scheffe's (5, 2) polynomial are expressed as follows. :

$$\beta_1 = P_1;\ \beta_2 = P_2;\ \beta_3 = P_3;\ \beta_4 = P_4;\ \beta_5 = P_5;\ \beta_{12} = 4P_{12} - 2P_1 - 2P_2;\ \beta_{13} = 4P_{13} - 2P_1 - 2P_3; \qquad \textbf{8(a-g)}$$

$$\beta_{14} = 4P_{14} - 2P_1 - 2P_4;\ \beta_{15} = 4P_{15} - 2P_1 - 2P_5;\ \beta_{23} = 4P_{23} - 2P_2 - 2P_3;\ \beta_{24} = 4P_{24} - 2P_2 - 2P_4; \qquad \textbf{9(a-d)}$$

$$\beta_{25} = 4P_{25} - 2P_2 - 2P_5;\ \beta_{34} = 4P_{34} - 2P_3 - 2P_4;\ \beta_{35} = 4P_{35} - 2P_3 - 2P_5;\ \beta_{45} = 4P_{45} - 2P_4 - 2P_5 \qquad \textbf{10(a-d)}$$

Where P_i = Response Function (Flexural Strength or Split Tensile Strength) for the pure component, *i*

2.2.5. SCHEFFE'S (5, 2) MIXTURE DESIGN MODEL FOR AFRC-FSTS

If we substitute Eqns. (8)-(10) into Eqn. (7), we obtain the mixture design model for the AFRC mixture based on Scheffe's (5, 2) lattice for the Flexural Strength [FS] and Split Tensile Strength [STS].

2.2.6. ACTUAL AND PSEUDO MIX RATIOS FOR THE AFRC SCHEFFE'S (5,2) DESIGN LATTICE AT INITIAL EXPERIMENTAL TEST POINT AND EXPERIMENTAL CONTROL TEST POINT

A. AT THE INITIAL EXPERIMENTAL TEST POINTS [IETP]

The requirement of simplex lattice design from Eqn.(1) makes it impossible to use the conventional mix ratios such as 1:2:4, 1:1.3:6, etc., at a given water/cement ratio for the actual mix ratio. This necessitates the transformation of the actual components (ingredients) proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix proportions were chosen for the five points/vertices.**:**

A_1 (0.67:1: 1.7: 2:0.5); A_2 (0.56:1:1.6:1.8:0.8);A_3 (0.5:1:1.2:1.7:1);A_4 (0.7:1:1:1.8:1.2)andA_5 (0.75:1:1.3:1.2:1.5),**(11)** which represent water/cement ratio, cement, fine aggregate, coarse aggregate and asbestos fibre. For the pseudo mix ratio, we have the following corresponding mix ratios at the vertices:

$A_1(1:0:0:0:0)$, $A_2(0:1:0:0:0)$, $A_3(0:0:1:0:0)$, $A_4(0:0:0:1:0)$, and $A_5(0:0:0:0:1)$ **(12)**

For the transformation of the actual component, Z to pseudo component, X, and vice versa , Eqns.(5)and (6) are used.. Substituting the mix ratios from point A_1 into Eqn. (5) gives:

$$\begin{Bmatrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{pmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \textbf{(13)}$$

Transforming the R.H matrix and solving , we obtain:

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} \qquad \textbf{(14)}$$

Thus

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} = \begin{pmatrix} 3.99 & 10.37 & -2.14 & -3.05 & -4.62 \\ -4.88 & -21.46 & 5.40 & 5.95 & 7.31 \\ -1.78 & 17.83 & -3.49 & -4.20 & -4.62 \\ 1.04 & -9.24 & 0.37 & 3.28 & 2.69 \\ 1.63 & 3.49 & -0.13 & -1.98 & -0.77 \end{pmatrix} \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \; . \\ Z_4 \\ Z_5 \end{Bmatrix} \qquad \textbf{(15)}$$

Considering the mix ratios at the midpoints of Eqn.(3), we have, after substituting these pseudo mix ratios in turn into Eqn. (15) for point A_{12}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.62 \\ 1 \\ 1.65 \\ 1.90 \\ 0.65 \end{Bmatrix} \qquad \textbf{(16)}$$

Solving, $Z_1 = 0.62$, $Z_2 = 1$, $Z_3 = 1.65$, $Z_4 = 1.9$, $Z_5 = 0.65$ and the rest are shown in Table 1. To generate the polynomial coefficients, fifteen experimental tests (each for Flexural Strength and Split Tensile Strength) will be carried out and the corresponding mix ratio is as depicted in Table 1.

Table 1: Pseudo (X) and Actual (Z) Mix Ratio For AFRC Based On Scheffe's (5,2) Simplex Lattice For IETP (For Flexural Strength And Split Tensile Strength).

S/N	IETP	PSEUDO COMPONENT					RESPONSE SYMBOL	ACTUAL COMPONENT				
		X_1	X_2	X_3	X_4	X_5		Z_1	Z_2	Z_3	Z_4	Z_5
1	E_I	1	0	0	0	0	P_1	0.67	1	1.70	2.00	0.50
2	E_2	0	1	0	0	0	P_2	0.56	1	1.60	1.80	0.80
3	E_3	0	0	1	0	0	P_3	0.50	1	1.20	1.70	1.00
4	E_4	0	0	0	1	0	P_4	0.70	1	1-00	1.80	1.20

5	E_5	0	0	0	0	1	P_5	0.75	1	1.30	1.20	1.50
6	E_{12}	0.50	0.50	0	0	0	P_{12}	0.62	1	1.65	1.90	0.65
7	E_{13}	0.50	0	0.50	0	0	P_{13}	0.59	1	1.45	1.85	0.75
8	E_{14}	0.50	0	0	0.50	0	P_{14}	0.69	1	1.35	1.90	0.85
9	E_{15}	0.50	0	0	0	0.50	P_{15}	0.71	1	1.50	1.60	1.00
10	E_{23}	0	0.50	0.50	0	0	P_{23}	0.53	1	1.40	1.75	0.90
11	E_{24}	0	0.50	0	0.50	0	P_{24}	0.63	1	1.30	1.80	1.00
12	E_{25}	0	0.50	0	0	0.50	P_{25}	0.66	1	1.45	1.50	1.15
13	E_{34}	0	0	0.50	0.50	0	P_{34}	0.60	1	1.10	1.75	1.10
14	E_{35}	0	0	0.50	0	0.5	P_{35}	0.63	1	1.25	1.45	1.25
15	E_{45}	0	0	0	0.5	0.5	P_{45}	0.73	1	1.15	1.50	1.50

B. AT THE EXPERIMENTAL (.CONTROL) TEST POINT [ECTP]

For the purpose of this research, fifteen different controls test (each for Flexural Strength and Split Tensile Strength) were predicted which according to Scheffes, their summation should not be more than one. Thus, the following pseudo mix proportions are applicable at the control points:

C_1 (0.25, 0.25, 0.25, 0.25, 0), C_2 (0.25, 0.25, 0.25, 0, 0.25), C_3 (0.25, 0.25, 0, 0.25, 0.25), C_4 (0.25, 0, 0.25, 0.25, 0.25), C_5 (0, 0.25, 0.25, 0.25, 0.25), C_{12} (0.20, 0.20, 0.20, 0.20, 0.20), C_{13} (0.30, 0.30, 0.30, 0.10, 0), C_{14} (0.30, 0.30, 0.30, 0, 0.10), C_{15} (0.30, 0.30, 0, 0.30, 0.1), C_{23} (0.30, 0, 0.30, 0.30, 0.1), C_{24} (0, 0.30, 0.30, 0.30, 0.10), C_{25} (0.10, 0.30, 0.30, 0.30, 0), C_{34} (0.30, 0.10, 0.30, 0.30, 0), C_{35} (0.30, 0.30, 0.10, 0.30, 0), C_{45} (0.10, 0.20, 0.30, 0.40, 0), **(17)**

Substituting into Eqn.(16) , we obtain the values of the actual mixes as follows, at Control 1 ,C_1

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.61 \\ 1 \\ 1.38 \\ 1.8 \\ 0.5 \end{Bmatrix} \qquad \textbf{(18)}$$

The rest are shown in Table 2

Table 2: Actual (Z_i) and Pseudo (X_i) Component of AFRC Scheffe's (5, 2) Simplex Lattice At ECTP (For Flexural Strength And Split Tensile Strength).

S/N	**RESONSE SYMBOL**	**PSEUDO COMPONENTS**					**EC TP**	**ACTUAL COMPONENTS**				
		Wat(X_1)	**Cem (X_2)**	**FA (X_3)**	**CA (X_4)**	**AF (X_5)**		**Water(Z_1)**	**Cem(Z_2)**	**FA (Z_3)**	**CA (Z_4)**	**AF (Z_5)**
1	P_1	0.25	0.25	0.25	0.25	0.00	C_1	0.61	1	1.38	1.83	0.50
2	P_2	0.25	0.25	0.25	0.00	0.25	C_2	0.62	1	1.45	1.68	0.80

3	P_3	0.25	0.25	0.00	0.25	0.25	C_3	0.67	1	1.40	1.70	1.00
4	P_4	0.25	0.00	0.25	0.25	0.25	C_4	0.66	1	1.30	1.68	1.20
5	P_5	0.00	0.25	0.25	0.25	0.25	C_5	0.63	1	1.28	1.63	1.50
6	P_{12}	0.20	0.20	0.20	0.20	0.20	C_{12}	0.64	1	1.36	1.70	0.65
7	P_{13}	0.30	0.30	0.30	0.10	0.00	C_{13}	0.59	1	1.45	1.83	0.75
8	P_{14}	0.30	0.30	0.30	0.00	0.10	C_{14}	0.59	1	1.48	1.77	0.85
9	P_{15}	0.30	0.30	0.00	0.30	0.10	C_{15}	0.65	1	1.42	1.80	1.00
10	P_{23}	0.30	0.00	0.30	0.30	0.10	C_{23}	0.64	1	1.30	1.77	0.90
11	P_{24}	0.00	0.30	0.30	0.30	0.10	C_{24}	0.60	1	1.27	1.71	1.00
12	P_{25}	0.10	0.30	0.30	0.30	0.00	C_{25}	0.60	1	1.31	1.79	1.15
13	P_{34}	0.30	0.10	0.30	0.30	0.00	C_{34}	0.62	1	1.33	1.83	1.10
14	P_{35}	0.30	0.30	0.10	0.30	0.00	C_{35}	0.63	1	1.41	1.85	1.25
15	P_{45}	0.10	0.20	0.30	0.40	0.00	C_{45}	0.61	1	1.25	1.79	0.50

2.2.2.7. MEASUREMENT OF QUANTITIES OF AFRC MATERIALS

The actual component as transformed from Eqn. (14) , Table (1) and (2) were used to measure out the quantities of Water/Cement Ratio (Z_1), Cement (Z_2), Fine Aggregate (Z_3), Coarse Aggregate (Z_4), and Asbestos Fibre (Z_5) in their respective ratios using a weighing balance of 50kg capacity for the eventual Concrete Beam Cube and Concrete Cylindrical specimen strengths at the laboratory.

Mathematically, Measured Quantity, M^Q of AFRC Mixture is given by Eqn.(19)

$$M^Q = \frac{X}{T} * Y \tag{19}$$

Where, X = Individual mix ratio at each test point = 0.67 for Z_1 at E_1 in Table 1, for example.

T = Sum of mix ratios at each test point = 5.87 at E_1 in Table 1, for example

And Y = Average weight of Concrete cube/beam/cylinder

For the Flexural Strength concrete beam mould of 15cm*15cm*60cm, Average Y from experience = 30kg

For the Split Tensile Strength Concrete cylinder mould of 15cm*30cm, Average Y from experience =12.5kg

Samples of measured quantities can be seen from the works of Nwachukwu and others 2024 (a and b).

2.3. METHOD

2.3.1. METHODS FOR AFRC FLEXURAL STRENGTH TEST

A. AFRC SPECIMEN PREPARATION / BATCHING/ CURING FOR FLEXURAL STRENGTH TEST

In this work, the standard size of specimen (mould) for the Flexural Strength measures 15cm*15cm*60cm. The mould is of steel metal with sufficient thickness to prevent spreading or warping. The mould is constructed with the longer dimension horizontal and in such a manner as to facilitate the removal of the moulded specimen without damage. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 30 mix ratios were to be used to produce 60 prototype concrete cubes. Fifteen (15) out of the 30 mix ratios were as control mix ratios to produce 30 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (10). Twenty-four (24) hours after moulding, curing commenced. Test specimens are stored in water at a temperature of 24^0 to 30^0 for 48 hours before testing. They are tested immediately on removal from the water whilst they are still in a wet condition. After 14 days and 28 days of curing respectively, the specimens were taken out of the curing tank for flexural strength determination.

B. AFRC FLEXURAL STRENGTH TEST PROCEDURE/CALCULATION

Flexural strength testing was done in accordance with BS 1881 – part 118 (1983) - Method of determination of Flexural Strength, ASTM C78/C78M-22 (2022) and ACI (1989) guideline. In this present study, two samples were crushed for each mix ratio. In each case, the Flexural Strength of each specimen/sample which is expressed as the Modulus of Rupture (MOR) was then calculated to the nearest 0.05 MPa using Eqn.(20)

$$\text{MOR} = \frac{PL}{bd^2} \qquad \textbf{(20)}$$

where b = measured width in cm of the specimen, d = measured depth in cm of the specimen at the point of failure, where L = Length in cm of the span on which the specimen was supported and P = maximum load in kg applied to the specimen.

2.3.2. METHODS FOR AFRC SPLIT TENSILE STRENGTH TEST

A. AFRC SPECIMEN PREPARATION / BATCHING/ CURING FOR SPLIT TENSILE STRENGTH TEST

The specimen for the Split Tensile Strength is Concrete Cylindrical specimen measuring diameter 150 mm and length 300 mm. They were cast with plastic fibres and the specimen was loaded for ultimate compressive load under Universal Testing Machine (UTM) for each mix. A total number of 30 mix ratios were to be used to produce 60 prototype concrete cylinders. Fifteen (15) out of the 30 mix ratios were as control mix ratios to produce 30 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (10).. After 28 days of curing the specimens were taken out of the curing tank for the Split Tensile Strength determination.

B. AFRC SPLIT TENSILE STRENGTH TEST PROCEDURE/CALCULATION

The cylindrical split tensile test was done using the universal testing machine in accordance with BS

EN 12390-6:2009 and ASTM C 496/ C 496 M-11 (2011). Two samples were crushed for each mix ratio and each case, the Split Tensile Strength of each specimen/sample was then calculated using Eqn. (21)

$$F_t = \frac{2P}{\pi D L} \qquad \textbf{(21)}$$

Where, F_t = Split Tensile Strength, MPa , P = maximum applied load (that is Load at failure, N) ; D = diameter of the cylindrical specimen (Dia. Of cylinder, mm); and L = Length of the specimen (Length of cylinder, mm),

3. RESULTS PRESENTATION AND DISCUSSION

3.1 AFRC RESPONSES (FLEXURAL STRENGTH) FOR THE IETP

The results of the Flexural Strength (responses) for the IETP based on Eqn. (20) are shown in Table 3

Table 3: AFRC Flexural Strength (Response) For The IETP Based on Eqn.(20)

S/N	IETP	EXPERIMENTAL NO.	RESPONSE SYMBOL	RESPONSE P_i, MPa		$\sum P_i$		AVERAGE RESPONSE P, MPa	
				14th day Results	28th day Results	14th day Results	28th day Results	14th day Results	28th day Results
	E_I	AFRC/FS/E_1 A		4.76	5.32	9.65	10.69	4.83	5.35
1		AFRC/FS/ E_1 B	P_1	4.89	5.37				
2	E_2	AFRC/FS/ E_2 A		5.32	5.86	10.68	11.89	5.34	5.95
		AFRC/FS/ E_2 B	P_2	5.45	6.03				
	E_3	AFRC/FS/ E_3 A		4.76	5.84	9.65	11.69	4.83	5.85
3		AFRC/FS/ E_3 B	P_3	4.91	5.85				
	E_4	AFRC/ FS/E_4 A		5.43	5.76	10.79	10.12	5.40	5.06
4		AFRC/FS/ E_4 B	P_4	5.36	5.78				
	E_5	AFRC/FS/ E_5 A		6.69	8.20	13.34	16.36	6.67	8.18
5		AFRC/FS/ E_5 B	P_5	6.65	8.16				
	E_{12}	AFRC/FS/ E_{12} A		6.21	7.65	12.53	15.28	6.27	7.64
6		AFRC/FS/ E_{12} B	P_{12}	6.32	7.63				

7	E_{13}	AFRC/FS/ E_{13} A AFRC/FS/ E_{13} B	P_{13}	5.88 5,68	7.32 7.34	11.56	14.66	5.78	7.33
8	E_{I4}	AFRC/FS/ E_{14} A AFRC/FS/ E_{14} B	P_{14}	4.98 4.84	6.98 6.94	9.82	13.92	4.91	6.96
9	E_{I5}	AFRC/FS/ E_{15} A AFRC/FS/ E_{15} B	P_{15}	4.79 4.80	5.72 5.86	9.59	11.58	4.80	5.79
10	E_{23}	AFRC/ FS/E_{23} A AFRC/FS/ E_{23} B	P_{23}	5.54 5.85	5.28 5.24	11.38	10.52	5.69	5.26
11	E_{24}	AFRC/FS/ E_{24} A AFRC/FS/ E_{24} B	P_{24}	4.77 4.73	5.75 5.45	9.50	11.20	4.75	5.60
12	E_{25}	AFRC/FS/ E_{25} A AFRC/FS/ E_{25} B	P_{25}	5.75 5.79	6.85 6.83	11.54	13.68	5.77	6.84
13	E_{34}	AFRC/FS/ E_{34} A AFRC/FS/ E_{34} B	P_{34}	4,98 5.00	5.45 5.43	9.98	10.88	4.99	5.44
14	E_{35}	AFRC/ FS/E_{35} A AFRC/FS/ E_{35} B	P_{35}	4.97 4.98	7.45 7.46	9.95	14.91	4.98	7.46
15	E_{45}	AFRC/FS/ E_{45} A AFRC/FS/ E_{45} B	P_{45}	5.34 5.45	6.43 6.54	10.79	12.37	5.40	6.49

3.2 AFRC RESPONSES (SPLIT TENSILE STRENGTH) FOR THE IETP

The results of the Split Tensile Strength (response) test based on Eqn. (21) are shown in Table 4

Table 4: AFRC Split Tensile Strength (Response) For The IETP Based on Eqn.(21)

S/N	IE TP	EXPERI MENTAL NO	RESPONSE SYMBOL	RESPONSE P_i, MPa		$\sum P_i$		AVERAGE RESPONSE P, MPa	
				14th day Results	28th day Results	14th day Results	28th day Results	14th day Results	28th day Results
1	E_I	AFRC/STS/E_1 A AFRC/STS/ E_1 B	P_1	3.02 3.05	4.34 4.42	6.07	8.74	3.04	4.37
2	E_2	AFRC/STS/ E_2 A AFRC/STS/ E_2 B	P_2	2.98 2.99	4.55 4.53	5.97	9.08	2.99	4.54
3	E_3	AFRC/STS/ E_3 A AFRC/STS/ E_3 B	P_3	2.35 2.43	3.87 3.89	4.78	7.74	2.39	3.87
4	E_4	AFRC/STS/E_4 A AFRC/STS/E_4 B	P_4	2.87 2.84	3.48 3.45	5.71	6.93	2.86	3.47

5	E_5	AFRC/STS/ E_5 A AFRC/STS/ E_5 B	P_5	3.06 3.10	4.74 4.70	6.16	9.44	3.08	4.72
6	E_{12}	AFRC/STS/ E_{12} A AFRC/STS/ E_{12} B	P_{12}	3.00 3.01	3.33 3.38	6.01	6.71	3.01	3.36
7	E_{13}	AFRC/STS/ E_{13} A AFRC/STS/ E_{13} B	P_{13}	3.01 2.98	4.38 4.46	5.99	8.84	3.00	4.42
8	E_{I4}	AFRC/STS/E_{14} A AFRC/STS/ E_{14} B	P_{14}	2.96 2.94	4.08 4.06	5.90	8.14	2.95	4.07
9	E_{I5}	AFRC/STS/ E_{15} A AFRC/STS/ E_{15} B	P_{15}	2.85 2.68	4.56 4.64	5.53	9.20	2.77	4.60
10	E_{23}	AFRC/ STS/E_{23} A AFRC/STS/ E_{23} B	P_{23}	2.87 2.89	4.44 4.39	5.76	8.83	2.88	4.42
11	E_{24}	AFRC/STS/ E_{24} A AFRC/STS/ E_{24} B	P_{24}	2.28 2.24	3.08 3.04	4.52	6.12	2.26	3.06
12	E_{25}	AFRC/STS/ E_{25} A AFRC/STS/ E_{25} B	P_{25}	2.56 2.45	4.42 4.47	5.01	8.89	2.51	4.45
13	E_{34}	AFRC/STS/ E_{34} A AFRC/STS/ E_{34} B	P_{34}	2.78 2.77	4.65 4.70	5.55	9.35	2.78	4.68
14	E_{35}	AFRC/ STS/E_{35} A AFRC/STS/ E_{35} B	P_{35}	2.98 2.79	4.43 4.38	5.77	8.81	2.89	4.41
15	E_{45}	AFRC/STS/ E_{45} A AFRC/STS/ E_{45} B	P_{45}	2.95 2.89	3.54 3.60	5.84	7.14	2.92	3.57

3.3. AFRC RESPONSES (FLEXURAL STRENGHT) FOR THE ECTP

The response (Flexural strength) from ECTP is shown in Table 5.

Table 5: AFRC Response (Flexural strength) For The ECTP.

S/N	**ECTP**	**EXPERIMENTAL NO.**	**RESPONSE MPa**		**Z_1**	**Z_2**	**Z_3**	**Z_4**	**Z_5**	**AVERAGE RESPONSE, MPa**		
			14th day Results	**28th day Results**						**14th day Results**	**28th day Results**	
1	C_1	AFRC/FS/C_1 A AFRC/FS/ C_1 B	4.65 4.64	5.27 5.28	0.61	1	1.38	1.83	0.5	4.65	5.28	10.42
2	C_2	AFRC/FS/ C_2 A AFRC/FS/ C_2 B	5.09 5.43	5.54 6.00	0.62	1	1.45	1.68	0.8	5.26	5.77	9.04
3	C_3	AFRC/FS/ C_3A AFRC/FS/ C_3 B	4.54 4.78	5.34 5.38	0.67	1	1.4	1.7	1	4.66	5.36	7.33
4	C_4	AFRC/ FS/C_4 A AFRC/FS/ C_4 B	5.41 5.42	5.70 5.70	0.66	1	1.3	1.68	1.2	5.42	5.70	7.89
5	C_5	AFRC/FS/ C_5 A AFRC/FS/ C_5 B	6.54 6.56	8.12 8.13	0.63	1	1.28	1.63	1.5	6.55	8.13	12.81
6	C_{12}	AFRC/FS/C_{12} A AFRC/FS/C_{12} B	6.09 6.08	7.34 7.45	0.64	1	1.36	1.7	0.65	6.09	7.40	10.77
7	C_{13}	AFRC/FS/C_{13} A AFRC/FS/C_{13} B	5.80 5.87	7.32 7.31	0.59	1	1.45	1.83	0.75	5.84	7.32	7.6
8	C_{14}	AFRC/FS/C_{14} A AFRC/FS/C_{14} B	4.54 4.56	6.43 6.48	0.59	1	1.48	1.77	0.85	4.55	6.46	8.1
9	C_{15}	AFRC/FS/C_{15} A AFRC/FS/C_{15} B	4.34 4.54	5.32 5.43	0.65	1	1.42	1.8	1	4.44	5.38	7.05
10	C_{23}	AFRC/FS/C_{23} A AFRC/FS/C_{23} B	5.65 5.76	5.37 5.38	0.64	1	1.3	1.77	0.9	5.71	5.38	7.25
11	C_{24}	AFRC/FS/C_{24} A AFRC/FS/C_{24} B	4.43 4.49	5.75 5.76	0.6	1	1.27	1.71	1	4.46	5.76	8.04

12	C_{25}	AFRC/FS/C_{25} A AFRC/FS/C_{25} B	5.86 5.97	6.45 6.56	0.6	1	1.31	1.79	1.15	5.92	6.51	7.96
13	C_{34}	AFRC/FS/C_{34} A AFRC/FS/C_{34} B	4.43 4.45	5.32 5.37	0.62	1	1.33	1.83	1.1	4.44	5.35	8.14
14	C_{35}	AFRC/FS/C_{35} A AFRC/FS/C_{35} B	4.87 4.84	7.23 7.21	0.63	1	1.41	1.85	1.25	4.86	7.22	10.54
15	C_{45}	AFRC/FS/C_{45} A AFRC/FS/C_{45} B	5.48 5.56	6.34 6.42	0.61	1	1.25	1.79	1.35	5.52	6.38	11.02

3.4. AFRC RESPONSES (SPLIT TENSILE STRENGHT) FOR THE ECTP

The response (Split Tensile Strength) from the ECTP is shown in Table 6.

Table 6: AFRC Response (Split Tensile Strength) For The ECTP

S/N	**ECTP**	**EXPERIMENTAL NO.**	**RESPONSE MPa**		**Z_1**	**Z_2**	**Z_3**	**Z_4**	**Z_5**	**AVERAGE RESPONSE MPa**		
			14th day Results	**28th day Results**						**14th day Results**	**28th day Results**	
1	C_1	AFRC/STS/C_1 A AFRC/STS/ C_1 B	3.00 3.01	4.30 4.38	0.61	1	1.38	1.83	0.5	3.01	4.34	10.42
2	C_2	AFRC/STS/ C_2 A AFRC/STS/ C_2 B	2.91 2.92	4.45 4.47	0.62	1	1.45	1.68	0.8	2.92	4.46	9.04
3	C_3	AFRC/STS/ C_3 A AFRC/STS/ C_3 B	2.34 2.37	3.77 3.86	0.67	1	1.4	1.7	1	2.36	3.82	7.33
4	C_4	AFRC/STS/C_4 A AFRC/STS/C_4 B	2.78 2.76	3.40 3.41	0.66	1	1.3	1.68	1.2	2.77	3.41	7.89
5	C_5	AFRC/STS/ C_5 A AFRC/STS/ C_5 B	3.02 3.08	4.72 4.71	0.63	1	1.28	1.63	1.5	3.05	4.72	12.81
6	C_{12}	AFRC/STS/ C_{12} A AFRC/STS/ C_{12} B	3.01 3.03	3.27 3.28	0.64	1	1.36	1.7	0.65	3.02	3.28	10.77

7	C_{13}	AFRC/STS/ C_{13} A	3.03	4.30								7.6
		AFRC/STS/ C_{13} B	2.99	4.34	0.59	1	1.45	1.83	0.75	3.01	4.32	
8	C_{14}	AFRC/STS/C_{14} A	3.03	4.02								8.1
		AFRC/STS/ C_{14} B	2.99	4.02	0.59	1	1.48	1.77	0.85	3.01	4.02	
9	C_{15}	AFRC/STS/ C_{15} A	2.89	4.51								7.05
		AFRC/STS/ C_{15} B	2.98	4.54	0.65	1	1.42	1.8	1	2.94	4.53	
10	C_{23}	AFRC/ STS/C_{23} A	2.78	4.36								7.25
		AFRC/STS/ C_{23} B	2.98	4.33	0.64	1	1.3	1.77	0.9	2.88	4.35	
11	C_{24}	AFRC/STS/ C_{24} A	2.34	3.06								8.04
		AFRC/STS/ C_{24} B	2.26	3.08	0.6	1	1.27	1.71	1	2.30	3.07	
12	C_{25}	AFRC/STS/ C_{25} A	2.47	4.37								7.96
		AFRC/STS/ C_{25} B	2.56	4.38	0.6	1	1.31	1.79	1.15	2.52	4.38	
13	C_{34}	AFRC/STS/ C_{34} A	2.73	4.58								8.14
		AFRC/STS/ C_{34} B	2.74	4.62	0.62	1	1.33	1.83	1.1	2.74	4.60	
14	C_{35}	AFRC/ STS/C_{35} A	2.86	4.34								10.54
		AFRC/STS/ C_{35} B	2.79	4.39	0.63	1	1.41	1.85	1.25	2.82	4.37	
15	C_{45}	AFRC/STS/ C_{45} A	2.86	3.59								11.02
		AFRC/STS/ C_{45} B	2.83	3.58	0.61	1	1.25	1.79	1.35	2.85	3.59	

3.5. SCHEFFE' S (5,2) POLYNOMIAL MODEL FOR THE AFRC RESPONSES (FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT).

A. FLEXURAL STRENGHT

By substituting the values of the flexural strengths (responses) from Table 3 into Eqns.(8) through (10), we obtain the coefficients (β_1 , β_2 ... β_{34} ,β_{35}…. β_{45}) of the Scheffe's second degree polynomial for AFRC. Substituting the values of these coefficients into Eqn. (7) yield the polynomial model for the optimization of the flexural strength of AFRC (at both 14th day or 28th day) based on Scheffe's (5,2) lattice as given under:

$$P^F = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{45}X_4X_5 \qquad \textbf{(22)}$$

B. SPLIT TENSILE STRENGHT

By substituting the values of the split tensile strengths (responses) from Table 4 into Eqns.(8) through (10), we obtain the coefficients (β_1 , β_2 ... β_{34} ,β_{35}…. β_{45}) of the Scheffe's second degree polynomial for AFRC. Substituting the values of these coefficients into Eqn. (7) yield the polynomial model for the optimization of the split tensile strength of AFRC (at both 14th day or 28th day) based on Scheffe's (5,2) lattice as given under:

$$P^S = ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 + ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{45}X_4X_5 \quad \textbf{(23)}$$

3.6. SCHEFFE'S (5,2) MODEL RESPONSES (FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT) FOR AFRC AT CONTROL POINTS.

A. FLEXURAL STRENGHT

By substituting the pseudo mix ratio of points C_1, C_2, C_3, C_4, C_5, … C_{45} of Table 5 into Eqn.(22), we obtain the Scheffe's second degree model responses (flexural strength) for the control points of AFRC.

B. SPLIT TENSILE STRENGHT

By substituting the pseudo mix ratio of points C_1, C_2, C_3, C_4, C_5, … C_{45} of Table 6 into Eqn.(23), we obtain the second degree model responses (split tensile strength) for the control points of AFRC.

1.1.1 3.7. VALIDATION OF AFRC MODEL RESULTS (FOR FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT) USING STUDENT'S – T -TEST

Here, our objective is to perform the test of adequacy so as to determine the percentage correlation between the flexural and split tensile strengths results (lab responses) given in Tables 5 and 6 and model responses from the control points based on Eqns.(22 and 23). By using the Student's – T – test as the means of validation, the result shows that there are no significant differences between the experimental results and model responses. The procedures/steps involved in using the Student's – T - test have been explained by Nwachukwu and others (2022 c). Thus, the models are adequate for predicting the flexural and split tensile strengths of AFRC based on Scheffe's (5,2) simplex lattice.

1.1.2 3.8. RESULTS DISCUSSION

The maximum flexural strengths of AFRC based on Scheffe's (5,2) lattice are **8.18** MPa and **5.60** MPa respectively for 28th and 14th day results. Similarly the maximum split tensile strengths of AFRC based on Scheffe's (5,2) lattice are **4.72** MPa and **3.08** MPa respectively for 28th and 14th day results .The corresponding optimum mix ratio is **0.75 :1:1.30:1.20:1.50** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Asbestos Fibre respectively. The minimum flexural strength and split tensile strength are **5.60** MPa, **4.75** MPa, **3.06** MPa and **2.26**MPa respectively for the 28th day and 14th day results. The minimum values correspond to the mix ratio of **0.63: 1:1.30: 1.80:1.00** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Asbestos Fibre respectively. Thus, the Scheffe's model can be used to determine the AFRC flexural and spilt tensile strength of all points (1 - 45) in the simplex based on Scheffe's Second Degree Model**.**

4. CONCLUSION

In this work so far, Scheffe's Second Degree Polynomial Model, Scheffe's (5,2) Model was presented and used to formulate a model for predicting the flexural and split tensile strengths of AFRC. Firstly, the Scheffe's model was used to predict the mix ratio for predicting both flexural and split tensile strengths of AFRC. Through the use of Scheffe's (5,2) simplex model, the values both strengths were determined at all 15 points (1- 45). The results of the student's t-test validated the strengths predicted by the models and the corresponding experimentally observed results. The maximum attainable strengths predicted by the model based on Scheffe's (5,2) model are as stated in the results discussion session, likewise the minimum values. Furthermore, with the Scheffe's (5,2) model, any desired strength, given any mix ratio can be easily predicted and evaluated and vice versa. Thus, the application of this Scheffe's optimization model has solved the problem of having to go through vigorous, time-consuming and laborious empirical mixture design procedures in order to obtain the desired strengths.

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