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Stochastic Modeling for Control of Hypertension Using Weibull Distribution

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ABSTRACT

To describe the temporal trends in prevalence and management status of hypertension between 1998 and 2015. Data of adults who were aged 30 year or older were extracted from the National Health and Nutrition Examination Survey, a nationwide representative population-based survey [6]. Hypertension was prevalent in 30.5% of Korean adults. The age and sex standardized prevalence showed little change between 1998 and 2015. The elderly population and men showed higher prevalence. We present some methods for estimating

Weibull parameters, namely, shape parameter (β) and scale parameter (Ƞ). The Weibull distribution is an important distribution especially for reliability and maintainability analysis. The presented methods can be classified into two categories: Weibull Probability Plotting Method and analytical methods.

Keywords: Logarithmic transformation, Cumulative density function, Hazard function, Hypertension, Plateaued.

INTRODUCTION

Hypertension is a leading health risk. The World Health Organization estimates that approximately 40% of adults have hypertension. Annually, 9.4 million deaths are attributable to complications from elevated blood pressure (BP). Elevated BP accounts for 45% of all cardiovascular mortalities, and 51% of stroke-related deaths. Studies have reported that the global prevalence of hypertension has continuously risen during recent decades along with the trend of global aging [7]. Because hypertension is the most important contributing risk fact for disease burden, the importance of adequate prevention, diagnosis, and control of hypertension is emphasized more than ever before. Globally, however, less than half of all individuals with elevated BP are aware of their diagnosis, and less than one third of those under treatment show adequately controlled BP. In probability theory and statistics, the Weibull distribution is a continuous probability distribution. It models a broad range of random variables, largely in the nature of a time to failure or time between events.

Weibull Probability Plotting Method

The Weibull distribution density function is given by:

$$F(x) = \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}},\ \beta>0,\ \eta>0,\ x\geq \gamma \geq 0 \qquad (1)$$

The cumulative Weibull distribution function is given by:

$$F(x) = 1-e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}} \qquad (2)$$

Where; β is the shape parameter, Ƞ is the scale parameter, and γ is the location parameter. To come up with the relation between the cumulative density function and the two parameters (β, γ), we take the double logarithmic transformation of the cumulative density function. From (2) and letting $\gamma = 0$, we have

$$1 - F(x) = e^{-\left(\frac{x}{\gamma}\right)^{\beta}}$$

$$\frac{1}{1-F(x)} = e^{-\left(\frac{x}{\gamma}\right)^{\beta}}$$

$$\ln\left[\frac{1}{1-F(x)}\right] = -\left(\frac{x}{\eta}\right)^{\beta}$$

$$\text{In } \ln\left[\frac{1}{1-F(x)}\right] = \beta \ln \eta - \beta \ln x$$

The last equation is an equation of a straight line. To plot $F(x)$ versus x, we apply the following procedure:

1. Rank failure times in ascending order.
2. Estimate F (x_i) of the i^{th} failure.

Plot F (x_i) vs. x in the Weibull distribution

1.1 Hazard Plotting Technique

The hazard plotting technique is an estimation procedure for the Weibull parameters. This is done by plotting cumulative hazard function H(x) against failure times on a hazard paper or a simple log-log paper.

The hazard function is given below:

$$h(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}$$

The cumulative hazard function is given below:

$$H(x) = \int h(x) = \left(\frac{x}{\eta}\right)^{\beta} \quad (3)$$

We can transform (3) by taking the logarithm as follows

$$1\text{n } H(x) = \beta\{In\ x - In\ \eta\}$$

$$\ln x = \frac{1}{\beta} \ln H(x) + \ln \eta \quad (4)$$

From (4), we can then plot 1n $H(x)$ versus 1n x using the following procedure:

1. Rank the failure times in ascending order.
2. For each failure, calculate $\Delta Hi = \frac{1}{(n+1)-1}$
3. For each failure, calculate $H=\Delta H_1+\Delta H_2+\ldots+\Delta H_i$
4. Plot 1n H vs. 1n x.
5. Fit a straight line.

Upon completing the plotting, the estimated parameters will be as follows:

$\beta = \frac{x}{y} = \frac{1}{slope}$

at H=1, η=x

2. Analytical Methods

Due to the high probability of error in using graphical methods, we prefer to use the analytical methods. This is motivated by the availability of high-speed computers. In the following, we discuss some of the analytical methods used in estimating Weibull parameters.

2.1 Maximum Likelihood Estimator (MLE)

The method of maximum likelihood [1, 2], is a commonly used procedure because it has very desirable properties. Let x_1, x_2 …x_n be a random sample of size n drawn from a probability density function f_x (x;θ) where θ is an unknown parameter. The likelihood function of this random sample is the joint density of the n random variables and is a function of the unknown parameter [2]. Thus

$$L = \prod_{i=1}^{n} \quad f\ xi\ (xi, \theta) \quad (5)$$

Is the likelihood function. The maximum likelihood estimator (MLE) of θ, say θ, is the value of θ that maximizes L or, equivalently, the logarithm of L. Often, but

$$\frac{d\ log\ log\ \ L}{d\theta} = 0$$

Where solutions that are not functions of the sample values x_1, x_2 … x_n are not admissible, nor are solutions which are not in the parameter space. Now, we are going to apply the MLE to estimate the Weibull parameters, namely the shape and the scale parameters. Consider the

Weibull pdf given in (1), then likelihood function will be

$$L(x_1, \dots, x_n; \beta, \eta) = \quad \prod_{i=1}^{n} \left(\frac{\beta}{\eta}\right)\left(\frac{x_i}{\eta}\right)^{\beta-1} e^{-\left(\frac{x_i}{\eta}\right)^{\beta}} \qquad (6)$$

On taking the logarithms of (6), differentiating with respect to β and Ŋ in turn equating to zero, we obtain the estimating equations

$$\frac{\partial \ln L}{\partial \eta} = \frac{n}{\beta} + \sum_{i=1}^{n} \ln x_i - \frac{1}{\eta}\sum_{i=1}^{n} x_i^{\beta} \ln x_i = 0 \qquad (7)$$

$$\frac{\partial \ \ln L}{\partial \eta} = -\frac{n}{\eta} + \frac{n}{\eta^2}\sum_{i=1}^{n} x_i^{\beta} = 0 \qquad (8)$$

On eliminating Ŋ between these two equations and simplifying, we have

$$\frac{\sum_{i=1}^{n} x_i^{\beta} \ln x_i}{\sum_{i=1}^{n} x_i^{\beta}} - \frac{1}{\beta} - \frac{1}{n}\sum_{i=1}^{n} \ln x_i = 0 \qquad (9)$$

Which may be solved to get the estimate of $\hat{\mu}_k = \beta$. This can be accomplished by the use of standard iterative procedures (i.e., Newton-Raphson method). Once β is determined, Ŋ can be estimated using equation (8) as

$$\eta = \frac{\sum_{i=1}^{n} x_i^{\beta}}{n} \qquad (10)$$

2.2 Method of Moments (MOM)

The method of moments is another technique commonly used in the field of parameter estimation. If the numbers $x_1, x_2 \dots x_n$ represent a set of data, then an unbiased estimator for the kth origin moment is

$$\widehat{m_k} = \frac{1}{n}\sum_{i=1}^{n} x_i^k \qquad (11)$$

Where; $\widehat{m}_k$ stands for the estimate of m_k. In Weibull distribution, the k^{th} moment readily follows from (1) as

$$\mu_k = \left(\frac{1}{\eta^{\beta}}\right)^{\frac{k}{\beta}} \Gamma\left(1 + \frac{k}{\beta}\right) \qquad (12)$$

Where Γ signifies the gamma function

$$\Gamma(s) = \int_0^{\infty} x^{s-1} e^{-x} dx, (s > 0)$$

Then from (12), we can find the first and the second moment as follows

$$m_1 = \widehat{\mu_k} = \left(\frac{1}{\eta}\right)^{\frac{1}{\beta}} \Gamma\left(1 + \frac{1}{\beta}\right) \qquad (13)$$

$$m_2 = \widehat{\mu_k^2} + \widehat{\sigma_k^2} = \left[\frac{1}{\eta}\right]^{\frac{1}{\beta}} \left\{\Gamma\left(1 + \frac{2}{\beta}\right) - \left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2\right\} \qquad (14)$$

When we divide m_2 by the square of m_1, we get an expression which is a function of β only

$$\frac{\widehat{\sigma}_k^2}{\widehat{\mu}_k^2} = \frac{\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)}{\Gamma^2\left(1+\frac{1}{\beta}\right)} \qquad (15)$$

On taking the square roots of (15), we have the coefficient of variation

$$\mathrm{CV} = \frac{\sqrt{\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)}}{\Gamma\left(1+\frac{1}{\beta}\right)} \qquad (16)$$

Now, we can form a table for various CV by using (16) for different β values. In order to estimate β and Ŋ, we need to calculate the coefficient of CV using the table. The corresponding β is the estimated one($\widehat{\beta}$). The scale parameter (Ŋ) can then be estimated using the following

$$\hat{\eta} = \{\bar{x}/\Gamma[(1/\hat{\beta}) + 1]\}^{\hat{\beta}}$$

Where $\underline{x}$ is the mean of the data.

2.3 Least Square Method (LSM)

The third estimation technique we shall discuss is known as the Least Squares Method. It is so commonly applied in engineering and mathematics problems that is often not thought of as an estimation problem [4]. We assume that a linear relation between the two variables. For the estimation of Weibull parameters, we use the method of least squares and we apply it to the results. Recall that

$$\text{In In}\left[\frac{1}{1-F(x)}\right] = \beta \text{ In } x - \beta \text{ In } \eta \qquad (17)$$

Equation (17) is a linear equation. Now, we can write

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} \ln\left\{\ln\left[\frac{1}{\left(1-\frac{i}{n+1}\right)}\right]\right\} \tag{18}$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} \ln x_i \tag{19}$$

$$\hat{\beta} = \frac{\left\{n.\sum_{i=1}^{n}(\ln x_i).\left(\ln\left\{\ln\left[\frac{1}{\left(1-\frac{i}{n+1}\right)}\right]\right\}\right)\right\}-\left\{\sum_{i=1}^{n}\ln\left(\ln\left[\frac{1}{\left(1-\frac{i}{n+1}\right)}\right]\right).\sum_{i=1}^{n}\ln x_i\right\}}{\{n.\sum_{i=1}^{n}(\ln x_i)^2\}-\{\sum_{i=1}^{n}(\ln x_i)\}^2} \tag{20}$$

$$\hat{\eta} = e^{(\bar{y}-\bar{x}/\hat{\beta})} \tag{21}$$

From equations (18)-(12), we can calculate the estimate of β and Ƞ.

3. Example:

The rate of hypertension treatment had a similar pattern as the awareness rate. While 20.4% of subjects with hypertension received in treatment [5]. The number went up to 63.6% trends in the treatment of hypertension (fig a). Using methods for estimating the parameters of the Weibull distribution for reliability and maintainability analysis (fig b).

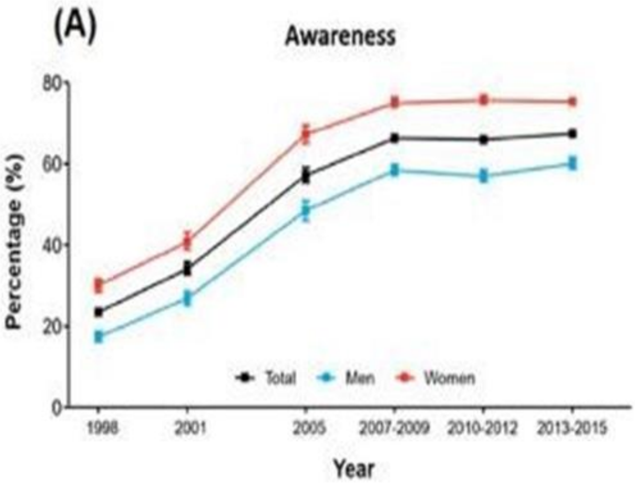

Fig (a)

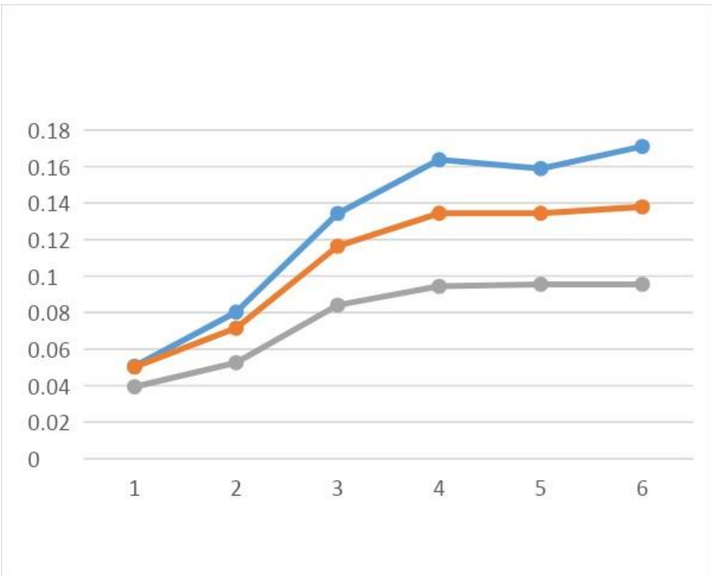

Fig (b)

Conclusion:

The present study investigated the temporal trends in prevalence and management status of hypertension. The temporal trends in the prevalence, awareness, treatment, and control of hypertension using the survey data from 1998 to 2015. We analysed the epidemiology among various demographic and socioeconomic subgroups in an attempt to identify methods to improve hypertension management by using Weibull probability plotting method and analytical methods for estimating the Weibull distribution parameters. Finally, from (fig b), we conclude that the results coincide with the mathematical and medical report.

Reference:

1) Cohen, A. C., "Maximum likelihood estimation in the Weibull distribution based on. complete and on censored samples", Technimetrics, Vol. 7 No. $, November1965.

2) Harter, H. L. and A. H. Moore,'' Point and interval estimators based on order statistics, for the scale parameter of Weibull population with known shape parameter" Technimetrics, Vol. 7, No.3, August 1965a.

3) Ivers, N.et al. Audit and feedback: effects on professional practice and healthcare outcomes. Cochrane Database Syst Rev, CD000256, https:// doi.org/10.1002/14651858. CD000256.pub3 (2017).

4) Mann, N. R. and K. W. Fertig, "Simplified efficient point and interval estimators of the Weibull parameters", Technometrics, Vol. 17. N.3, August 1975.

5) N.et al. Audit and feedback: effects on professional practice and healthcare outcomes. Cochrame Database Syst Rev, CD000256, https:// doi.org/10.1002/14651858. CD000256.pub3(2017).

6) World Health Organization. Global status rport on non communicable diseases 2010(2011).

7) Roy, A. et al. Changes in hypertension prevalence, awareness,treatment, and control rates over 20 years in the National Capital of India: results from a repeat cross-sectional study. BMJ open 7, e015639, https://doi.org/10.1136 /bmjopen-2016-015639(2017).