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Fixed Point Results Using Implicit Mid Point Rule Iterative Process

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ABSTRACT:

The present work is related to the study of fixed point results via Picard-Mann iteration process of implicit midpoint rule for one nonlinear mapping. More over we shall show that Picard-Mann iteration process of implicit midpoint rule for one nonlinear mapping has same rate of convergence Picard- Kranoselskii iteration process of implicit midpoint rule for one nonlinear mapping. Our analytical results are supported by suitable numerical examples.

Keywords: Picard-Mann iteration process of implicit midpoint rule, Picard- Kranoselskii iteration process of implicit midpoint rule, contractive condition of rational expression, stability.

1. Introduction and Preliminaries

The aim of present work is to present some convergence and stability results for Picard-Mann iteration process of implicit midpoint rule for one nonlinear mapping. The study of fixed point results for iterative processes is one of the most fascinating areas in analysis. Iterative processes are developed to solve the equations arising in the physical formulation of the mathematical problems. In recent time some notable work is done by [6, 8-10]

We shall now discuss some the iterative schemes that are relevant to our work. Let H be a real normed linear space and $T : H \to H$ be a mapping. A point θ is called the fixed point if $T(\theta) = \theta$. Throughout the paper $F(T)$ will represent the set of fixed points of mapping T. For $x_0 \in H$, the sequence $\{x_n\}_{n=0}^{\infty}$ generated by

$$x_{n+1} = Tx_n, \; x_0 \in H. \qquad (1.1)$$

is called the Picard iteration process.

Khan in 2013[3], introduced the Picard-Mann hybrid iterative process by the method

$$x_{n+1} = Ty_n, \qquad y_n = (1-\rho_n)x_n + \rho_n Tx_n, \qquad (1.2)$$

Where $x_0 \in H$ and $\{\rho_n\}_{n=0}^{\infty} \in [0,1]$.

Khan [3] with suitable examples proved that iteration (1.2) has better rate of convergence than the Picard, Mann and Ishikawa iterative process for contractive type mappings in the sense of Berinde [2].

In this direction Okeke and Abbas [4] introduced the concept of Picard- Kranoselskii hybrid iterative process by

$$x_{n+1} = T_{y_n}, \qquad y_n = (1-\delta)x_n + \delta Tx_n, \qquad (1.3)$$

Where $x_0 \in H$ and $\delta \in [0,1]$.

Further Okeke and Abbas [4] claimed the higher rate of convergence of their iterative process than Picard, Mann, Ishikawa and Kranoselskii iterative process for contractive type mappings in the sense of Berinde [2].

Motivated by the above work Li and Lan [1] in 2019, introduced the notion of Picard-Mann iteration process of implicit midpoint rule for one nonlinear mapping as

$$x_{n+1} = T\left(\frac{x_n+y_n}{2}\right), \qquad y_n = (1-\alpha_n)x_n + \alpha_n T\left(\frac{x_n+y_n}{2}\right), \qquad (1.4)$$

x_0 is the initial approximation such that $x_0 \in H$ and $\{\alpha_n\}_{n=0}^{\infty} \in [0,1]$.

In the similar way we shall introduce the concept of Picard- Kranoselskii iteration process of implicit midpoint rule for one nonlinear mapping as

$$x_{n+1} = T(\frac{x_n+y_n}{2}),$$

$$y_n = (1-\lambda)x_n + \lambda T\left(\frac{x_n+y_n}{2}\right), \quad (1.5)$$

x_0 is the initial approximation such that $x_0 \in H$ and $\lambda \in [0,1]$.

We shall prove the convergence and stability results for the iterative process (1.4) and compare its rate of convergence with (1.5).

Definition 1.1[2]:- Let H be a real normed linear space and Let $\{x_n\}_{n=0}^{\infty}$ and $\{u_n\}_{n=0}^{\infty}$be the sequence converging to l_1 and l_2 respectively. Assume that $\lim_{n\to\infty} \frac{|sx_n - l_1|}{|u_n - l_2|} = l$. Then

1. If $l = 0$ then the sequence $\{x_n\}_{n=0}^{\infty}$ converges faster to l_1 than $\{u_n\}_{n=0}^{\infty}$ to l_2.
2. $0 < l < \infty$, then both the iterative process have same rate of convergence.

Definition1.2 [7]:- Let $\{z_n\}_{n=0}^{\infty}$ be the sequence in X. Then the iterative process $x_{n+1} = f(T, x_n)$ which converges to a fixed point q of T is said to be stable with respect to T if for $t_n = \| z_{n+1} - f(T, z_n) \|, n = 0, 1, 2, \ldots,$ we have $\lim_{n\to\infty} t_n = 0$ if and only if $\lim_{n\to\infty} z_n = q$.

Definition 1.3[2] :- Let H be a real normed linear space and $T : H \to H$ be a mapping. T is called a contraction mapping if $\| Tx - Ty \| \leq \delta \ \| x - y \|$, $\delta \in (0,1)$ and for all $x, y \in H$.

Lemma 1.4 [11]:- If l be a real number satisfying $0 \leq l < \infty$ and $\{\vartheta_n\}_{n=0}^{\infty}$be the sequence of positive numbers such that $\lim_{n\to\infty} \vartheta_n = 0$ and for $u_{n+1} \leq l\, u_n + \vartheta_n, n = 1,2, \ldots$ then $\lim_{n\to\infty} u_n = 0$.

2. Main Results

Theorem 2.1:- Let H be a closed convex subset of a real normed linear space X and $T : H \to H$ be a contraction mapping. Let $\{x_n\}_{n=0}^{\infty}$ and $\{u_n\}_{n=0}^{\infty}$be the sequence generated by the iterative process (1.4) and (1.5) respectively with sequence $\{\alpha_n\}_{n=0}^{\infty} \in [0,1]$ and $0 < \lambda < \alpha_n$. Then the Picard –Mann hybrid iteration process of implicit mid point rule as same rate of convergence as the Picard Kransoselskii hybrid iteration process of implicit mid point rule.

Proof : Let p be the fixed point of the mapping T. Then for iteration process (1.4) we have,

$$\| x_{n+1} - p \| = \| T\left(\frac{x_n+y_n}{2}\right) - p \| \leq \delta \ \| \frac{x_n+y_n}{2} - p \| \leq \frac{\delta}{2}(\| x_n - p \| + \| y_n - p \|) \quad (2.1)$$

Now

$$\| y_n - p \| = \| (1-\alpha_n)x_n + \alpha_n T\left(\frac{x_n+y_n}{2}\right) - p \|$$

$$\leq (1-\alpha_n) \| x_n - p \| + \alpha_n \| T\left(\frac{x_n+y_n}{2}\right) - p \|$$

$$\leq (1-\alpha_n) \| x_n - p \| + \alpha_n \delta \| \left(\frac{x_n+y_n}{2}\right) - p \|$$

$$\leq (1-\alpha_n) \| x_n - p \| + \alpha_n \frac{\delta}{2}(\| x_n - p \| + \| y_n - p \|)$$

$$\leq (1-\alpha_n) \| x_n - p \| + \alpha_n \frac{\delta}{2} \| x_n - p \| + \alpha_n \frac{\delta}{2} \| y_n - p \|$$

$$\left(1 - \frac{\alpha_n \delta}{2}\right) \| y_n - p \| \leq (1-\alpha_n) \| x_n - p \| + \alpha_n \frac{\delta}{2} \| x_n - p \|$$

$$\leq \left(1-\alpha_n+\frac{\alpha_n\delta}{2}\right) \| x_n-p \|$$

$$\Rightarrow \quad \| y_n-p \| \leq \frac{\left(1-\alpha_n+\frac{\alpha_n\delta}{2}\right) \|x_n-p\|}{\left(1-\frac{\alpha_n\delta}{2}\right)} \tag{2.2}$$

Using (2.2) in (2.1) we have,

$$\| x_{n+1}-p \| \leq \frac{\delta}{2}\left(1+\frac{\left(1-\alpha_n+\frac{\alpha_n\delta}{2}\right)}{\left(1-\frac{\alpha_n\delta}{2}\right)}\right) \| x_n-p \|$$

$$\| x_n-p \| \leq \frac{\delta}{2}\left(1+\frac{\left(1-\alpha_{n-1}+\frac{\alpha_{n-1}\delta}{2}\right)}{\left(1-\frac{\alpha_{n-1}\delta}{2}\right)}\right) \| x_{n-1}-p \|$$

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$$\| x_1-p \| \leq \frac{\delta}{2}\left(1+\frac{\left(1-\alpha_0+\frac{\alpha_0\delta}{2}\right)}{\left(1-\frac{\alpha_0\delta}{2}\right)}\right) \| x_0-p \|$$

Combining all the above inequalities we have

$$\| x_{n+1}-p \| \leq \left(\frac{\delta}{2}\right)^{n+1} \| x_0-p \| \prod_{k=0}^{n+1s}\left(1+\frac{\left(1-\alpha_k+\frac{\alpha_k\delta}{2}\right)}{\left(1-\frac{\alpha_k\delta}{2}\right)}\right) \tag{2.3}$$

Using the fact that $0<\lambda<\alpha_n$ in (2.3) we have

$$\| x_{n+1}-p \| \leq \left(\frac{\delta}{2}\right)^{n+1} \| x_0-p \| \prod_{k=0}^{n+1}\left(1+\frac{\left(1-\lambda+\frac{\lambda\delta}{2}\right)}{\left(1-\frac{\lambda\delta}{2}\right)}\right)$$

$$\Rightarrow \| x_{n+1}-p \| \leq \left(\frac{\delta}{2}\right)^{n+1} \| x_0-p \| \left(1+\frac{\left(1-\lambda+\frac{\lambda\delta}{2}\right)}{\left(1-\frac{\lambda\delta}{2}\right)}\right)^{n+1} \tag{2.4}$$

Let $\vartheta_n = \left(\frac{\delta}{2}\right)^{n+1} \| x_0-p \| \left(1+\frac{\left(1-\lambda+\frac{\lambda\delta}{2}\right)}{\left(1-\frac{\lambda\delta}{2}\right)}\right)^{n+1}$

By similar arguments we have for Picard- Kransoselskii hybrid iteration process of implicit mid point rule

$$\| u_{n+1}-p \| \leq \left(\frac{\delta}{2}\right)^{n+1} \| u_0-p \| \left(1+\frac{\left(1-\lambda+\frac{\lambda\delta}{2}\right)}{\left(1-\frac{\lambda\delta}{2}\right)}\right)^{n+1} \tag{2.5}$$

Let $\varphi_n = \left(\frac{\delta}{2}\right)^{n+1} \| u_0-p \| \left(1+\frac{\left(1-\lambda+\frac{\lambda\delta}{2}\right)}{\left(1-\frac{\lambda\delta}{2}\right)}\right)^{n+1}$ (2.6)

Now $$\frac{\vartheta_n}{\varphi_n} = \frac{\left(\frac{\delta}{2}\right)^{n+1} \|x_0-p\| \left(1+\frac{\left(1-\lambda+\frac{\lambda\delta}{2}\right)}{\left(1-\frac{\lambda\delta}{2}\right)}\right)^{n+1}}{\left(\frac{\delta}{2}\right)^{n+1} \|u_0-p\| \left(1+\frac{\left(1-\lambda+\frac{\lambda\delta}{2}\right)}{\left(1-\frac{\lambda\delta}{2}\right)}\right)^{n+1}} \tag{2.7}$$

Since $u_0 \neq p$ and $0<\| x_0-p \|<\infty$ and $0<\| u_0-p \|<\infty$ so by (2.7) we have $\lim_{n\to\infty} \frac{\vartheta_n}{\varphi_n} = l$, with $0<l<\infty$. Hence by definition (1.1) both iterative process have same rate of convergence.

Example 2.2:- Let H and and $T: H \to H$ be a contraction mapping defined by $\| Tx-Ty \| < \frac{x}{2}$. Then 0 is the only fixed point of the mapping T. Consider the initial approximation $x_0 = 0.1$. Let $\alpha_n = \frac{1}{2}$ and $\delta = \frac{1}{3}$.

Convergence pattern of iterative process (1.4) and (1.5) is shown in the Table 1.

x_n	Iteration (1.4)	Iteration (1.5)
x_0	0.1	0.1
x_1	0.04285714286	0.04375
x_2	0.01836734694	0.019140624
x_3	0.00787172012	0.00837402344
x_4	0.00337359434	0.00366363525
x_5	0.00144582615	0.00160284042
x_6	0.00061963978	0.00070124268
x_7	0.0002655599	0.00030679367
x_8	0.00011381139	0.00013422223
x_9	0.00004877631	0.00005872223
x_{10}	0.00002090413	0.00002569098
x_{11}	0.00000895891	0.00001123980
x_{12}	0.00000383953	0.00000491741
x_{13}	0.00000164551	0.00000215137
x_{14}	0.00000070522	0.00000094122
x_{15}	0.00000030224	0.00000041178
x_{16}	0.00000012953	0.00000018016
x_{17}	0.00000005551	0.00000007882
x_{18}	0.00000002379	0.00000003448
x_{19}	0.00000001020	0.00000001509
x_{20}	0.00000001020	0.00000000660
x_{21}	0.00000000437	0.00000000289
x_{22}	0.00000000187	0.00000000126
x_{23}	0.00000000080	0.00000000055
x_{24}	0.0000000034	0.00000000024
x_{25}	0.00000000015	0.00000000011
x_{26}	0.00000000006	0.00000000005
x_{27}	0.00000000002	0.00000000002
$\mathbf{x_{28}}$	**0**	**0**

Table 1 : Comparison of rate of convergence of among iterative processes

Table 1 shows that both the iterative process has same rate of convergence.

Remark 2.3:- In above example the value of parameters α_n and δ is taken different because if we take $\alpha_n = \delta$ then both the iterative process (1.4) and (1.5) become identical.

Theorem 2.4:- Let H be a closed convex subset of a real normed linear space X and $T : H \to H$ be a contraction mapping. Let $\{x_n\}_{n=0}^{\infty}$ be the sequence generated by the iterative process (1.4) with sequence $\{\alpha_n\}_{n=0}^{\infty} \in [0,1]$ and $\sum \alpha_n = \infty$. Then the sequence $\{x_n\}_{n=0}^{\infty}$ converges to fixed point of mapping T.

Proof:- Banach contraction principle guarantees the existence of unique fixed point p of mapping T. Now we prove the convergence of iteration (1.4) to p. Using (2.2) in (2.1) we have,

$$\| x_{n+1} - p \| \leq \frac{\delta}{2}(1 + \frac{\left(1-\alpha_n+\frac{\alpha_n\delta}{2}\right)}{\left(1-\frac{\alpha_n\delta}{2}\right)}) \| x_n - p \|$$

$$\leq \frac{\delta}{2}(\frac{1-\frac{\alpha_n\delta}{2}+1-\alpha_n+\frac{\alpha_n\delta}{2}}{1-\frac{\alpha_n\delta}{2}}) \| x_n - p \|$$

$$\leq \frac{2\delta}{2-\alpha_n\delta}(1 - \frac{\alpha_n}{2}) \| x_n - p \|$$

From the above inequality we have the following estimate

$$\| x_{n+1} - p \| \leq \frac{2\delta}{2-\alpha_n\delta}(1 - \frac{\alpha_n}{2}) \| x_n - p \|$$

$$\| x_n - p \| \leq \frac{2\delta}{2-\alpha_{n-1}\delta}(1 - \frac{\alpha_{n-1}}{2}) \| x_{n-1} - p \|$$

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$$\| x_1 - p \| \leq \frac{2\delta}{2-\alpha_0\delta}(1 - \frac{\alpha_0}{2}) \| x_0 - p \|$$

Combining the above inequalities we have

$$\| x_{n+1} - p \| \leq (2\delta)^{n+1} \| x_0 - p \| \prod_{i=0}^{n+1} \frac{1}{2-\alpha_i\delta} \prod_{i=0}^{n+1} 1 - \frac{\alpha_i}{2} \quad (2.8)$$

Since $1 - b \leq e^{-b}$ for all $b \in [0,1]$. Now $\{\alpha_n\}_{n=0}^{\infty} \in [0,1]$ so we have $0 < 1 - \frac{\alpha_i}{2} < 1$, hence (2.8) becomes

$$\| x_{n+1} - p \| \leq \| \leq (2\delta)^{n+1} \| x_0 - p \| \prod_{i=0}^{n+1} \frac{1}{2-\alpha_i\delta} e^{-\sum_{i=0}^{n+1}\frac{\alpha_i}{2}} \quad (2.9)$$

Taking limit $n \to \infty$ in (2.9) we obtain $\lim_{n\to\infty} \| x_n - p \| = 0$. Which is the desired result.

Okake [5] introduced the following contractive condition of rational expression

$$\| Tx - Ty \| \leq \frac{\psi(\|x-Tx\|+b\|x-y\|}{1+\kappa\|x-Tx\|} \quad (2.10)$$

Where $x, y \in H, b \in [0, 1], \kappa \geq 0$ and $\psi: H \to H$ be a monotone increasing function such that $\psi(0) = 0$.
Now we prove some fixed point results related to contractive condition (2.10).

Theorem 2.5:- Let H be a closed convex subset of a real normed linear space X and $T : H \to H$ be a contraction mapping satisfying condition (2.10). Let $\{x_n\}_{n=0}^{\infty}$ and $\{u_n\}_{n=0}^{\infty}$ be the sequence generated by the iterative process (1.4) and (1.5) respectively with sequence $\{\alpha_n\}_{n=0}^{\infty} \in [0,1]$ and $0 < \lambda < \alpha_n$. Then both the iterative processes has same rate of convergence.

Proof :- Let q be the fixed point of the mapping T. Now by (1.4) and (2.10) we have

$$\| x_{n+1} - q \| = \| T\left(\frac{x_n+y_n}{2}\right) - q \| = \| Tq - T\left(\frac{x_n+y_n}{2}\right) \|$$

$$\leq \frac{\psi(\|q-Tq\|+b\|\frac{x_n+y_n}{2}-q\|}{1+\kappa\|q-Tq\|} \leq \frac{\psi\|0\|+\frac{b}{2}[\|x_n-q\|+\|y_n-q\|]}{1+\kappa\|0\|} \leq \frac{b}{2}[\| x_n - q \| + \| y_n - q \|] \quad (2.11)$$

Now

$$\| y_n - p \| = \| (1 - \alpha_n)x_n + \alpha_n T\left(\frac{x_n+y_n}{2}\right) - q \|$$

$$\leq (1 - \alpha_n) \| x_n - q \| + \alpha_n \| T\left(\frac{x_n+y_n}{2}\right) - q \|$$

$$\leq (1 - \alpha_n) \| x_n - q \| + \alpha_n [\frac{\psi(\|q-Tq\|+b\|\frac{x_n+y_n}{2}-q\|}{1+\kappa\|q-Tq\|}]$$

$$\leq (1-\alpha_n) \parallel x_n - q \parallel + \frac{\alpha_n b}{2}[\parallel x_n - q \parallel + \parallel y_n - q \parallel]$$

$$\leq \left(1-\alpha_n + \frac{\alpha_n b}{2}\right) \parallel x_n - q \parallel + \frac{\alpha_n b}{2} \parallel y_n - q \parallel$$

$$\Longrightarrow (1-\frac{\alpha_n b}{2}) \parallel y_n - q \parallel \leq \left(1-\alpha_n + \frac{\alpha_n b}{2}\right) \parallel x_n - q \parallel$$

$$\parallel y_n - q \parallel \leq \frac{\left(1-\alpha_n+\frac{\alpha_n b}{2}\right)}{(1-\frac{\alpha_n b}{2})} \parallel x_n - q \parallel \qquad (2.12)$$

Using (2.12) in (2.11) we have

$$\parallel x_{n+1} - q \parallel \leq \frac{b}{2}\left[\parallel x_n - q \parallel + \frac{\left(1-\alpha_n+\frac{\alpha_n b}{2}\right)}{(1-\frac{\alpha_n b}{2})} \parallel x_n - q \parallel\right] \leq \frac{b}{2}\left[1+\frac{\left(1-\alpha_n+\frac{\alpha_n b}{2}\right)}{(1-\frac{\alpha_n b}{2})}\right] \parallel x_n - q \parallel$$

By the above inequality we have

$$\parallel x_{n+1} - q \parallel \leq \frac{b}{2}\left[1+\frac{\left(1-\alpha_n+\frac{\alpha_n b}{2}\right)}{(1-\frac{\alpha_n b}{2})}\right] \parallel x_n - q \parallel$$

$$\parallel x_n - q \parallel \leq \frac{b}{2}\left[1+\frac{\left(1-\alpha_{n-1}+\frac{\alpha_{n-1} b}{2}\right)}{(1-\frac{\alpha_{n-1} b}{2})}\right] \parallel x_{n-1} - q \parallel$$

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$$\parallel x_1 - q \parallel \leq \frac{b}{2}\left[1+\frac{\left(1-\alpha_0+\frac{\alpha_0 b}{2}\right)}{(1-\frac{\alpha_0 b}{2})}\right] \parallel x_0 - q \parallel$$

Combining above all the inequalities we have

$$\parallel x_{n+1} - q \parallel \leq (\frac{b}{2})^{n+1} \parallel x_0 - q \parallel \prod_{i=0}^{n+1}\left[1+\frac{\left(1-\alpha_i+\frac{\alpha_i b}{2}\right)}{(1-\frac{\alpha_i b}{2})}\right] \qquad (2.13)$$

Using the fact that $0 < \lambda < \alpha_n$, we have from (2.13)

$$\parallel x_{n+1} - q \parallel \leq (\frac{b}{2})^{n+1} \parallel x_0 - q \parallel \prod_{i=0}^{n+1}\left[1+\frac{\left(1-\lambda+\frac{\lambda b}{2}\right)}{(1-\frac{\lambda b}{2})}\right] \leq (\frac{b}{2})^{n+1} \parallel x_0 - q \parallel \left[1+\frac{\left(1-\lambda+\frac{\lambda b}{2}\right)}{(1-\frac{\lambda b}{2})}\right]^{n+1} \qquad (2.14)$$

Similarly by using above arguments we have for (1.5)

$$\parallel u_{n+1} - q \parallel \leq (\frac{b}{2})^{n+1} \parallel u_0 - q \parallel \left[1+\frac{\left(1-\lambda+\frac{\lambda b}{2}\right)}{(1-\frac{\lambda b}{2})}\right]^{n+1} \qquad (2.15)$$

Let $\rho_n = (\frac{b}{2})^{n+1} \parallel x_0 - q \parallel \left[1+\frac{\left(1-\lambda+\frac{\lambda b}{2}\right)}{(1-\frac{\lambda b}{2})}\right]^{n+1}$ and $\theta_n = (\frac{b}{2})^{n+1} \parallel u_0 - q \parallel \left[1+\frac{\left(1-\lambda+\frac{\lambda b}{2}\right)}{\left(1-\frac{\lambda b}{2}\right)}\right]^{n+1}$

Then $$\frac{\rho_n}{\theta_n} = \frac{(\frac{b}{2})^{n+1}\parallel x_0 - q\parallel\left[1+\frac{\left(1-\lambda+\frac{\lambda b}{2}\right)}{(1-\frac{\lambda b}{2})}\right]^{n+1}}{(\frac{b}{2})^{n+1}\parallel u_0 - q\parallel\left[1+\frac{\left(1-\lambda+\frac{\lambda b}{2}\right)}{\left(1-\frac{\lambda b}{2}\right)}\right]^{n+1}} \qquad (2.16)$$

Since $u_0 \neq q$ and $0 < \parallel x_0 - p \parallel < \infty$ and $0 < \parallel u_0 - p \parallel < \infty$ so by (2.16) we have $\lim_{n\to\infty} \frac{\rho_n}{\theta_n} = l$, with $0 < l < \infty$. Hence by definition (1.1) both iterative process have same rate of convergence.

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