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Application of Bernoulli's Differential Equation to Rate of Dissolution

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A B S T R A C T

This study was about one of the real life application of Bernoulli's differential equation. In this paper, we have discussed about different types of differential equations, order, degree, general form of first order and first degree differential equation and its application in the field of chemistry particularly for finding the number of undissolved grams of a solute in a solution at any time't'. Further graphical interpretation was also given in this paper.

Keywords: Bernoulli's Differential Equation, Solvent, Solute, Time, Concentration of the solution

1. INTRODUCTION:

An equation involving dependent variables, independent variables and the differential coefficients (derivatives) of one or more dependent variables with respect to one or more independent variables is called a differential equation. Differential equations have remarkable ability to predict the world around us. They are used in a wide variety of disciplines, from physics, biology, economics, chemistry, and engineering. Basically, there are two types of differential equations. That are ordinary and partial differential equations. A differential equation is said to be an ordinary if it contains only one independent variable and ordinary derivatives with respect to this independent variable whereas the partial differential equation contains at least two independent variables and partial derivatives with respect to either of these independent variables. The order of a differential equation is the order of the highest derivative occurring in it. The degree of the highest derivative involved in an ordinary differential equation, when the equation has been expressed as a polynomial in the highest derivative by eliminating radicals and fraction exponents of the derivatives is called the degree of the differential equation. The first order and firs degree differential equation general form is dy/dx $= \boldsymbol{f(x,y)}$. One of the method of solving this first order and first degree differential equations is Bernoulli's differential equation. In this paper, we have been applied Bernoulli's differential equation to find out the number of undissolved grams of a solute in a solution at any time t.

2. BERNOULLI'S DIFFERENTIAL EQUATION:

An equation of the form $\frac{dy}{dx} + Py = Qy^n$,where P and Q are functions of x only, is called a Bernoulli's equation.

PROCEDURE TO SOLVE $\boldsymbol{\frac{dy}{dx} + Py = Qy^n}$:

Step 1: Divide by y^n

Step 2: Put $y^{1-n} = v$ so that $y^{-n}\frac{dy}{dx} = \frac{1}{1-n}\frac{dv}{dx}$

Step 3: Substitute these and write in the form $\frac{dv}{dx} + P'v = Q$

Step 4: Solve as a linear equation in v.

SOLVENT AND SOLUTE:

A liquid capable of dissolving another substance is known as a solvent,

and the dissolved substance is called the solute.

3. FORMULA FOR CONCENTRATION OF THE SOLUTION:

The concentration of the solution at any time't = solute / solvent ' at time 't'. If the concentration attains its maximum possible value then the solution is said to be saturated

NOTATIONS:

Let U denote the number of undissolved grams of a solute in a solution at time t. In many important cases, the rate at which U decreases with respect to t is proportional to the amount U at t and to the difference between the saturation concentration and the concentration at t.

$$\frac{dU}{dt} \propto U(saturation\ concentration - concentration)$$

$\frac{dU}{dt} = kU(saturation\ concentration - concentration)$ where k is the constant of proportionality.

STATEMENT OF THE PROBLEM: 100g of a certain solvent is capable of dissolving 50g of a particular solute. Given that 25g of the undissolved solute is contained in the solvent at time t=0 and that 10g dissolves in 2hour, find the amount U of the undissolved solute at any time t.

APPLICATION OF BERNOULLI'S D.E TO THE PROBLEM:

The saturation concentration is $\frac{50}{100}$

The concentration at time t is $\frac{25-U}{100}$

Thus, the differential equation of the problem is

$$\frac{dU}{dt} = kU(\frac{50}{100} - \frac{25-U}{100})$$

where proportionality constant k is negative.

$$\frac{dU}{dt} = kU(\frac{50-25+U}{100})$$

$$\frac{dU}{dt} = kU(\frac{25+U}{100})$$

$$\frac{dU}{dt} = \frac{kU}{4} + \frac{kU^2}{100}$$

$\frac{dU}{dt} - \frac{kU}{4} = \frac{kU^2}{100}$ ……(i) which is Bernoulli's Differential Equation

Dividing (i) by U^2,we get

$$\frac{1}{U^2}\frac{dU}{dt} - \frac{1}{U^2}\frac{kU}{4} = \frac{1}{U^2}\frac{kU^2}{100}$$

$\frac{1}{U^2}\frac{dU}{dt} - \frac{k}{4}\frac{1}{U} = \frac{k}{100}$ ……(ii)

Put $\frac{1}{U} = Y$ ……(iii)

Differentiating w.r.t to 't'

$$-\frac{1}{U^2}\frac{dU}{dt} = \frac{dY}{dt}$$

$\frac{1}{U^2}\frac{dU}{dt} = -\frac{dY}{dt}$……(iv)

From (ii), (iii) and (iv),we get

$$-\frac{dY}{dt} - \frac{k}{4}Y = \frac{k}{100}$$

$\frac{dY}{dt} + \frac{k}{4}Y = -\frac{k}{100}$ …(v) which is a linear differential equation in 'Y'.

Integrating Factor, I.F= $e^{\int\frac{k}{4}dt} = e^{\frac{kt}{4}}$

General solution of (v) is

Y (I.F) = $\int(-\frac{k}{100})$.(I.F) dt + c

$$Y\, e^{\frac{kt}{4}} = -\frac{k}{100}\int e^{\frac{kt}{4}}\,dt + c$$

$$Y\, e^{\frac{kt}{4}} = -\frac{k}{100}\frac{e^{\frac{kt}{4}}}{\frac{k}{4}} + c$$

$$Y\, e^{\frac{kt}{4}} = -\frac{1}{25}e^{\frac{kt}{4}} + c$$

$$\frac{1}{U}e^{\frac{kt}{4}} = -\frac{1}{25}e^{\frac{kt}{4}} + c$$

$$\frac{1}{U} = -\frac{1}{25} + ce^{-\frac{kt}{4}}$$

$$\frac{1}{U} + \frac{1}{25} = ce^{-\frac{kt}{4}}$$

$\frac{25+U}{25Uc} = e^{-\frac{kt}{4}}$ …………….(vi)

Put t=0 and U=25 in (vii), we get $c = \frac{2}{25}$

Substituting the value of c in (vi), we get $\frac{25+U}{2U} = e^{-\frac{kt}{4}}$ ………..(vii)

Again, put t=2 and U=25-10=15 in (vii), we get $e^{-\frac{kt}{4}} = \frac{4}{3}$

$$e^{-\frac{k}{4}} = \left(\frac{4}{3}\right)^{\frac{1}{t}} = \left(\frac{4}{3}\right)^{\frac{1}{2}}$$

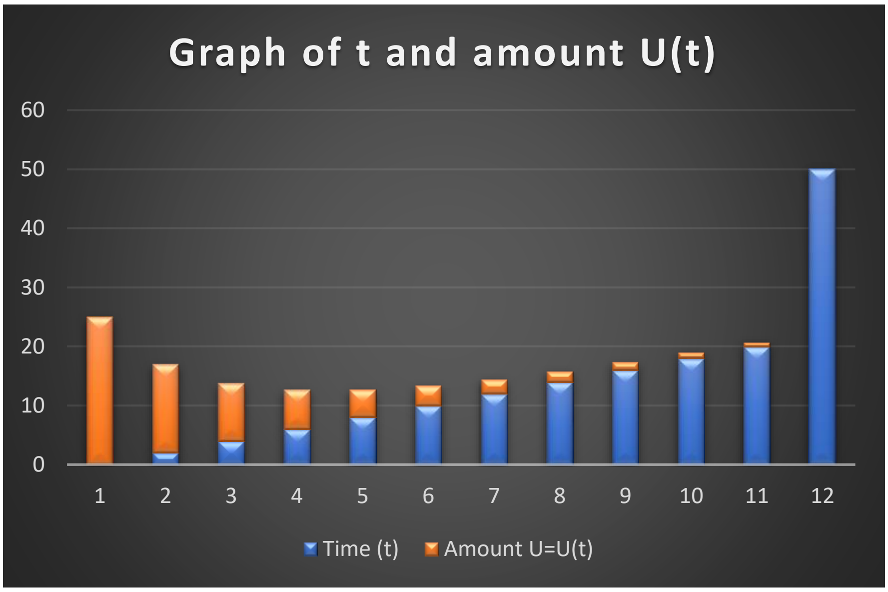
Hence $\frac{25+U}{2U} = e^{-\frac{kt}{4}}$ becomes $\frac{25+U}{U} = 2\left(\frac{4}{3}\right)^{\frac{t}{2}}$

Therefore, $U = U(t) = \frac{25}{2\left(\frac{4}{3}\right)^{\frac{t}{2}} - 1}$

4. TABULAR VALUES:

Time (t)	**Amount U=U(t)**
0	25
2	15
4	9.7826
6	6.6831
8	4.6983
10	3.3656
12	2.4420
14	1.7878
16	1.3173
18	0.9751
20	0.7243
50	0.0094
100	0.000007
200	0.000000000004

GRAPHICAL INTERPRETATION:



RESULTS:

From the table,the amount Q = 25,15,9.7826,6.6831, ……. are the number of grams of undissolved solute at time t = 0,2,4,6,8…… . and.Also graphically, we observed that, as t tends to infinity, U tends to 0.

Conclusion:

In this paper, we have seen that the application of Bernoulli's differential equation and was useful in mathematics and chemistry for instance in analysing problems involving rate of dissolution. This procedure was used for calculating number of grams of undissolved solute at any time t.

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