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Mathematical Modeling through Calculus of Variations and Dynamic Programming

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ABSTRACT

in this paper we discuss here the concepts of calculation of variation and Dynamic programming and also discuss the application of Dynamic programming with the help of calculus of variations and Dynamic programming the equation of Euler's language and maximum entropy distribution caused out and the last we discuss optimigation theory and their types.

Key words: Calculus of variation, Dynamic programming, Euler's Language equation.

INTRODUCTION

The calculus of variation (or variational calculus) is a fold of mathematical analysis that uses variations, which are small changes in functions and functional to find maxima and minima of functional:

Mapping from set of functions to the real numbers. Functional are often expressed as definite involving functions and their derivatives. Functions that maximize or minimize functional may be found using the Euler to langrage equation of the calculus of variations.

A simple example of such a problem is to find the cause of shortest length connecting two points if there are no constraints, the solution is a straight line between the points. However, if the curve is constrained to lie on a surface in space, then the solution is less obvious, and possible many situations may exist. Such solutions are known ass geodesics. A related problem is posed be fermatas Principle: light follow the path of shortest optical length connecting two points, which depends upon the material of the medium. One corresponding concepts in mechanics is the principle of least or stationary action.

Differential dynamic programming is an optical control algorithm of the trajectory optimization class. The algorithm was introduced in 1966 by mayne and subsequently analyzed in Jocobson and mayne'book. The algorithm uses locally-quadratic models of the dynamics and cost functions, and displays quadratic convergence. It is closely related to Newton's Method.

It can be used in the study of dynamic programming and other new mathematical formalisms in optical control problems, such as the determination of rocket trajectories, the correction of lunch error and inflight disturbances of spacecrafts and in the problems of optimal control found in economics, biology and the Social Sciences.

1. Mathematical Modelling through calculus of variations.

1.a : <u>Euler Langrage Equation:</u>

Consider $I = \int_a^b + \left(x, y, \frac{dy}{dx}\right) dx.$ 1

For every well-behaved function y of x, we can find I as a real number so that I depends on what function y is of x. the problem of calculus of variations is to find that function y(x) for which I is maximum of minimum. The answer is given by the solution of Euler – Langrage's equation.

$\frac{df}{dy} - \frac{d}{dx}\left(\frac{df}{dy}\right) = 0$ 2

Which is an ordinary differential equation of the second order. A proof of this result will be obtained in the next section by using dynamic programming.

$if \ \ I = \iint f\left(x, y, z, \frac{d2}{dx}, \frac{d2}{dy}\right) dx\, dy \cong \iint f\ (x, y, z, p, g), dxdy$ 3

Then I is maximum of minimum when

$\frac{df}{d2} - \frac{d}{dx}\left(\frac{df}{dp}\right) - \frac{d}{dy}\left(\frac{df}{dq}\right) = 0$ 4

1.b: Maximum Entropy Distributions:

a. We want to find that probability distribution for a variety varying over the range (-00, 00) which has maximum entropy out of all distributions having a given mean 'm' and a given variance 6^2.

Let f(x) be the probability density function, then we have to maximize entropy defined by.

$$s = -\int_{-00}^{\infty} f(x) in\, f(x)\, dx \quad 5$$

Subject to $\int_{-00}^{\infty} f(x)dx = 1$, $\int_{-00}^{\infty} x\ xf(x)dx = m$

$$\int_{-00}^{\infty} x^2 f(x)dx = 6^2 + m^2 \quad 6$$

We form the lagrangian

$$L = \int_{-00}^{\infty} -f(x)\ln\ f(x) - \lambda \int_{-00}^{\infty} f(x)dx - M\int_{-00}^{\infty} x f(x)dx - v \iint_{-00}^{\infty} x^2 f(x)dx \quad 7$$

Here the integral contains only x and y = f(x) and there is no y in it. As such equation 2

$$-(1 + \ln f(x)) - \lambda - Mx - Vx^2 = 0 \quad 8$$

or $f(x) = Ae^{Mx+vx2}$ 9

We use equation (5) to calculate A, M, V to get

$$f(x) = \frac{1}{\sqrt{2z6}}\ e^{-[1/2}\,)x - {}^{m)^2}/_{6^2}\,] \quad 10$$

This shows that out of all distributions with a given mean m and a given variance 6^2, the normal distribution $N(m, 6^2)$ has the maximum entropy.

Now mean and variance are simplest moments and the maximum entropy distribution for which these moments have prescribed values is the normal distribution. This gives one reason for the importance of the normal distribution.

1. We now want to find the distribution over the interval [o, ∞] which has maximum – entropy, out of all those which have given arithmetic and geometric means.

Hence we have to maximize

$$\int_{-00}^{\infty} f(x) lnf(x) dn \quad II$$

Subject to

Using lagrange's method and equation 2 we get

$$\int_{0}^{\infty} f(x)dx = 1, \int_{0}^{\infty} xf(x)dx - m, \int_{0}^{\infty} lnx\, f(x)dx = lng \quad 12$$

Using lagrange's method and equation 2 we get

$$f(x) = A\ 2^{-ax} x^{r-1} \quad 13$$

A, d, r are determined by using equation 2

Thus gamma distribution has the maximum entry out of all distributions which have given arithmetic and geometric means.

C. We want to find the maximum entropy bi-variate distribution when x, y vary from -∞ to ∞ and when means, variances and covariance are prescribed

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) lx\, f(x,y)dxdy \quad 14$$

Subject to

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1,\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(x,y)dxdy = m_1,$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} yf(x,y)dxdy = m_2, \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2(x,y)dxdy = 6_1^2 + m_1^2$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y^2 f(x,y)dxdy = 6_2^2 + m_2^2 \quad 15$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy + (x,y)dxdy = P6_1 6_2 + m_1 m_2$$

Forming the Lagrangian and using equation 4

We get

$$f(x,y) = Ae^{-a,x - a_2 y - a_1 x^2 - b_2 y^2 - cxy} \quad 16$$

Using equation 15 to find a_1, a_2, b_1, b_2, c

We get

$$f(x,y) = \frac{1}{a\pi 6,6_{2}\sqrt{1-p^2}} \ exp \left(-\frac{1}{2(1-p^2)} \left(\frac{x-m_1}{6_1}\right)^2 - 2p \left(\frac{(x-m_1)(y-m_2)}{6_1 6_2} + \left(\frac{y-m_2}{6_2^2}\right)^2 \right) \right) \quad 17$$

d. We want to find the multivariate distribution for $x_1, x_2, \ldots x_n$ where $0 \le x_1 \le I$,

$0 \le x_1 \le I, \ldots, 0 \le x_n \le I; x_2 + \ldots + x_n = 1$ 18

For which E $(l_n x_1), \ldots$ E $(l_n x_1)$ have prescribed values and for which entropy is maximum.

Using the principal entropy, we get,

$$f(x_1, x_1, \ldots, x_1) = \frac{T(m_1+m_2+\ldots+m_n)}{T(m_1)T(m_2)\ldots T(m_n)} + x_1^{m_{2-1}} x_2^{m_{2-1}} x_{n-1}^{m_{n-1}}$$

$$(1 - x_1 - x_2 \ldots - x_{n-1})m_{n-1} \quad 19$$

Which is dirichlet distribution.

2. a Optimization theory and their types, advantages of mathematical Modeling

Mathematical optimization or mathematical programming is the selection of the best element from a set of available alternatives based on some criterion optimization problems of various types arises in all quantitative discipline, ranging from Computer Science and Engineering to operations research and economics and the development of solution methods has long been a topic of study in mathematics.

In its most basis form, an optimization problem consists of maximizing or minimizing a real function by systemically selecting impact values from an allowed set and computing the function's value. A large area of applied mathematics is the generalization of optimization theory and techniques to other formulations.

Some techniques for optimization:

1. Method of inequalities: it can be shown that

$f(x_1 x_2, \ldots x_n) \ge$ A or $\le$ B 20

Whenever $gi(x_1, x_2, \ldots x_n) = b_1 \quad I = 1, 2, \ldots m$ 21

And for some $x_1, x_2, \ldots x_n, f(x_1, x_2, \ldots x_n) = A$

Or B, then

A gives the maximum of the function and B is the maximum value of the function. Subject to constraints.

Some other techniques are Method of Differential Calculus, Method of Calculus of Variations, Methods of Dynamic Programming, Method Based on Maximum Principle, Mathematical Programming etc.

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