



Application of the Kalman Filter in Air Defense Radar to Track the Target's Trajectory

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ABSTRACT

The air defense radar system serves as a vital apparatus for obtaining crucial information about airborne targets through the reception and analysis of radio waves. The target of air defense radar can be unmanned aircraft, reconnaissance aircraft, transport aircraft, cruise missiles, winged missiles... However, it faces significant challenges such as dealing with nonlinear measurement models, non-Gaussian noise, and electronic warfare jamming measures, which impact accurate target trajectory tracking. To address these challenges, this research proposes a comprehensive solution utilizing the Kalman filter approach. By leveraging the Kalman filter, the study aims to mitigate the impacts of noise and nonlinearities. This adaptive strategy promises to enhance the precision of target trajectory tracking in air defense radar systems. The Kalman filter-based solution has the potential to improve trajectory prediction quality, thereby enhancing the overall effectiveness of air defense radar in safeguarding airspace.

Keywords: Radar, Kalman filter, target trajectory

1. Introduction

Orbit tracking is the process of associating newly acquired data points with one of the detected and tracked orbits during the previous observation cycle and calculating orbit parameters with higher accuracy than during the detection process. To achieve this, the orbit parameters must first be calculated, followed by extrapolating coordinates for the next observation cycle and establishing a tracking gate for extrapolated data points, as illustrated by the concentric circles in Figure 1.

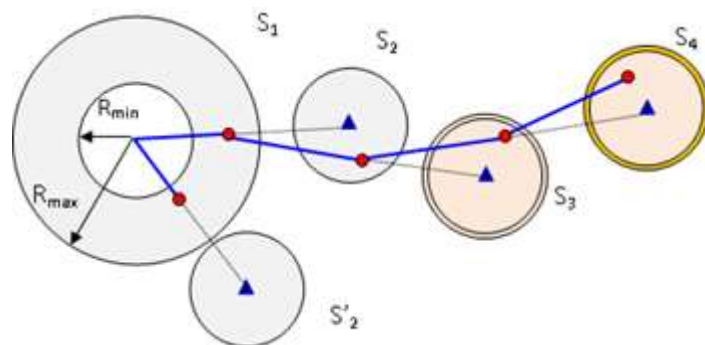


Fig.1. Radar information level 2 processing

An additional task involves the selection of reference points within the tracking gate. This can result in three scenarios: a single reference point, multiple reference points, or no reference point within the tracking gate. The approach to maintaining trajectory tracking is as follows:

In the case of a single reference point within the tracking gate, the provided reference point naturally becomes the next point on the initiated trajectory.

If no reference point exists within the tracking gate, the extrapolated point is considered the continuation of the tracked trajectory.

When multiple reference points are present within the tracking gate, a selection process is necessary to choose one reference point for the ongoing trajectory tracking.

From here, it can be observed that If there is a foundation regarding the target's motion model and the form of the noise, it is entirely possible to estimate and predict the parameters within that motion model. This means the trajectory of the target can be forecasted at different time points. In other words, the fundamental issue of target tracking revolves around the approach to predicting and estimating the target's trajectory, or, in essence, extrapolating the trajectory.

The method of extrapolating coordinates based on the target's motion parameters is illustrated in Figure 2. Let's assume we need to extrapolate coordinates at time $t = t + T_0$. Given the conditions that the last two observed coordinates of the target are (X_{N-1}, Y_{N-1}) and (X_N, Y_N) , if we hypothesize a linear target trajectory, the extrapolation algorithm can be readily inferred from Figure 2.

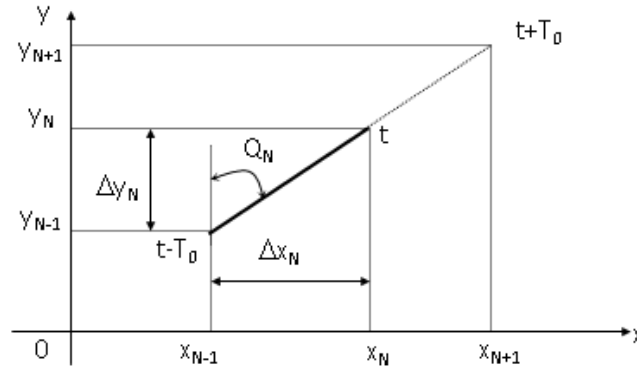


Fig.2. Extrapolation of coordinates according to the motion parameter of the target

$$\left. \begin{aligned} x_{N+1} &= x_N + V_N T_0 \sin Q_N = x_N + V_{x_N} T_0 \\ y_{N+1} &= y_N + V_N T_0 \cos Q_N = y_N + V_{y_N} T_0 \end{aligned} \right\}$$

In which:

$$\left. \begin{aligned} V_N &= \frac{1}{T_0} \sqrt{\Delta x_N^2 + \Delta y_N^2} \\ Q_N &= \text{artg} \frac{\Delta x_N}{\Delta y_N} \end{aligned} \right\}$$

For the problem of tracking the target's trajectory, we can apply the Kalman filter algorithm to build an estimator of the target's state. The construction of the estimator is based on modeling the target's motion trajectory, along with the target's range and azimuth measurement signals, along with process noise and measurement noise. The task of the estimator is to predict the X-Y displacement on the two OX-OY axes and the V_x - V_y speed, thereby solving the orbital extrapolation problem.

2. Building a kalman filter for extrapolating target trajectories

To construct a Kalman filter, two conditions must be satisfied:

- The observed object can be modeled using a system of state equations.
- The observed object can be measured.

Based on the model form of the observed object and the input signals, there are three fundamental types of Kalman filters: Kalman filter for linear objects with discrete signal form; extended Kalman filter for nonlinear objects with discrete signal form; Kalman-Bucy filter for objects with continuous signal form.

Applying this to the target tracking problem, firstly, we assume that the motion model of the target follows a linear form as follows:

$$\left\{ \begin{aligned} X_n &= X_{n-1} + V_{x_{n-1}} \cdot \Delta t \\ V_{x_n} &= V_{x_{n-1}} \\ Y_n &= Y_{n-1} + V_{y_{n-1}} \cdot \Delta t \\ V_{y_n} &= V_{y_{n-1}} \end{aligned} \right.$$

In which:

X_n and X_{n-1} represent the displacement of the target along the X-axis in Cartesian coordinates at time $t = n$ and the preceding time $t = n-1$

Y_n and Y_{n-1} denote the displacement of the target along the Y-axis in Cartesian coordinates at time $t = n$ and the preceding time $t = n-1$.

V_{xn} and V_{yn} stand for the velocities along the X and Y axes, respectively.

Δt signifies the sampling time.

Furthermore, considering the radar-derived information of the target, which includes range r and azimuth angle θ , a coordinate transformation is required from polar coordinates to Cartesian coordinates as follows:

$$\begin{cases} r = \sqrt{X_n^2 + Y_n^2} \\ \theta = \tan^{-1}\left(\frac{Y_n}{X_n}\right) \end{cases}$$

Thus, describing the object by the following system of equations:

$$[T_n] = \begin{bmatrix} X_n \\ V_{x_n} \\ Y_n \\ V_{y_n} \end{bmatrix} = [F] \cdot \begin{bmatrix} X_{n-1} \\ V_{x_{n-1}} \\ Y_{n-1} \\ V_{y_{n-1}} \end{bmatrix}$$

$$[O_n] = \begin{bmatrix} r \\ \theta \end{bmatrix} = [H] \cdot \begin{bmatrix} X_{n-1} \\ V_{x_{n-1}} \\ Y_{n-1} \\ V_{y_{n-1}} \end{bmatrix}$$

In which:

$$[F] = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $[H]$ is the Jacobi matrix to linearize the nonlinear relationship.

$$[H] = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \end{bmatrix}$$

$$\begin{cases} h_{11} = \frac{dr}{dX_n} = \cos \theta; h_{12} = \frac{dr}{dV_{x_n}} = 0; \\ h_{13} = \frac{dr}{dY_n} = \sin \theta; h_{14} = \frac{dr}{dV_{y_n}} = 0 \\ h_{21} = \frac{d\theta}{dX_n} = \frac{-\sin \theta}{r}; h_{22} = \frac{d\theta}{dV_{x_n}} = 0; \\ h_{23} = \frac{d\theta}{dY_n} = \frac{\cos \theta}{r}; h_{24} = \frac{d\theta}{dV_{y_n}} = 0 \end{cases}$$

From here, we build the system of state equations for the object. The object here is the transformation process between two coordinates. Suppose we can determine the statistical characteristics of the process noise and the measured noise by the covariances Q and R .

The system of state equations describes the object:

$$\begin{cases} [T_n] = [F] \cdot [T_{n-1}] \\ [O_n] = [H] \cdot [T_n] \end{cases}$$

The Kalman filter algorithm consists of two fundamental components: the state prediction phase and the state correction phase.

In the prediction phase, the state vector T at time (k) is forecasted based on the state T at the immediately preceding time (k-1). The prediction algorithm within the Kalman filter's prediction phase comprises three equations:

$$\begin{cases} \bar{T}_k = F \cdot \hat{T}_{k-1} \\ \bar{P}_k = F \cdot \hat{P}_{k-1} \cdot F^T + Q \\ K = (\bar{P}_k H_k^T) / (H_k \cdot \bar{P}_k H_k^T + R) \end{cases}$$

In the correction phase of the prediction, after time k, the measured outcomes are compared with the predicted results to adjust errors, forming the basis for the next-time prediction. Consequently, after several iterations, errors tend to converge towards zero, leading to accurate predictions. The algorithm within the Kalman filter's correction phase involves three equations:

$$\begin{cases} SSO = \tilde{O}_k - H \cdot \bar{T}_k \\ \hat{T}_k = \bar{T}_k + K \cdot SSO \\ \hat{P}_k = (1 - K \cdot H) \cdot \bar{P}_k \end{cases}$$

3. Simulation Verification

3.1. Target Motion Trajectory Generator

Figure 3 illustrates the target motion trajectory generator. The assumed motion of the target along the Ox axis maintains a constant velocity of $V_x = 100$ m/s, while the target's motion along the Oy axis remains stationary. The initial coordinates are set at (0, 0). The target's motion in Cartesian coordinates is transformed into polar coordinates. Thus, the target's motion in polar coordinates encompasses the range and azimuth angle, which correspond to positive inputs for the radar system.

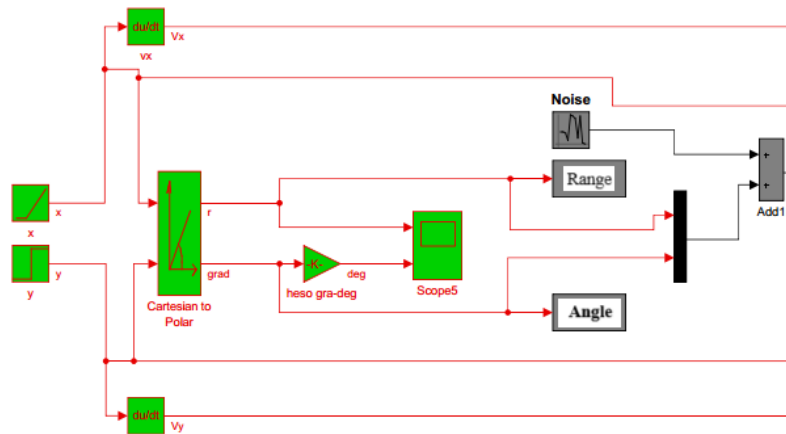


Fig.3. Target's trajectory generator

3.2. Target Trajectory Prediction Unit

Upon obtaining target information, the polar coordinates of the target are relayed to the secondary information processing module of the radar system for trajectory detection and tracking tasks. Constructing the target trajectory prediction unit aims to extrapolate the target's trajectory, accounting for signal noise in radar measurements. This prediction unit is founded on the estimation of trajectories utilizing the Kalman filter algorithm, facilitating trajectory tracking.

In this context, the Kalman filter is employed to predict the motion trajectory of the target, as in Figure 4.

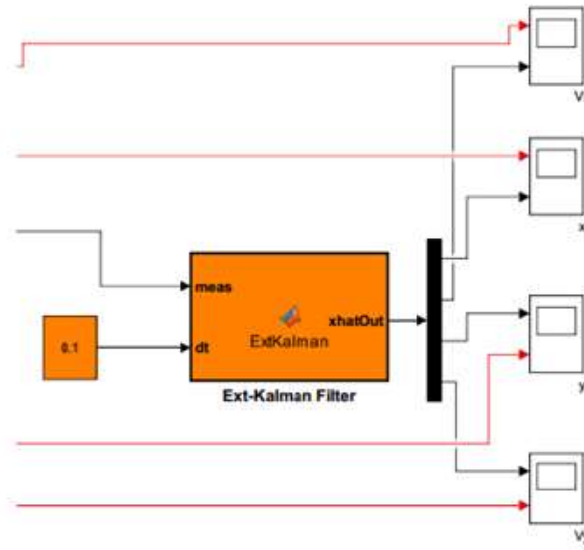


Fig.4. Target trajectory predictor using the Kalman filter algorithm

3.3. Target Trajectory Tracking Results

The target trajectory tracking results are displayed on four scope screens for a comparative analysis between the predicted trajectory and the actual trajectory for each corresponding pair: \hat{V}_x and V_x , \hat{V}_y and V_y , \hat{X} and X , \hat{Y} and Y .

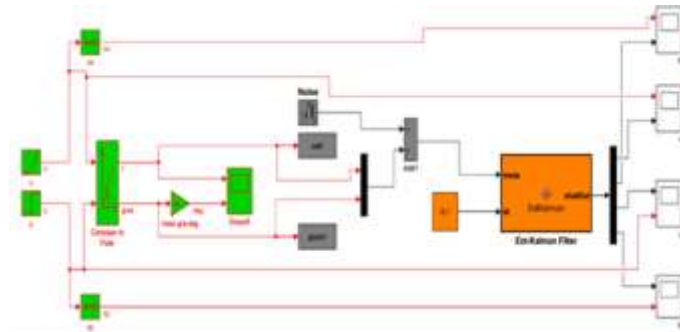


Fig.5. Overall diagram of anti-aircraft radar using a Kalman filter to track target trajectory

In the absence of noise interference, Figure 6 illustrates the target's position and velocity along the Ox axis. Figure 7 depicts the target's position and velocity along the Oy axis. In both figures, the actual values are represented by the blue line, while the predicted values are denoted by the red line.

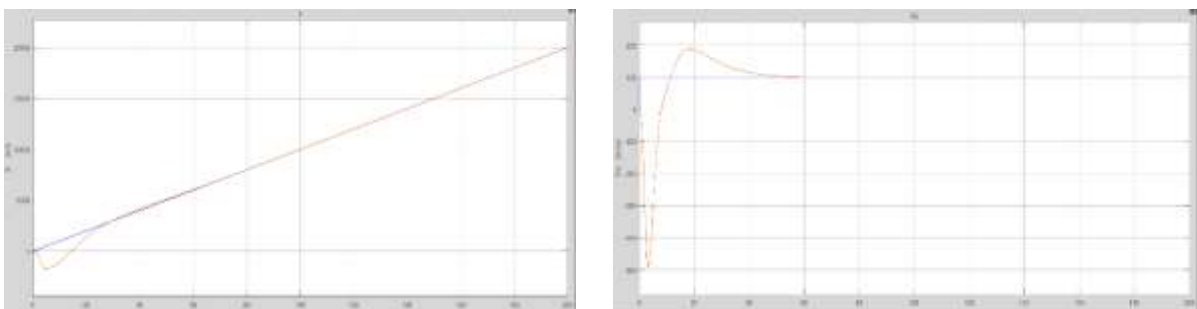


Fig.6. Target position and speed along the OX axis

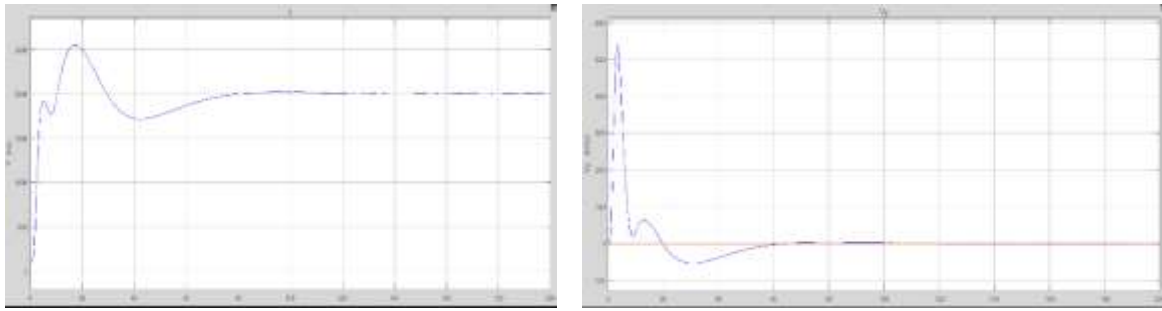


Fig.7. Target Position and Velocity along the OY Axis

In the presence of noise interference: Figure 8 illustrates the target trajectory in the Cartesian coordinate system. In this depiction, the actual trajectory is represented by the red line, the measured trajectory (the actual trajectory affected by noise) is denoted by the green line, and the predicted trajectory is shown by the blue line.

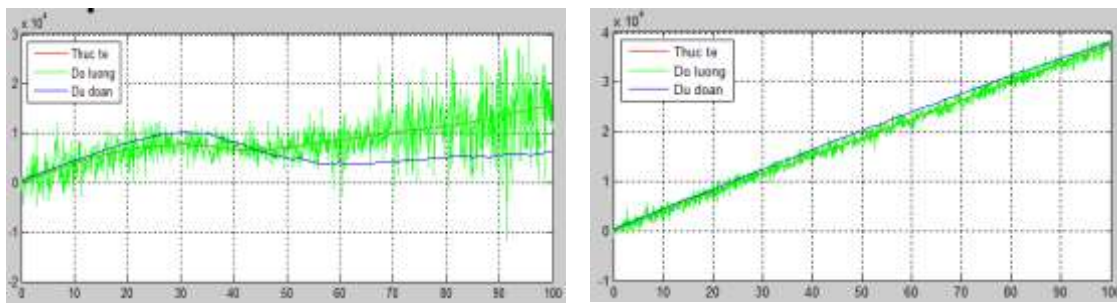


Fig.8. The target trajectory in the cartesian coordinate system

- a) When not using the Kalman filter to follow the target trajectory
- b) When using the Kalman filter to follow the target trajectory

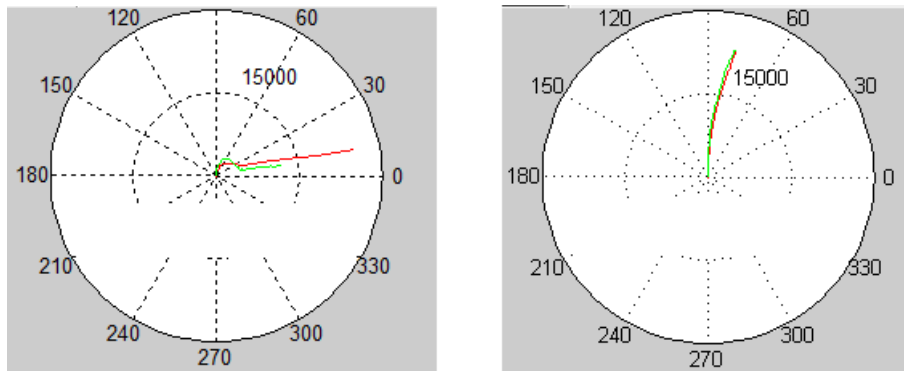


Fig.9. Target trajectory in the polar coordinate system

- a) When not using the Kalman filter to follow the target trajectory
- b) When using the Kalman filter to follow the target trajectory

6. Conclusion

In this study, the authors propose the application of the Kalman filter as a replacement for conventional tracking devices in air defense radar systems. In cases with significant noise interference, traditional tracking devices have proven insufficient for maintaining precise target trajectory tracking. Constructing a target trajectory prediction unit based on the Kalman filter algorithm allows us to mitigate noise effects and effectively track the target trajectory within permissible deviations. This opens the possibility of utilizing the Kalman filter as a substitute for conventional tracking devices in air defense radar systems to ensure accurate target trajectory tracking.

By introducing the Kalman filter, the research demonstrates its potential to address the limitations of traditional tracking methods. The filter's adaptability in handling noise and its ability to provide accurate trajectory prediction contribute significantly to enhanced target tracking performance. Consequently,

this work underscores the viability of leveraging the Kalman filter to augment or replace conventional tracking devices, thereby advancing the capabilities of air defense radar systems.

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