International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Delineation of Integer Solutions to Non-Homogeneous Quinary Quintic Diophantine Equation $(x^3 - y^3) - (x^2 + y^2) + (z^3 - w^3) = 2 + 87T^5$

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ABSTRACT

This paper aims at determining varieties of non-zero distinct integer solutions to non-homogeneous quinary quintic diophantine equation

$(x^3 - y^3) - (x^2 + y^2) + (z^3 - w^3) = 2 + 87T^5$.

Keywords: quinary quintic , non-homogeneous quintic ,integer solutions

Introduction

It is well-known that the Diophantine equations, homogeneous or non-homogeneous, have aroused the interest of many mathematicians. In particular, one may refer [1-18] for quintic equations with three, four and five unknowns.

While collecting problems on fifth degree Diophantine equations ,the problem of getting integer solutions to the non-homogeneous quinary quintic diophantine equation given by $(x^3 - y^3) - (x^2 + y^2) + z^3 - w^3 = 2 + 87T^5$ [19] has been noticed. The authors of [19] have presented two sets of integer solutions to the quintic equation considered in [19]. The main thrust of this paper is to exhibit other sets of integer solutions to quinary non-homogeneous quintic equation given by $x^3 - y^3 - (x^2 + y^2) + z^3 - w^3 = 2 + 87T^5$ in [19] by using elementary algebraic methods. The outstanding results in this study of diophantine equation will be useful for all readers.

Method of analysis

The non-homogeneous quinary quintic diophantine equation to be solved is given by

$$(x^3 - y^3) - (x^2 + y^2) + (z^3 - w^3) = 2 + 87T^5 \quad (1)$$

The process of obtaining different sets of integer solutions to (1) is illustrated below :

Illustration 1

Introduction of the linear transformations

$$x = kv + 1, y = kv - 1, z = v + 1, w = v - 1 \quad (2)$$

in (1) leads to

$$(4k^2 + 6)v^2 = 87T^5 \quad (3)$$

which is satisfied by

$$v = 87^3(4k^2 + 6)^2 s^5 \quad (4)$$

and

$$T = 87(4k^2 + 6)s^2 \quad (5)$$

Using (4) in (2) , we get

$x = 87^3k(4k^2 + 6)^2s^5 + 1, y = 87^3k(4k^2 + 6)^2s^5 - 1,$
$z = 87^3(4k^2 + 6)^2s^5 + 1, w = 87^3(4k^2 + 6)^2s^5 - 1$ (6)

Thus, (5) & (6) represent the integer solutions to (1).

Illustration 2

Introduction of the linear transformations

$$x = u + 1, y = u - 1, z = ku + 1, w = ku - 1 \quad (7)$$

in (1) leads to

$$(6k^2 + 4)u^2 = 87T^5$$

which is satisfied by

$$u = 87^3(6k^2 + 4)^2s^5 \quad (8)$$

and

$$T = 87(6k^2 + 4)s^2 \quad (9)$$

Using (8) in (7) ,we get

$$x = 87^3(6k^2 + 4)^2s^5 + 1, y = 87^3(6k^2 + 4)^2s^5 - 1, z = 87^3k(6k^2 + 4)^2s^5 + 1, w = 87^3k(6k^2 + 4)^2s^5 - 1 \quad (10)$$

Thus , (9) & (10) represent the integer solutions to (1).

Illustration 3

Taking

$$x = u + 1, y = u - 1, z = 32ks^2 + 1, w = 32ks^2 - 1, T = 4s \quad (11)$$

in (1) , it is written as

$$u^2 = 256s^4(87s - 6k^2) \quad (12)$$

It is possible to choose the values of s so that the R.H.S. of (12) is a perfect square and hence the corresponding values of u are obtained.

Substituting these values of s, u in (11) , the respective integer solutions to (1) are found. The above process is exhibited below:

Let

$$\alpha^2 = 87s - 6k^2 \quad (13)$$

which is satisfied by

$$s_0 = k^2, \alpha_0 = 9k$$

Assume

$$\alpha_1 = h - \alpha_0, s_1 = h + s_0 \quad (14)$$

to be the second solution to (13). Substituting (14) in (13) and simplifying,

we have

$$h = 2\alpha_0 + 87$$

In view of (14) , one has

$$\alpha_1 = \alpha_0 + 87, s_1 = 2\alpha_0 + 87 + s_0$$

The repetition of the above process leads to the general solution to (13) as

$\alpha_n = \alpha_0 + 87n = 9k + 87n,$
$s_n = 2n\alpha_0 + 87n^2 + s_0 = 18kn + 87n^2 + k^2$

From (12) , it is seen that

$$u_n = 16(87n + 9k)(87n^2 + 18kn + k^2)^2$$

In view of (11) ,the integer solutions to (1) are given by

$$x_n = 16(87n + 9k)(87n^2 + 18kn + k^2)^2 + 1,$$
$$y_n = 16(87n + 9k)(87n^2 + 18kn + k^2)^2 - 1,$$
$$z_n = 32k(87n^2 + 18kn + k^2)^2 + 1,$$
$$w_n = 32k(87n^2 + 18kn + k^2)^2 - 1,$$
$$T_n = 4\,(87n^2 + 18kn + k^2).$$

Illustration 4

Taking

$$z = v + 1, w = v - 1, x = 108ks^2 + 1, y = 108ks^2 - 1, T = 6s$$

in (1) , it is written as

$$v^2 = (36)^2 s^4 (87s - 6k^2)$$

Following the analysis as in Illustration 3 , the corresponding integer solutions to (1) are given by

$$z_n = 36(87n + 9k)(87n^2 + 18kn + k^2)^2 + 1,$$
$$w_n = 36(87n + 9k)(87n^2 + 18kn + k^2)^2 - 1,$$
$$x_n = 108k(87n^2 + 18kn + k^2)^2 + 1,$$
$$y_n = 108k(87n^2 + 18kn + k^2)^2 - 1,$$
$$T_n = 6\,(87n^2 + 18kn + k^2).$$

Conclusion:

In this paper, we have made an attempt to find infinitely many non-zero distinct integer solutions to the non- homogeneous quintic equation with five unknowns given by $x^3 - y^3 - (x^2 + y^2) + z^3 - w^3 = 2 + 87T^5$. To conclude, one may search for other choices of solutions to the considered quintic equation with five unknowns and higher degree diophantine equations with multiple variables.

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