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Fractional Mathematical Modeling of Pulsatile Non-Newtonian Blood Flow and Heat Transfer in a Blood Vessels

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ABSTRACT

This study employs a fractional Oldroyd-B fluid model to describe blood as a non-Newtonian fluid, aiming to investigate how a magnetic field affects dynamic blood flow in a narrow artery exposed to pulsatile pressure gradients. The research also explores the flow's behavior in terms of heat transfer, considering factors like electromagnetic coupling, viscous dissipation, and radiative heat flux. Numerical solutions for the coupled, nonlinear equations governing velocity and temperature were obtained using a finite difference approach. Extensive analyses were conducted to validate the numerical algorithm's stability and convergence, which were confirmed to be satisfactory. The study highlights significant differences between predictions made by Newtonian fluid dynamics and the fractional Oldroyd-B fluid model. Notably, the fractional Oldroyd-B model predicts higher blood velocities and temperatures compared to the Newtonian model. Furthermore, the research reveals that increasing thermal radiation or applying a magnetic field has opposing effects on blood flow parameters. Specifically, a magnetic field reduces flow rate, while higher thermal radiation increases it. These findings offer insights into the intricate interplay of fluid dynamics, heat transfer, and external influences in blood flow within small arteries.

1.0 Introduction

The study of blood flow dynamics has been a subject of extensive research in understanding cardiovascular diseases and developing effective treatment strategies. Traditional models for blood flow have typically assumed a Newtonian behavior, which assumes constant viscosity and homogeneity. However, it is increasingly recognized that blood exhibits non-Newtonian behavior due to its composition of various cellular and plasma components. Research has indicated that blood can be better represented as an Oldroyd-B fluid, which incorporates both elastic and viscous components. The Oldroyd-B model has been widely used to describe the non-Newtonian behavior of blood flow. For example, Yannopoulos et al. (2014) employed the Oldroyd-B model to simulate blood flow in stenosis arteries, revealing the importance of non-Newtonian behavior in determining the flow patterns and wall shear stress distributions. Additionally, blood flow in the circulatory system is inherently pulsatile due to the contraction of the heart. Pulsatile flow exhibits periodic variations in velocity, pressure, and wall interactions within the blood vessels. These pulsations induce heat transfer and chemical reactions, which affect the overall physiological processes in the human body.

According to Chakravarty et al. (2019), blood is a suspension of various cellular and plasma components, making it a non-Newtonian fluid. The presence of suspended red blood cells, white blood cells, platelets, and plasma proteins leads to the development of shear-thinning or shear-thickening behavior, where the viscosity of blood changes with the shear rate. This behavior significantly affects the flow dynamics and can impact the transport processes occurring within the blood vessels. Numerous studies have explored the impact of non-Newtonian rheology on blood flow. For instance, Fung (2013) demonstrated that the deformation and motion of red blood cells significantly affect the viscosity and flow behavior of blood. Similarly, Kulkarni et al. (2015) investigated the influence of non-Newtonian behavior on the hemodynamics of blood flow, highlighting the importance of accurate modeling.

Furthermore, mathematical modeling approaches using fractional calculus have gained attention as a tool to capture memory effects, time-varying behavior, and long-range correlations in complex systems. Fractional calculus has been successfully applied in various biomedical applications, including blood flow modeling. Despite significant advancements in blood flow modeling, there remains a critical gap in developing comprehensive mathematical models that integrate the Oldroyd-B fluid model, fractional calculus, pulsatile flow, heat transfer, and chemical reactions in blood vessels. The existing models either oversimplify the rheology of blood or do not adequately account for the effects of fractional calculus and pulsatile flow on heat transfer and chemical reactions. This limitation hinders our ability to accurately understand and predict the behavior of pulsatile non-Newtonian blood flow, heat transfer, and chemical reactions within blood vessels, particularly when incorporating the Oldroyd-B fluid model. Without an accurate mathematical model, it is challenging to develop reliable diagnostic tools, treatment strategies, and medical devices to effectively address cardiovascular diseases.

Thus, the problem at hand is to develop a comprehensive mathematical model that integrates the Oldroyd-B fluid model, fractional calculus, pulsatile flow, heat transfer, and chemical reactions to provide an accurate representation of the dynamics within blood vessels. Such a model would enable a

deeper understanding of the physiological processes involved in cardiovascular diseases and facilitate the development of advanced diagnostic techniques and targeted therapeutic interventions.The specific objectives of this study include: Conducting a comprehensive review of the literature on blood flow modeling, non-Newtonian rheology, heat transfer, chemical reactions, the Oldroyd-B fluid model, and fractional calculus to establish a foundation for the model development, Formulating a mathematical model that integrates the Oldroyd-B fluid model, fractional calculus, pulsatile flow, heat transfer, and chemical reactions within blood vessels, Implementing numerical techniques to simulate the mathematical model and validate its predictions against experimental data and existing theoretical models, analyzing the model outputs to gain insights into the interplay between pulsatile non-Newtonian blood flow, heat transfer, and chemical reactions within blood vessels and assessing the capability of the model in predicting the dynamics of blood flow and its potential in clinical applications for cardiovascular disease diagnosis and treatment.

By addressing these research objectives, this study aims to contribute to the development of an advanced mathematical model that accurately represents the interplay of fractional non-Newtonian blood flow, heat transfer, and chemical reactions in blood vessels using the Oldroyd-B fluid model. This will enhance our understanding of cardiovascular diseases and potentially lead to advancements in personalized medicine in this field.

2.0 The mathematical modeled of the problem

Assuming blood behaves as a conductive non-Newtonian fluid under the influence of a consistent magnetic field (MF), it results in the generation of an electrical current and the creation of an electromagnetic force. The motion of the blood is induced by both the electromagnetic force and a pulsatile pressure gradient. To describe this phenomenon, we utilize cylindrical coordinates (r, θ, z), where 'r' represents the radial direction, 'θ' represents the circumferential direction, and 'z' represents the flow direction.

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) - \sigma_e{\beta_o}^2 u + \rho g\beta_t(T - T_O) + \rho g\beta_c(C - C_w)(1)$$

$$\rho c_p\frac{dT}{dt} = k\frac{1}{r}\frac{\partial}{dr}\left(r\frac{\partial T}{dr}\right) - \frac{\partial qr}{\partial r} + \sigma_e\beta_0^2 u^2 + \mu\left(\frac{\partial u}{\partial r}\right)^2, (2)$$

$$\frac{\partial \bar{c}}{\partial \bar{t}} = Dm\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\left(r\frac{\partial c}{\partial r}\right)\right] - k_O(C - C_O)(3)\right.$$

where u and T are fluid velocity and temperature, which are the functions of cylindrical coordinate r and time t, $\partial p/\partial z$ is the pulsatile pressure gradient ρ, g, β, and T_0 are constants representing the fluid density, gravity acceleration, thermal coefficient, and initial temperature of blood, respectively. S_{rz}denotes the tangential stress of fractional Oldroid-B fluid, expressed by C. Friedrich, (1991), W.C. Tan, W.X. Pan, M.Y. Xu (2003),

$$(1 + {\lambda_1}^{\alpha}{D_t}^{\alpha})\overline{S}_{rz} = \mu\left(1 + {\lambda_2}^{\beta}{D_t}^{\beta}\right)\frac{\partial u}{\partial r}(4)$$

Here, λ_1, $\lambda_2$and express the relaxation time, retardation time and viscosity constant. ${D_t}^{\alpha}$is Caputo's time-fractional derivative defined as;

$$D_t^{\alpha}u(r,t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{u'(r,\tau)}{(r,\tau)^{\alpha}}d\tau, (5)$$

Note that the classical Maxwell fluid model can be recovered when $\lambda_2 = 0$ and $\alpha = 1$.The pulsatile pressure gradient given by, shah, N.A.D.Vieru, C.fetecau,(2016),

$\frac{\partial p}{\partial r} = A_o + A_1 Cos(\omega t)(6)$

where A_0 andA_1 represent the constant and pulsatile pressure gradient amplitudes, respectively, and ω is the pulsatile frequency.

where $\gamma^2 = \pi\int_0^{\infty}\gamma\nu\frac{dB_\nu}{dT}$|d $d\nu$, in whichγv and B_ν represent the frequency dependent absorption coefficient and the Planck function, respectively.

Initial along with boundary conditions considered for the presented problem are

$$u(r,0) = 0, \frac{\partial u(r,0)}{\partial t} = 0$$

$$T(r,0) = T_0, \frac{\partial T(r,0)}{\partial t} = 0, (7)$$

$\frac{\partial u(0,t)}{\partial r} = 0, u(R,t) = 0, \frac{\partial T(0,t)}{\partial r} = 0, T(R,t) = T_{w,}$

To simplify the above equations, the following dimensionless variables will be introduced:

$$\left.\begin{aligned} r = R\bar{r}, \partial r = R\partial\bar{r}, \underline{\bar{t}}R = ut, \frac{R\partial\bar{t}}{u} = u\partial t, \bar{\lambda}_1\frac{R}{u} = \lambda_1, \bar{\lambda}_2\frac{R}{u} = \lambda_2, \\ \frac{\varpi u}{R} = \omega, \frac{\bar{A}_O\mu U}{R^2} = A_{\,o}, \frac{\bar{A}_O\mu u}{R^2} = A_1, C - C_o = (C_W - C_o). \end{aligned}\right\}(8)$$

Combining equation (4) with one; we obtained:

$$\left.\begin{aligned} (1 + {\lambda_1}^{\alpha}{D_t}^{\alpha})\frac{\partial u}{\partial t} = (1 + {\lambda_1}^{\alpha}{D_t}^{\alpha})\left(A_o + A_1 Cos(\omega t)\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\left(1 + {\lambda_2}^{\beta}{D_t}^{\beta}\right)\right)\frac{\partial u}{\partial r} \\ +\left(\rho g\beta_T(T - T_O))(1 + \lambda_1^{\alpha}D_t^{\alpha}) + \rho g\beta_c(C - C_w)(1 + \lambda_1^{\alpha}D_t^{\alpha})\right) \end{aligned}\right\}(9)$$

Introducing equation (8) to equations (2), (3), (7), (9) and dropping the bars we have:

$$Re(1+\lambda_1{}^{\alpha}D_t{}^{\alpha})\frac{\partial u}{\partial t}=(1+\lambda_1{}^{\alpha}D_t{}^{\alpha})\left(A_o+A_1Cos(\omega t)\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\left(1+\lambda_2{}^{\beta}D_t{}^{\beta}\right)\right)\frac{\partial u}{\partial r}\\ +Grh(1+\lambda_1^{\alpha}D_t^{\alpha}))+Grc(1+\lambda_1^{\alpha}D_t^{\alpha})\Bigg\}(10)$$

$$p_e\frac{\partial\theta}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)+Ra\theta+Ha^2Bru^2+Br\left(\frac{\partial u}{\partial r}\right)^2\ (11)$$

$$Sc\frac{\partial c}{\partial t}=\frac{1}{r}\frac{\partial c}{\partial r}+\frac{\partial c}{\partial r^2}-Sc\vartheta C(12)$$

$$u(r,0)=0,\frac{\partial u(r,0)}{\partial t}=0,\theta(r,0)=0\ \ ,\frac{\partial\theta(r,0)}{\partial t}=0,\frac{\partial u(r,0)}{\partial r}=0,u(r,0)=0,\frac{\partial\theta(r,0)}{\partial r}=0,T(r,0)=\ 1 \qquad (13)$$

Initial along with boundary conditions considered for the presented problem are

are proposed to denote the Schmitt number, Chemical reaction parameter, Reynolds number, Hartmann number, Grashof thermal number, Grashof mass number radiation parameter, Prandtl number, Brinkman number, and Peclet number, respectively

$$\left.\begin{array}{l}Sc=\frac{U}{Dm},Cr=\frac{Rk_0}{U},Re=\frac{R\rho U}{\mu},Ha^2=B_0R\sqrt{\frac{\sigma_e}{\mu}},Grh=\frac{\rho g\beta_T(T_W-T_o)R^2}{\mu u},\\ Grc=\frac{\rho g\beta c(C_W-C_O)R^2}{\mu u},Ra=\frac{4\gamma^2R^2}{k},Pr=\frac{C_p\mu}{k},Br=\frac{\mu U^2}{k(T_W-T)},Pe=RePr\end{array}\right\}(14)$$

By employing the initial and boundary conditions of equation (13), the solutions of equations (10) -(12) was obtained with the aid of finite difference algorithm and L1 approximation, and the result is presented as follows:

3.0 Results and Discussion

Velocity profile of the flow

The velocity profile of fractional blood flow refers to the distribution of blood velocities at different points within a blood vessel when considering non-integer (fractional) derivatives in the mathematical model of blood flow. Fractional calculus is used to describe complex behaviors in blood flow that may not be adequately captured by traditional integer-order derivatives. A typical velocity profile for fractional blood flow might exhibit characteristics that deviate from the standard parabolic profile seen in Newtonian fluids. Instead, it could vary based on the specific fractional order used in the mathematical model, vessel geometry, blood viscosity, and other factors. Here's a general description of what a fractional blood flow velocity profile might entail. In figure 1 below, an increasing Hartman number Ha, tends to flatten the blood flow velocity profile, particularly near the center of the vessel. This is because the magnetic field interacts with the conductive blood, leading to a more uniform distribution of velocities across the vessel cross-section. The presence of a magnetic field, as quantified by the Hartman number (Ha), has significantly decreases the blood flow velocity profiles in blood vessels. These changes have broad implications for heat transfer, chemical reactions, and various physiological processes. Understanding these effects is essential for both fundamental research and the development of medical interventions that involve the application of magnetic fields in the vascular system. In figure2, an increase in Reynolds number altered velocity profile which physically affect the rates of chemical reactions occurring in the bloodstream. This revealed that Chemical species are transported differently in the presence of a magnetic field, potentially leading to variations in reaction kinetics. In a medical context, this could be crucial for drug delivery systems that rely on chemical reactions in the bloodstream. The rate at which drugs are delivered to specific tissues or cells may be influenced by the presence of the magnetic field. Figure 3, shows that an increases in Grashof number Grm, signifies a greater influence of buoyancy forces due to temperature variations. This can alter the blood flow pattern in the vessel. A low Grm (e.g., 0.5) may indicate that buoyancy forces are relatively weak compared to viscous forces, resulting in a flow pattern dominated by viscosity. As Grm increases, buoyancy forces become more significant, potentially leading to a transition from natural convection to forced convection. In Figure 4, it was revealed that as the fractional parameter (α) increases from 0.1 to 0.7, it signifies a higher degree of non-integer differentiation applied to the blood flow model. This can lead to deviations from traditional velocity profiles. Transition from Integer to Fractional Behavior: Lower α values (e.g., 0.1 and 0.3) might result in velocity profiles that are more similar to those described by traditional integer-order derivatives (Newtonian behavior), while higher α values (e.g., 0.5 and 0.7) could exhibit more pronounced fractional effects. From figure, as α increases from 0.1 to 0.7, it signifies a higher degree of non-integer differentiation applied to the blood flow model. This can lead to deviations from traditional velocity profiles. Transient Behavior: At t=0.5 seconds, the system exhibited a transient behavior as it reaches a steady state or approaches a quasi-steady state. The impact of α on transient behavior is essential to understand, as it can influence the time it takes for the system to stabilize. Influence on Steady-State Behavior: Different α values may lead to variations in the steady-state velocity profile. The steady-state profile represents the long-term behavior of blood flow after transient effects have subsided.

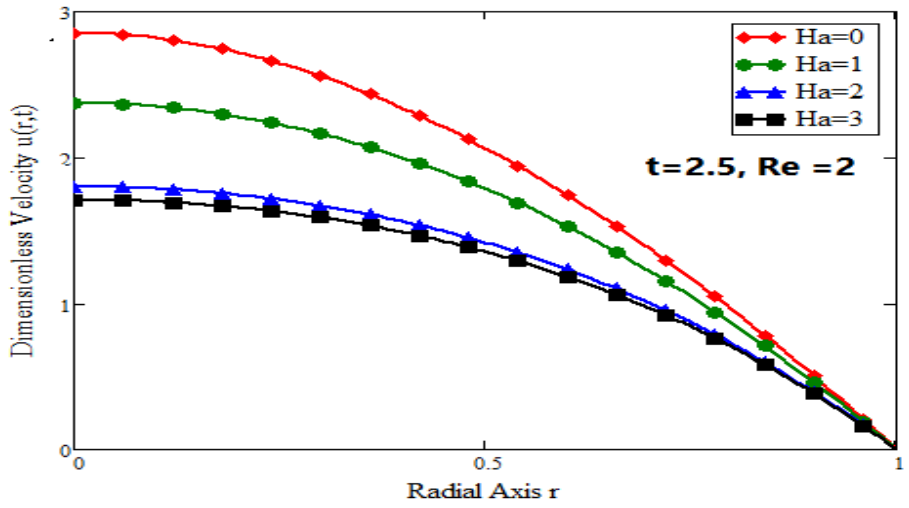

Fig 1: Variation of velocity with different Hartman number Ha

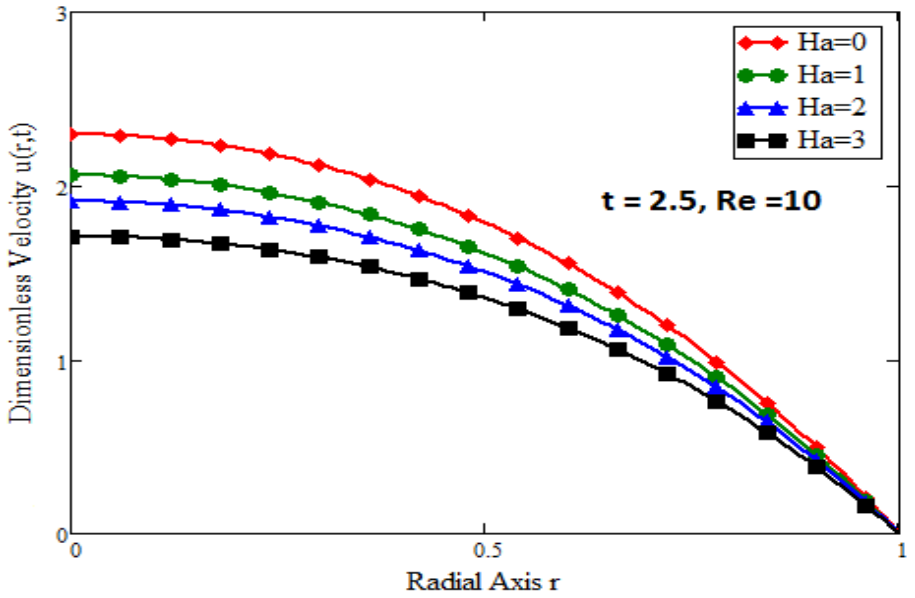

Fig 2: Variation of velocity with different Hartman number Ha

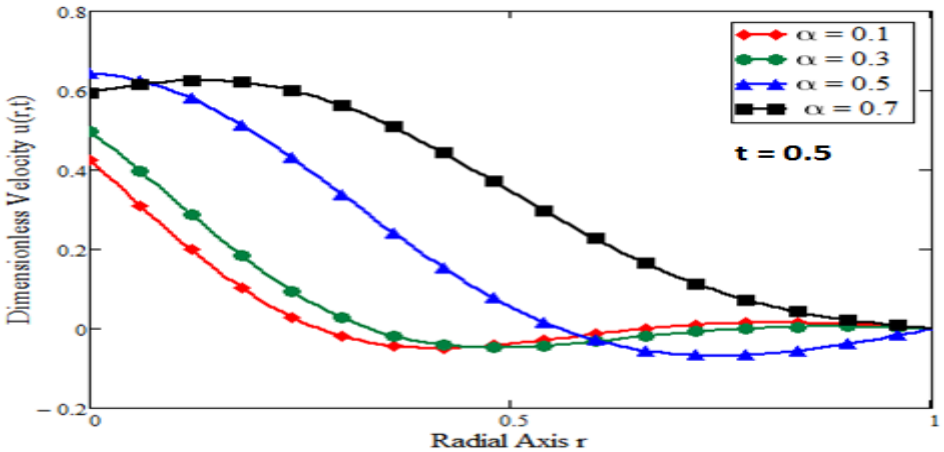

Fig 3: Variation of velocity with different Reynolds Number Re

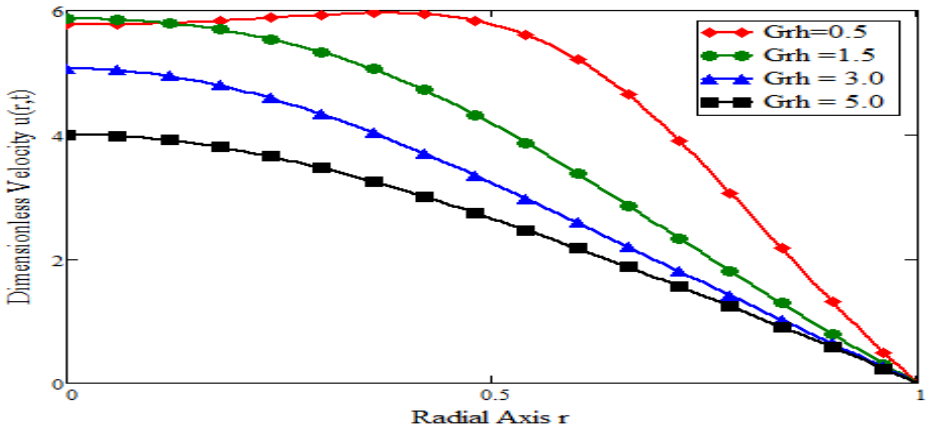

Fig 4: Variation of velocity with different fractional parameter

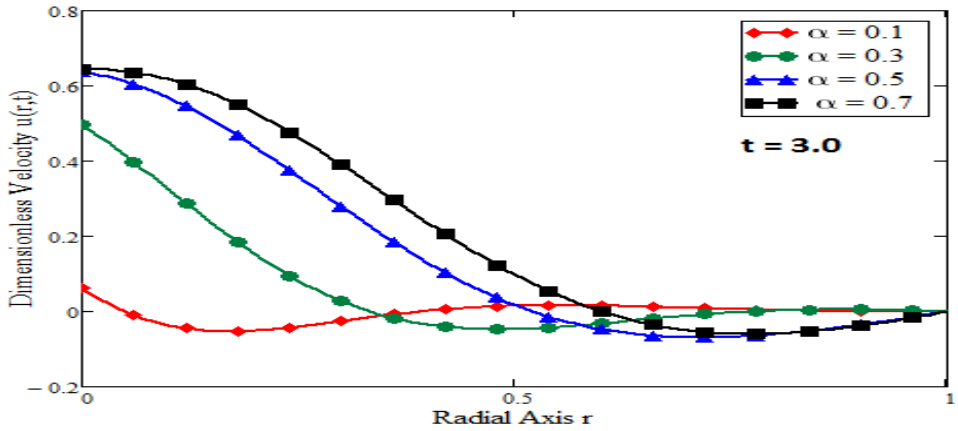

Fig 5: Variation of velocity with different Grashop Number Re

In conclusion, the presence of a magnetic field, as quantified by the Hartman number (Ha), can significantly alter blood flow velocity profiles in blood vessels. These changes have broad implications for heat transfer, chemical reactions, and various physiological processes. Understanding these effects is essential for both fundamental research and the development of medical interventions that involve the application of magnetic fields in the vascular system. Tshe Grashof number for mass transfer (Grm) plays a significant role in shaping the blood flow velocity profile within blood vessels. Analyzing the variations in velocity profiles for different Grm values is important for understanding the fluid dynamics and heat transfer in the circulatory system. These findings may have implications for various physiological processes and medical applications, warranting further investigation and experimentation. The fractional parameter (α) significantly influences the blood flow velocity profile within blood vessels when fractional calculus is applied. Analyzing the variations in velocity profiles for different α values at a specific time (t=0.5 seconds) provides insights into the complexity of blood flow dynamics and its implications for heat transfer and chemical reactions. These findings contribute to a better understanding of non-Newtonian blood flow and the impact of fractional calculus on modeling physiological processes.

REFERENCES

Adamu, G. T., Kwami, A. M., Abdulhameed, M. & Yakubu, D. G. (2020), Effect of retardation time on non-Newtonian Electro-osmotic flow in a micro-channel. *Journal of Engineering Fluid Flows and Heat Transfer Analysis*, 26, 39-52.

Escandon, J. Jimenez, E. Hernandez, O. Bautista, O. & Mendez, F. (2015), Transient electroosmotic flow of Maxwell Fluids in a slit microchannel with asymmtric zeta potentials. *European Journal of Mechanics B/Fluids, 53*, 180-189.

Masliyah, J. H. & Bhattacharjee, S. (2006), Electrokinetic and Colloid Transport Phenomena. Wiley-Interscience.

Muhammad, H., Reza, N., Ahmad, N. & Mehrdad, R. (2011), Numerical study of Electroosmotic micromixing of non-Newtonian fluid. 166:965-971.

Wang, X, Huanying Xu, and Haitao Qi (2020). "Transient magnetohydrodynamic flow and heat transfer of fractional Oldroyd-B fluids in a microchannel with slip boundary condition Phys. 32,103104

Probstein, R. F. (2003), Physicochemical Hydrodynamics: An Introduction, second edition. Wiley-Interscience.

Yang, C., Ng, C. B. & Chan, V. (2002), Transient Analysis of Electroosmotic Flow in a Slit Microchannel, *Journal of Colloid and Interface Science*, 248, 524-527.