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The Non-Homogenous Biquadratic Equation with Four Unknowns $xy(x+y)+54zw^3=0$

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ABSTRACT:

The Non-homogenous biquadratic equation with four unknowns given by $xy(x+y)+54zw^3=0$ is analyzed for obtaining different sets of non-zero integer solutions through employing the linear transformations.

KEYWORD: Non-homogenous biquadratic, Biquadratic with four unknowns , Integer solutions.

INTRODUCTION:

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, biquadratic diophantine equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity [1-5]. In this context, one may refer [6-20] for various problems on the biquadratic diophantine equations with four and five variables. However, often we come across non-homogeneous biquadratic equations and as such one may require its integral solution in its most general form. It is towards this end, this paper concerns with the problem of determining non-trivial integral solutions of the non-homogeneous equation with four unknowns given by $xy(x+y)+54zw^3=0$is analyzed for obtaining different sets of non-zero integer solutions through employing the linear transformations.

NOTATION :

$$t_{3,n}=\frac{n(n+1)}{2}$$

METHOD OF ANALYSIS

The Non-homogenous biquadratic equation with four unknowns under consideration is,

$$xy(x+y)+54zw^3=0 \tag{1}$$

The process of obtaining non-zero integer solutions to (1) is illustrated below:

Illustration 1 :

Introduction of the linear transformations

$$\left.\begin{aligned} x&=3(u+v)\\ y&=3(u-v)\\ z&=u \end{aligned}\right\} \tag{2}$$

in (1) leads to

$$v^2-u^2=w^3 \tag{3}$$

Employing the method of factorization and performing some algebra , the following two sets of integer solutions to (1) are obtained :

Set 1:

$$x=24k^3+36k^2+18k+3, y=-3, z=4k^3+6k^2+3k, w=2k+1$$

Set 2 :

$$x=3k^2, y=-3k, z=t_{3,k-1}, w=k$$

Note 1:

The choice $u = kw, k \geq 0$ in (2) leads to the integer solutions to (1) given by

$$x = 3(a+k)(a^2-k^2), y = 3(k-a)(a^2-k^2), z = k(a^2-k^2), w = (a^2-k^2)$$

Note 2:

The choice $v = kw, k \geq 0$ in (2) leads to the integer solutions to (1) given by

$$x = 3(k+a)(k^2-a^2), y = 3(a-k)(k^2-a^2), z = a(k^2-a^2), w = (k^2-a^2)$$

Note 3 :

The choice $v = ku, k \geq 1$ in (2) leads to the integer solutions to (1) given by

$$x = 3(k+1)(k^2-1)\alpha^{3s}, y = 3(1-k)(k^2-1)\alpha^{3s}, z = (k^2-1)\alpha^{3s}, w = (k^2-1)\alpha^{2s}$$

Illustration 2:

Introduction of the linear transformations

$$\left.\begin{aligned} x &= (u+v) \\ y &= (u-v) \\ z &= u \end{aligned}\right\} \quad (4)$$

in (1) leads to

$$v^2 - u^2 = 27w^3 \quad (5)$$

which is expressed as the system of double equations as given below:

System	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
$v+u=$	$27w^3$	$9w^3$	$3w^3$	w^3	$27w^2$	$9w^2$	$3w^2$	w^2	$27w$	$9w$	$3w$	w
$v-u=$	1	3	9	27	w	$3w$	$9w$	$27w$	w^2	$3w^2$	$9w^2$	$27w^2$

Solving each of the above system of double equations ,the values of u, v, w are obtained. In view of (4) ,the respective values of $x, y, z$are found. For simplicity and brevity , the corresponding integer solutions to (1) are exhibited below :

Solutions from System I :

$$x = 27(2k+1)^3, y = -1, z = 108k^3 + 162k^2 + 81k + 13, w = 2k+1$$

Solutions from System II :

$$x = 9(2k+1)^3, y = -3, z = 36k^3 + 54k^2 + 27k + 3, w = 2k+1$$

Solutions from System III :

$$x = 3(2k+1)^3, y = -9, z = 12k^3 + 18k^2 + 9k - 3, w = 2k+1$$

Solutions from System IV :

$$x = (2k+1)^3, y = -27, z = 4k^3 + 6k^2 + 3k - 13, w = 2k+1$$

Solutions from System V :

$$x = 27k^2, y = -k, z = 13k^2 + +t_{3,k-1}, w = k$$

Solutions from System VI :

$$x = 9k^2, y = -3k, z = 4k^2 - k + +t_{3,k-1}, w = k$$

Solutions from System VII :

$$x = 3k^2, y = -9k, z = k^2 - 4k + +t_{3,k-1}, w = k$$

Solutions from System VIII :

$$x = k^2, y = -27k, z = -13k + t_{3,k-1}, w = k$$

Solutions from System I X:

$$x = 27k, y = -k^2, z = 13k - t_{3,k-1}, w = k$$

Solutions from System X:

$$x = 9k, y = -3k^2, z = -k^2 + 4k - t_{3,k-1}, w = k$$

Solutions from System XI:

$$x = 3k, y = -9k^2, z = -4k^2 + k - t_{3,k-1}, w = k$$

Solutions from System XII:

$$x = k, y = -27k^2, z = -13k^2 - t_{3,k-1}, w = k$$

Note 4 :

The choice $v = ku, k \geq 1$ in (5) leads to the integer solutions to (1) given by

$$x = 27^2(k+1)(k^2-1)\alpha^{3s}, y = 27^2(1-k)(k^2-1)\alpha^{3s}, z = 27^2(k^2-1)\alpha^{3s}, w = 27(k^2-1)\alpha^{2s}$$

CONCLUSION:

In this paper, an attempt has been made to determine non-zero distinct integer solutions to the non-homogeneous biquadratic equation with four unknowns given by$xy(x+y) + 54zw^3 = 0$. The researchers in this field may search for other sets of integer solutions to the equation under consideration and other forms of biquadratic equations with four or more variables.

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