

International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

The Indian Mathematical Philosophy and its Overlooked Connections: Fibonacci Numbers and their Fractal Properties

Vaishnnavi Purram^a, Dr. G. Srinivasa Rao^{b*}, Dr. Panem Charanarur^c

^a Student, Resonance Jr. College, West Marredpally, Hyderabad, India

^{b*}Assistant Professor, Department of Digital Forensics, School of Sciences, Malla Reddy University, Dhulapally, Hyderabad, Telangana, India

^c Assistant Professor, Department of Cyber Security and Digital Forensics, National Forensic Sciences University Tripura campus, Tripura, India

ABSTRACT

Numerous disciplines, from biology to physics to chemistry to technology, make use of the Fibonacci numbers, a basic integer sequence. The name "Fibonacci" was initially used in the 2nd century BCE, although it was coined by Italian mathematician Leonardo of Pisa. The name "Fibonacci Sequences" was first used in 1877 by French mathematician Edouard Lucas. Pingala's obscure laws inspired the Fibonacci sequence, whose limiting ratio is the golden mean. Ohm's "golden section" formula was first employed in 1835. Based on the Chandastra of Pigala and comments from the 12th century by Gopla and Hemachandra Sr, Sanskrit composer Virahka established a complete grasp of series development in prosody. Natural patterns such as those seen in rabbit breeding, leaf arrangement, pine cone packing, seed head packing, flower petal arrangement, and plant growth often follow the Fibonacci sequence. Due to widespread recursionism, they may be found in inanimate, biological, and cognitive systems. The Mahamrutyunjaya Mantra is a phrase from the Rugveda and Yajurveda that invokes Shiva's incarnation as Tryambaka, the Three-eyed One. The Fibonacci sequence is related with spiritual development, and yantras, which are diagrammatic representations of mantras, help resonance good energies and scatter bad ones.

Keywords: Acharya Pingala, Chandas Shāstra, Fibonacci numbers, Golden ratio, Indian text, Integer sequence, Meru Prastara, Pascal's triangle, Simatic patterns, Sophisticated mechanisms, Virahanka, Yantras

Whenever self-organization mechanisms are at work and/or minimal energy configurations are being expressed, Fibonacci numbers are present. We include some instances from the fields of biology, physics, astronomy, chemistry, and technology; nevertheless, this list is in no way complete. [1] The Fibonacci sequence is easily understood on its own. It suggests that the sequential number for an integer series beginning with 0 or 1 is the sum of the two previous numbers. $F_i=F_{i-1}+F_{i-2}$ with $F_1=1$ and $F_2=1$. Generally, Fibonacci series have two differences such as, one type of a series starts with 1 and the other starts with 0.

Type -1 Series: 1,1,2,3,5,8,13,21,34,55,89,144 up to infinity

Type -2 Series: 0,1,1,2,3,5,8,13,21,34,55,89,144 up to infinity

The rule of golden ratios is another mathematical principle that may be deduced from the Fibonacci sequence. This is essentially an observation that the ratio of each pair of consecutive Fibonacci numbers is around 1.618, the value often symbolized by the Greek letter phi (). The approximation of 1.618 improves as the size of the successive integers increase

 $\Phi = 1/(\Phi - 1)$

The Fibonacci sequence starts as an arithmetic progression and ends up being a geometric progression (Φ^n). In a way, we could call it quantized logarithmic scaling (in between +1 and x^2).

 $\phi = \lim_{i \to \infty} [f_i / f_{i-1}] = (1 + \sqrt{5})/2 \sim 1.618$

The ratios of Fibonacci numbers for the sequences starts with 1, the ratios approximate Fibonacci value of Phi =1.618.

Similarly, When,

 F_{N+1} is 2 and F_N is 1 then $(F_{N+1})/F_N$ will be 2

 $F_{N\!+\!1}$ is 3 and F_N is 2 then $(F_{N\!+\!1})\!/F_N$ will be 1.5000

 $F_{N\!+\!1}$ is 5 and F_N is 3 then $(F_{N\!+\!1})\!/F_N$ will be 1.6666

 $F_{N^{\rm +1}}$ is 8 and F_N is 5 then $(F_{N^{\rm +1}})/F_N$ will be 1.6000

- $F_{N^{\pm}1} \text{ is } 21 \text{ and } F_N \text{ is } 13 \text{ then } (F_{N^{\pm}1})/F_N \text{ will be } 1.6154$
- $F_{N^{+1}} \mbox{ is } 34 \mbox{ and } F_N \mbox{ is } 21 \mbox{ then } (F_{N^{+1}}) / F_N \mbox{ will be } 1.6190$
- $F_{N\!+\!1}$ is 55 and F_N is 34 then $(F_{N\!+\!1})\!/F_N$ will be 1.6176

 $F_{N\!+\!1}$ is 89 and F_N is 55 then $(F_{N\!+\!1})\!/F_N$ will be 1.6182

 F_{N+1} is 144 and F_N is 89 then $(F_{N+1})/F_N$ will be 1.6179 (The nearest approximation of Phi)

According to Richard Merrick's "Interference Theory", the Fibonacci series is found in the Harmonic and Damping components of everything that emerges from and returns to equilibrium.

1.1 Etymology and Origin

The Italian mathematician and physicist Leonardo of Pisa, who lived around the 1300s and is credited with developing the Fibonacci sequence, is where the term "Fibonacci" originates from. However, Acharya Pingala, who lived in India around the second century BCE, was the first person to write about what are now known as the "Fibonacci numbers". An Indian scientist by the name of Pingala is credited with being the first to recognise the Fibonacci pattern. Between the years 300 and 200 BCE, he wrote a treatise on language called The Chanda(s) Shastra. In it, he discusses how to employ long and short vowels appropriately. [2] Pingala is also the author of a book on the proper use of long and short sounds in linguistic expression. This was once known as Mt. Meru but is now more often referred to in the East as the Gopala-Hemachandra Number or the Virahanka Number. In Western culture, this pattern is known as the Fibonacci order. Edouard Lucas, a French physicist, came up with the term "Fibonacci Sequences" in 1877. However, the term wasn't really used until 1877. This was done in commemoration of Leonardo Pisano.

In addition to this, Fibonacci is credited for popularizing the usage of Indian numerals in Western culture. He said in his historical work Liber Abaci, which means "Book of Calculations" in English and is also called "Book of Calculations", that he wished to comprehend the Modus Indorum, which is the Indian style of calculating arithmetic. Liber Abaci is also known as "Book of Calculations". After the publication of the book Liber Abbaci in 1202, the Hindu-Arabic letters and number system became widespread across all of Western Europe. In addition to that, many of the mathematical procedures that are still in use today were first presented to the public. There are a total of 15 distinct components to this whole. This tale examines how Fibonacci came to comprehend "the method of the Indians", which is often referred to as the Modus Indorum [3].

"As my father was a public official away from our homeland in the Bugia customshouse established for the Pisan merchants who frequently gathered there, he had me in my youth brought to him, looking to find for me a useful and comfortable future; there He wanted me to be in the study mathematics and to be taught for some days. They're from a marvelous instruction in the art of nine Indian figures; the introduction and knowledge of the art pleased me so much above all else, and I learned from them, whoever has learned it, from nearby Egypt, Syria, Greece, Sicily, and Provence, and their various methods, to which locations of business I travelled considerably afterwards for much study, and from the assembled disputations. But, on the whole, the algorithm and even the Pythagorean arcs still constituted almost an error compared to the Indian method. Therefore, strictly embracing the Indian method and being attentive to the study of it, from my own sense adding some and some more still from the subtle Euclidean geometric art, applying the sum that I was able to perceive to this book, I worked to put it together in xv distinct chapters, showing certain proof for almost everything that I put in, so that further, this method perfected above the rest, this science is instructed to the eager and to the Italian people above all others, who up to now are found without a minimum. If, by chance, something less or more proper or necessary I omitted, your indulgence for me is entreated, as there is no one who is without fault and in all things is altogether circumspect".

Fibonacci was the son of a customs official, and when he was a child, he travelled all throughout North Africa with his father. Fibonacci is credited with developing the Fibonacci sequence. During his travels, Fibonacci gained an understanding of Arabic mathematics. When he went back to Italy, he immediately began doing all in his power to spread the news. As a direct consequence of this, mathematics, which had been dormant in Europe for a significant amount of time as a direct result of the influence of the Church, soon had a rebirth. He is considered to have been one of the pioneers who worked in the sector. In the latter part of the eighth century, Muhammad ibn Musa Al-Khwarizmi made his debut on the international scene. He was a scholar and jurist. Al-Khwarizmi travelled all the way to Baghdad under the reign of Al-Mamun. He was born in the safe haven known as Khwarazm, which is located in the modern-day city of Khiva in the country of Uzbekistan. He taught at the renowned academic institution known as Bayt al-Hikma, sometimes referred to as the House of Wisdom. This establishment is focused on research and translation. His compositions often make use of mathematical formulas originating from the Indus Valley. The author discusses the significance of this book in Kitab al-Jabr wa al-Muqabala, which is also known as The Book of Manipulation and Restoration.

It was in the seventh century CE that the Indus Valley system faced competition from various systems from other regions of the globe. In the seventh century, a Syrian monk and scientist named Severus Sebokht made the following statement: "I will not talk about the scientific achievements of Hinduism." This was one of the issues that were discussed all during the lecture. I'll simply state that in order to accomplish mathematics, they employ a set of nine signs that are more "subtle", "valuable", "and" "creative" than anything the Greeks or Babylonians ever thought of. I'm going to leave it at that for now. If you believe that understanding Greek automatically gives you access to the most cutting-edge scientific material, you should take the following into consideration.

"Teach what is easiest and most useful in math, such as what people always need in cases of inheritance, legacies, division, lawsuits, trade, and everything else they do with each other; or when it comes to measuring land, digging canals, doing geometric calculations, and other things of many different kinds". "Teach what is easiest and most useful in math". The objective of this lesson is to "teach what is easiest and most useful in math", and that is the focus of the session.

Al-Khwarizmi took the explanations that he had received in India in the form of poetry and transformed them into explanations that were written in prose [3]. Throughout the course of history, these responses have been repeated on several occasions. This essay became a popular place to begin one's education in what was eventually going to become the subject of mathematics.

1.2 The Indian origin

The ancient Indian literature, Chandas Shastra is one of many remarkable discoveries in the Indian mathematical universe. Written by Acharya Pingala (about 200 B.C., but maybe 450 B.C. if the many literary notes about him being the younger brother of the famous grammarian Panini are correct), this work not only created the binary numeric system, but also paved the way for the development of the Fibonacci sequence. Pingala is described as Panini's younger brother by Sadgurasisya in his commentary on Rganu Kramani (1187 A.D.). His possible birthplace on the west coast of India has sparked much conjecture.

'*Chandognanaidhim jaghana makaro velatate Pingalam*' is a line from the Panchatantra, and it provides indirect evidence that he lived in a coastal area. The metre expert Pingala was slain by a crocodile while bathing in the ocean [4]. When it comes to prosody, Pingala's formal theory is as useless as Panini's Sanskrit grammar. His method, however, was mathematical. Mathematical developments find resonance in new ideas. Pingala often employs recursion in his algorithms, continuing a lineage that began with Panini (c. 600 BCE) and continued through Aryabhata (c. 500 CE) and Madhava (c. 1400 CE). Pingala's approach uses words meaning "Laghu" (light) and "Guru" (heavy) to represent the binary numbers 0 and 1, respectively. This ground-breaking idea has been used in many other areas, including music, poetry, and even contemporary Tabla. According to the essay, a poet may craft a beautiful poem by carefully considering the number of different permutations of the Laghu and Guru syllables inside each shloka. Pingala's syllabary has an unexpected relationship to the Fibonacci sequence, which may be seen in its underlying numerical patterns. Fibonacci's sequence is reflected in the total number of permutations for each "shloka". A shloka with a single beat, for instance, might be represented by the number zero (0). The combinations follow the Fibonacci sequence as the number of beats grows, starting with 00 and 1 for 2 beats, 000 and 01 for 3 beats, and so on. There are five possible permutations for a four-beat shloka (0000, 001, 100, 010, and 11). There are eight possible options for a shloka with five beats (00000, 1000, 110, 0001, 011, 0010, 0100, 101). The surprising connection between ancient Indian poetry and a well-known mathematical sequence is a fascinating aspect of Pingala's work.

When Narayana was writing in Ganitakaumudi about the 14th century, he reproduced virtually precisely Pingala's method for summing geometric series. The recursive method developed by the Indian mathematician Pingala has been replaced by the current formula developed by Sridhara. This formula is used to calculate the binomial coefficients. The prosodists continued to use Pingala's method in their work. Sarngadeva improved the algorithms devised by Pigala so that they could handle rhythms with four different kinds of beats (Sagtaratnakara, about 1225 C.E.): druta, laghu, guru, and pluta, each of which had lengths of 1, 2, 4, and 6, known as Druta, Laghu, Guru and Pluta respectively. On the other hand, Narayana made substantial development in Ganitakaumudi with a number of different series and the applications of those series. The precise manner in which Pingala determined nCk is clouded in mystery. As documented by Halayudha in the tenth century C.E., Pingala's final sutra, "Pare Purnamiti", proposes the construction of Meru Prastara (what is now known as "Pascal's triangle") for the purpose of calculating nCk. However, with the exception of Virahanka, none of the authors (from Bharata the 1st century C.E. onwards) until Halayudha mention such a structure or even use the title Meru Prastara for the architecture that they do describe. Halahyudha is the only author to do so. It's odd, especially when you realize that other authors have been using and improving upon Pigala's ideas for almost a thousand years already. In 1933, Alsdorf argues that Albrecht Weber misread Halayudha and that Weber's assertion is without merit. In 1835, Weber responded to Halyudha's interpretation by stating unequivocally, "that our author (Pigala) may have had in mind something like Meru Prastara does not follow from his words in any way." He then suggests, without offering any evidence to support his claim, that the last repetition of the sutra "Pare Purnam" is an afterthought and was inserted in order to make a connection to the later-invented Meru Prastara. Several writers, starting with Bharata, describe the same triangle. They do not fill the triangle working from the bottom up; rather, they work diagonally from left to right as they fill the triangle. The Suchi ("needle") and Meru ("mountain") Prastara [5] structures both end up producing a triangle, despite the fact that they are relatively distinct from one another in terms of the algorithms that they use. This is known as the "Maatra Meru" or the "Sumeru," and it is a pyramid consisting of stacked numbers that averages out. His pyramidal design, which is evocative to Pascal's triangle (which was invented centuries later), demonstrates that Pingala understood the importance of the golden ratio, which is proved by the fact that he used it. Pingala exhibits his great knowledge of mathematics by illustrating how proportions that are very near to one another tend to converge to the golden ratio. Pingala's Chandas Shastra served as a foundation for the subsequent scholarly work that came after it. The music of Carnatic is based on the Fibonacci sequence, which is a basic notion.

In his theory of binomial coefficients, the mathematician Varahamihira, who flourished in the fifth century, refers to Pingala as an important source. In the commentary of Halayudha, which dates back to the ninth century, Meru Prastara is first mentioned. As a result, Pascal's triangle may also be referred to by its alternative name, the Halayudha triangle. Virahanka, who lived some 700 years before the common era, offers a clear and straightforward description of the Fibonacci sequence. His original work has been lost, and much of what is known about him now comes from a quotation that was found in the works of a mathematician called Gopala who lived in the 12th century. Chandoratnakara by Ratnakarashanti, which was composed in the 11th century, is a further example. The publications of Pingala and Varahamihira were pertinent to their professions, but Acharya Hemchandra, a Jain scholar who served at the court of the Chalukya ruler Kumarapala in the 12th century, produced a detailed commentary on the Fibonacci numbers. Acharya

Hemchandra was known for his work on the Fibonacci numbers. We are now around 50 years before the time of Fibonacci [6]. It is also known as Pascal's triangle (Meru-Prastara, which literally translates to "steps of Mount Meru", while Bhaskara II in his Lilavati, which was written in 1150, referred to it as Meru-Khanda, which means "portion of Mount Meru").

The sum of the shallow diagonals on Mount Meru is equal to the golden mean, which is the limiting ratio of the Fibonacci sequence. Pingala's abstruse regulations were eventually made clear by later commentators such as Kedara (who lived in the seventh century) and Halayudha (who lived in the tenth century). Virahanka, a mathematician from the seventh century, arranges the numbers 3, 5, 8, 13, and 21 in a certain sequence. Gopala, who lived about the year 1135, and Hemachandra Sri, a polymath who lived between 1089 and 1172, offered more explanation on the number notions. Hemachandra (the spelling that is most often used) was contemplating the topic, "If each line takes the same amount, what combination of short (S) and long (L) 3 syllables can it have?" This is based on the hypothesis that pronouncing large syllables takes twice as much time as pronouncing short syllables. In a broad sense, there are F(n) many alternative configurations for a line that must be completed in n different time units.

In his literature around the year 1150 AD, Hemachandra made this argument quite obvious. Ganitakaumudi (1356) was written by Narayana Pandita, and it focuses on the study of each word in an additive sequence. This sequence is one in which the last q words are added together. He explains the paradox in the following manner: "Every year a cow has a calf. When the calves reach the age of three, they have reached maturity as young adults and can start breeding. I am curious as to how many calves a single cow would be able to produce over the duration of twenty years, so please enlighten me". [7] This will become clearer as we go through the series.

Virahanka's The real challenge is counting the possible poetry metres with eight beats, where the syllables may be either short (lasting for one beat) or long (lasting for two beats). For example: (Shardulavikridita) 19 syllables, 15 beats = L L L S S L S L S L S L L S L L S L.

"ya kun den du tu sha r ha r dha va la ya shubh r vas tra vru ta ya vee na va ra dan d man di ta k ra ya shwe t pad ma s na Aa ji chya ja wa lii gha dya l ka sa lay aa he ch mat kaa ri ka De ii the vu ni te ku the a ju n hii na hi ku na tha u ka"

To determine whether a word has a long or short last syllable, as suggested by Virahanka's solution, one just needs to use the Pingala approach. [8] Surprisingly, however, neither mathematician provided any prerequisites or explanations for the solution, suggesting that this information was previously common knowledge and therefore indicating that the series was not a new discovery.

Martin Ohm seems to have coined the phrase "golden section" (Goldene Schnitt) in the 1835 edition of his textbook Die Reine Elementar-Mathematik. James Sulley, writing on aesthetics for the 9th edition of the Encyclopedia Britannica in 1875, is credited for popularizing the phrase in English.

1.3 The formulation of series

Virahāňka (viraha = separation, aňka = mark) is believed to have lived in the 6th or 7th century. His work on prosody builds on the Chandas Shastra of Pińgala (4th to second century BCE) and was the basis for the 12th-century commentaries by Gopāla and Hemacandra Sūrī (1089–1172). Although the sequence is implicit in the Meru, Prastāra of Pińgala, which provides considerable historical background), it is reasonable to call the sequence after Virahāňka since he explicitly describes it in his Prakrit work Vrttajātisamuccya, and provides the justification [9].

A long syllable is indicated by the letter G (for "Guru") and a short one by the letter L (for "Laghu").

Sanskrit is home to two distinct types of metre:

Aksharachanda (1): The number of syllables in each of these is a criterion for classification. The number of syllables is all that's needed to define the Vedic akarachanda, also known as Chanda. The later aksharachanda, Vttachanda, is divided into four stanzas (psda), with each stanza having a unique pattern of long and short syllables.

Those are quantified by how many Matras they hold. Simply said, an Matra is a unit of time. One Matra is used for a single syllable, whereas two are used for two syllables. These metres come in two varieties:

(a) Ganachanda: Metres in which one foot (Gana) is equal to a certain amount of Matras.

(b)Matrachanda: When just the total number of Matras is given, the metre is called an Matrachanda.

Only the Vrttachanda is processed by Pingala's algorithms. One may choose from one of three distinct varieties. Sama (equal) is the form in which the lengths of the syllables in each of the four feet are the same. The short and long syllables in each foot of Ardhasama (half-equal) are arranged differently in the odd feet than in the even feet, but in every other pair of feet. Vishama (unequal) refers to shapes that are neither sama nor Ardhasama. While Pingala did note the use of Ganachanda and Matrachanda in prosody, these techniques had not yet reached their full potential until they were further refined by Prakrta prosodists. The combinatorics of Ganachanda and Matrachanda have been discussed by every prosodists following Pingala. The following six formal issues and their answers (known as Pratyayas) have historically been identified by Sanskrit prosodists. (Pingala has not given them any names.)

Name	Literal Translation	Functions		
Prastara	Spread	Systematically lists all theoretically possible forms of a meter with a fixed number of syllables		
Nastam	Annihiliated, lost	Recovers the form of a meter when its serial number in the list is given		
Uddistam	Indicated	Determine the serial number of a given form		
Lagakriya	Short-long exercise	Calculates the number of forms with a specified number of short syllables or long syllables		
Sankhya	Count	Calculates the total number of theoretically possible forms of a meter		
Adhvayoga	Space measure	Determines the amount of space needed to write down the entire list of the forms of a meter		

Sankhyā: In his sixth method, Pingala employs recursion to get the total number of metre types, Sn. He points out that there are twice as many n-syllable metre forms as there are (n-1) syllable metre forms.

Sn can be found by doubling it over and over again, but Pingala provides a recursive approach for finding 2^n based on the assumption that if n is even, $2^n = \{2^n(n/2)\} ^2$, and if n is odd, then $2^n = 2^2(n-1)$.

Pingala lays forth the scenario as follows: |tvadardhe tadguitam/ dvirardhe/ rpe nyam/ dvi nye/

"two in the case of half. (If n can be halved, write 'twice'.) In case one (which must be subtracted in order to halve), write 'zero'. (going in reverse order), twice if 'zero'. In the case where the number can be halved, multiply by itself (that is, square the result)."

Consider the case when n = 6. Build the first two columns of the table below, then build the third by working your way back up:

When n=6 twice (2.2²)²=64

When n=3 zero $(2.2^2)=8$

When n=2 twice 2²=4

When n=1 zero 2=2

Therefore, there are 64 distinct forms of length 6, or S6. Later, prosodists introduced still another approach: Sn = lagakriya's outputs summed together.

Importantly, Virahanka used this recursion to calculate the total number of Matra-meter forms: Sn = Sn-1+Sn-2, which of course yields the Fibonacci sequence. [5]

If you follow the comments step by step, you'll end up with Pascal's triangle (Meru Prastara) and the Fibonacci sequence (Matra Meru) [10].

Sutra -1

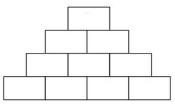
अतोऽनेकद्वित्रिलघुक्रियासिद्यर्थयावदभिमतंप्रथमप्रस्तारवन्मेरुप्रस्तारंदर्शयति- परेपूर्णमिति॥८।३५॥

उपरिष्टादेकंचतुरस्रकोष्ठंलिखित्वातस्याधस्तादुभयतोऽर्धनिष्क्रान्तंकोष्ठकद्वयंलिखेत्।

तस्याप्यधस्तात्रयंतस्याप्यधस्ताच्चतृष्टयंयावदभिमतंस्थानमितिमेरुप्रस्तारः॥

Meaning: Below the top of one square (Prathama Prasthava), two squares are drawn, with half of each square extending to the left and right. To make the necessary pyramid shape, three squares are drawn below it, then four squares, and so on.

Thus Development of Pingala's Meru Prastara is achieved.



Sutra -2

तस्यप्रथमेकोष्ठेएकसंख्यांव्यवस्थाप्यलक्षणमिदंप्रवर्तयेत्।तत्रपरेकोष्ठेयद्वृत्तसंख्याजातंतत्पूर्वकोष्टयोःपूर्णंनिवेशयेत्। तत्रोभयोःकोष्ठकयोरेकैकमंगंदद्यात्, मध्येकोष्ठेतुपरकोष्टद्वयांकमेकीकृत्यपूर्णनिवेशयेदितिपूर्णशब्दार्थः। Meaning: The first box must include a 'l' sign. After that, you'll need to place one in each of the squares formed by the second line. Then, two should be placed on the furthest squares in the third row. Then, place one on each of the two corners of the fourth row.

Sutra -3

चतुर्थ्यांपंक्तावपिपर्यन्तकोष्ठयोरेकैकमेवस्थापयेत्।मध्यमकोष्ठयोस्तुपरकोष्ठद्वयांकमेकीकृत्यपूर्णंत्रिसंख्यारूपंस्थापयेत्।

उत्तरत्राप्ययमेवन्यासः ।तत्रद्विकोष्ठायांपंक्तौएकाक्षरस्यविन्यासः ।तत्रैकगुर्वेकलघुवृत्तंभवति।तृतीयायांपंक्तौद्वयक्षरस्यप्रस्तारः ।

Meaning: The total from the two squares above must be entered into the centre square of the third line. Three are put in the centre squares of the two rows above. This procedure is repeated for subsequent squares. This is shown by the fact that the second line provides an extension of syllable combinations; the third line follows suit for words with two syllables; the fourth line does the same for words with three syllables; and so on.

Following these procedures yields the famous Pascal's triangle, also known as Pingala's Meru Prastara.

Using this method, Pascal's triangle, or Pingala's Meru Prastara, may be constructed.

			1	Ê			
		1		1			
	1	Ľ.	2	2	1		
1		3	3	3	3	1	

Sutra-4

तत्रैकंसर्वगुरु,द्वेएकलघुनी, एकंसर्वलघ्वितिकोष्ठक्रमेणवृत्तानिभवन्ति॥चतुर्थ्यांपंक्तौत्यक्षरस्यप्रस्तारः।

तत्रैकंसर्वगुरुत्रीण्येकलघूनित्रीणिद्विलघूनिएकंसर्वलघु॥तथापंचमादिपंक्तावपिसर्वगुर्वादिसर्वलघ्वन्तमेकद्वयादिलघुद्रष्टव्यमिति॥

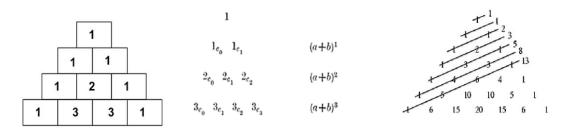
Meaning: According to the preceding lines of the Halayudha commentary, the second line of the Meru Prastara is the development of a one-syllable metre. A meter's third line is the result of expanding two syllabi, while the meter's fourth line does the same for three syllabi. Similar expansions of four curricula are illustrated for the fifth line of the metre. Therefore, the Pascal triangle, Binomial coefficients, emerged as a result of attempts to arrange all possible patterns of short and long syllables along a line with "n" syllables.

Thus, Pingala's Meru Prastara results in a growth in the Binomial coefficient.

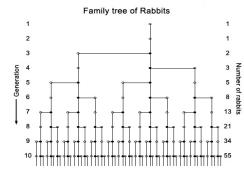
Finding the number of sequences of a certain total length when long syllables are allocated with length 2 and short syllables are assigned with length 1 led to the development of the Maatra Meru, also known as the Fibonacci numbers. Maatra Meru, often known as the Fibonacci sequence, is obtained by adding the numbers along the diagonal of the Meru Prastara, which is 1, 1, 2, 3, 5, 8, 13,....

Therefore, the Fibonacci series is equal to the sum of the Meru Prastara's shallow diagonals.

Pascal's triangle, the Binomial expansion, and the Fibonacci sequence are all described in Pingala's Chandas Shastra sutra. Numerous recursive algorithms are also investigated, as well as the calculation of Binomial coefficients, the use of repeated partial sums of sequences, the formula for summing a geometric series, and many more. [5]

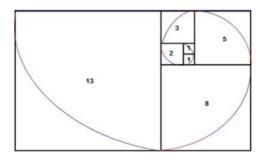


Fibonacci series in connection with nature and routine



(Image source: https://fractalfoundation.org/OFC/OFC-11-1.html)

Let's check out some examples of Fibonacci sequences in the wild. Take, for example, the practice of raising rabbits. Consider a basic set of rabbit reproduction principles, such as these: Rabbits can only reproduce with another rabbit. Once rabbits reach sexual maturity after two months, they reproduce once each month. Obviously oversimplified, a litter of rabbits will consist of a male and a female. The graphic below, read from top to bottom, depicts the evolution of the rabbit family tree. Two bunnies, not just one, are represented by each dot. Dots that are empty reflect rabbit pairings who are still too young to breed successfully.



(Image source: https://bfpcs-journals.rtu.lv/article/view/bfpcs.2016.007)

By removing a square from the middle of a golden rectangle, it may be transformed into another golden rectangle. By continuing in this fashion and sketching circular arcs, a curve resembling a naturally occurring logarithmic spiral is generated. Many plants employ these numbers under ideal conditions, such as leaf arrangement around the stem, pine cone packing, seed head packing, and flower petal arrangement. Because physical factors lie at the root of this natural occurrence of the golden mean, it is intriguing to study the structural basis for why it is a cognitively appealing percentage.

The pleasantness of the golden mean would be obvious if the stimulation inside the brain corresponded to an input distributed by a spiral function. However, since neurophysiological systems are not purely two-dimensional, the spiral would contain a third-dimensional component that would deviate from the "normative" golden mean and generate many other comparable numbers, such as Wilson's Meru 2 through 9 or other more generic numbers. Because everyone's anatomy is distinct, there is a large variety of alternatives for what defines an optimum arrangement.

However, as a result of ubiquitous recursionism [11], non-living, living, and cognitive systems all exhibit comparable behaviors. The logarithmic spiral is used in snail and nautilus shells and may also be found in the inner ear's cochlea. It's also reflected in the horns of certain goats and the structure of some spider webs. If you look at the length of our fingers from the tip of the base to the wrist, you'll see that each segment is proportionally bigger than the one before it. Technical analysts utilize "Fibonacci retracement" to inform their forecasts of future market buying and selling activity. The introduction of the sequence in the world of finance is another evidence of its ability to grab the human imagination, even if utilizing these numbers to anticipate market movements is far less certain than using them to compute sunflower seed patterns. The Fibonacci retracement levels are horizontal lines based on the Fibonacci sequence that show where support and resistance are expected to be encountered. The Fibonacci sequence, fractal branching, and the golden ratio all play significant roles in the organization of these plant parts. Many seed bracts exhibit the Fibonacci sequence and golden spirals in explicit detail. White pine (Pinus strobus) has a 3:5 order, yellow pine (5:8), and Oregon pine (8:13) spirals. The flower's head follows the same Fibonacci sequence as the rest of the flower. in most cases, seeds germinate at the core and spread outward to cover the rest of the area. Pineapples and cauliflower both include spiral formations but to a lesser extent. There are three on a lily, five on the buttercups (seen at left), twenty-one on the chicory, thirty-four on the daisy, and so on. Each petal is positioned at 0.618034 per turn (out of a 360° circle), providing for the optimum potential exposure to sunlight and other variables; this optimal packing arrangement, determined by Darwinian processes, manifests as phi in petals.

The Fibonacci sequence emerges in the creation and splitting of tree branches. A main stem expands until it splits into two growth points. One new stem split into two, while the other becomes dormant. These branching patterns are repeated with each new stem. Sneezewort is one example. Algae and roots exhibit this pattern.

Our bodies, too, have Fibonacci proportions. The golden ratio is the distance between the navel and the floor, as well as the distance between the head and the navel. Dolphins (whose eyes, fins, and tails fall at Golden Sections), starfish, sand dollars, sea urchins, ants, and honey bees all show similar tendencies.

The Golden Ratio may be seen in both human and non-human faces. The lips and nose are golden components located between the eyes and the bottom of the chin. Proportions are comparable from the side and even the spiral eye and ear.

In other interesting ways, honey bees follow Fibonacci. The most obvious example is dividing a colony's females by males (females usually outnumber men). 1.618 is the most common response. A pattern may also be seen in honey bee family trees. Males have one (female) parent, while females have two. Males had two, three, five, and eight grandparents, great-grandparents, gr-gr-grandparents, and gr-gr-gr-grandparents, respectively. Females use the same pattern with 2, 3, 5, 8, 13, and so on. As previously stated, bee physiology closely follows the Golden Curve. Fibonacci has an impact on the microcosm. DNA double helix spiral cycles are 34 angstroms long and 21 angstroms wide. Fibonacci numbers 34 and 21 have a ratio of 1.6190476 which is near to Phi, 1.6180339.

The layout of the keyboard also displays this in a visually simple way. According to Richard Merrick, the real magic lies in the acoustic interference pattern found between the root note and another sweeping tone (from root to octave), creating 12 gaps in the octave that can be described mathematically using Fibonacci numbers. Another variation of this system is the 22-note system in India, which is really an extension of the 12-note system. The Indian system uses a 5-limit JI system with 10 alternate derivations of all 12 notes except C and G, using higher powers of 2, 3, and 5. This system (of 22 'shrutis, as they are known) uses a syntonic comma of 81/80 = 22c to generate all the alternate derivations. This is one such example of a 22-Shruti system. So, a scale typically made of 7 notes is picked from a set of 22 notes.

1.4 The Indian Philosophy of devotion and its connection with Fibonacci series

The Mahamrityunjaya Mantra, also known as the Rudra Mantra or Tryambakam Mantra, is a verse of the Rugveda (7.59.12). It is addressed to Tryambaka, "The Three-eyed One", an epithet of Rudra, who is identified with Shiva in Shaivism. The verse also recurs in the Yajurveda. Maha Mrutyunjaya Mantra, by definition, protects us from our deepest fear of death.

ॐ त्र्यम्बकं यजामहे सुगन्धिं पुष्टिवर्धनम्।

उर्वारुकमिव बन्धनान्मृत्योर्मुक्षीय माऽमृतात् 🏻

Meaning:

1. Om, We Worship the Tryambaka (the Three-Eyed One),

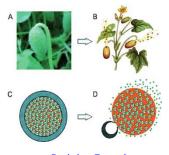
2. Who is Fragrant (as the Spiritual Essence), Increasing the Nourishment (of our Spiritual Core);

3. From these many Bondages (of Samsara) similar to Cucumbers (tied to their Creepers),

4. May I be Liberated from Death (Attachment to Perishable Things) So that I am not separated from the perception of Immortality (Immortal Essence pervading everywhere).

The reason I associate this mantra with the Virahanka numbers is due to many reasons.

"Urvarukam" is a species of cucumber known as the Squirting Cucumber (Ecballium elaterium), although I would assume it might be its Indian relative, the Spiny Gourd (Momordica dioica). The interesting bit about this is that the arrangement of seeds in this cucumber is the Fibonacci pattern, which, although difficult to visualise in a 3D view, one can understand after looking at a few pictures. Seeds are a metaphor for life, reproduction, and immortality; another fascinating relationship is that the seeds of this species follow the Fibonacci spiral.



Squirting Cucumber

(Image source: https://www.researchgate.net/figure/A-A-picture-of-squirting-cucumber-huanbao-c1-200905-7861html-B-A-schematic_fig1_248843 402)

(I think that the dispersal of seeds when the fruit bursts is the same as the Fibonacci spiral, with the spin of the fruit due to propulsion being the cause.)



Spiny Gourd

(Image source: https://cartkochi.com/product-detail/kantolaspiny-gourd-500g--0407-Z1BYSFhnZGhsMTIGdkFjSmJhN0NDUT09)

For the Seed shape quantification in the order Cucurbitales, one of the Models used was the outline of Fibonacci's spiral. an ellipse with the ratio x/y = 1.76 was used.

Image and Information source



Ecballium elaterium

(Image Source: https://digital.csic.es/bitstream/10261/161946/1/Seed%20shape%20quantification%20in%20the%20order%20Cucurbitales.pdf)

Most Cucurbitaceae seeds, such as Ecballium elaterium, adapt better to an ellipse with an x/y ratio of 1.76. (This is quite close to 1.618.) [12]

Saundarya Lahiri by Adi Sankaracharya is an authoritative text on Yantra. Yantra means "to control, kerb, bind, or influence" in Sanskrit. They may also be seen as diagrammatic representations of various Mantras, and so aid in the resonance of good energies, the dispersal of bad energies, and the spiritual elevation of individuals. Yantra is a Sanskrit word that means "chant that awakens energy when placed

anywhere in the environment around you."



Fig.1. Maha Mrutyunjaya Yantram and Sri Rudra Yantram

The geometry of Maha Mrityunjaya Yantra is very interesting, noting that the prominent numbers starting from the centre are 1, 3, 5, 8, all Fibonacci numbers. Not so surprisingly, the rituals related to this yantra are advised to be performed on a Trayodashi, i.e., the 13th day of the lunar fortnight in the Hindu calendar. 13 is also a Fibonacci number. This Tithi is loved by Lord Shiva, which is why the natives born on this Tithi specifically worship Lord Shiva. Observing Pradosh fast on Trayodashi Tithi holds great importance. This fast is observed to get a male child relief from debt, happiness, health, etc.

Other interpretations of the Mrityunjaya yantra have a similar pentagram-based approach, like the Rudra yantra and several Kali yantras, such as Guhya Kali and Shyama Kali yantras.

In addition, the central pentagram in the Yantra plays a big role in its meaning.

We know the famous Shiva mantra. It is called Siva Panchakshara, Shiva Panchakshara, or simply Panchakshara, meaning the "five-syllable" mantra (viz., excluding the Om), and is dedicated to Shiva. This Mantra appears as 'Na, 'Ma, 'Śi' 'Vā' and 'Ya' in the Shri Rudra Namakam Chamakam.

ॐ नमः शिवाय

Frederick Bunce explained the geometry and meaning behind several yantras; his explanation of this yantra is evident in the connection revealed. [13] "Mrit-Sanjivani is one of the companions of the Ashvini-Devatas. The Ashvini-devalas (aka Ashvinikumaras) are twin deities who were important Devas during the Vedic period. They are the personification of heaven and earth and are the heavenly physicians. Since the employment of the Mrit-Sanjivani Yantra is for the protection from disease, the use of this demi-deity Mrit-Sanjivani, the personification of the herb, is an appropriate intercessor. The herb mritasanjivani is said to restore life and is to be found on Mt. Meru."



(Image Source: https://betweenthepillars.com/2016/08/03/the-pentagram-and-its-many-meanings/)

Na Ma Shi Va Ya – these five syllables indicate the five elements (known as Pancha Bhoota in Sanskrit): – Earth, Water, Fire, Air, and Ether. The mantra begins with earth and ends with spirit. While drawing the star, we start from Earth, move to spirit, to fire, air, and water, and then return to Earth. This symbolises the mortality of living things, being born from the ground and returning to it. The five elements are the building blocks of everything in Creation, including the human body, and Lord Shiva is the master of these five elements.[14] Each element has its opposing element but the spirit element does not have any. This indicates that the soul or spirit is the purest and supreme element.

Yash Narendra Chaudhari has authored an excellent paper, "The Comparative Study of Dark Matter -Dark Energy and MahamrityunjayYantra." He believes that this Yantra is associated with the realities of Modern Physics - The Dark Cosmic energy and proof that there is a purely scientific concept in the Mahamrityunjay Yantra. It is a well-designed structural and geometrical figure and a powerful mathematical tool similar to the Shri Yantra. By comparison, the main goal is that spirituality is not only a belief but it is a pure science. Attempt in establishing the fact that Mahamrityunjay Yantra is generated by using great scientific phenomena as well as mathematical models.

The shapes of five-ness are the pentagon and the pentagram. The pentagon has five equal sides and angles, whilst the pentagram is a five-pointed star. The pentagram has a somewhat tainted history and is often associated with witchcraft and the dark arts. The pentagram seems like an appropriate choice for this ordeal because of its fractal properties, which reflect the immortality of the mantra. Five-fold geometry governs the human form - a head, two arms and two legs. We also have evolved five fingers on each hand and five toes on each foot. However, five is not just found throughout the human body, it is found throughout the natural world, most prevalently in living forms. [15] In mathematics, a fractal is a geometric shape that contains detailed structure at arbitrarily small scales, usually having a fractal dimension strictly exceeding the topological dimension. Fractals are created by repeating a simple process over and over in an ongoing feedback loop, and they exist in between our familiar dimensions. Unsurprisingly, the fractal dimension of a pentagram/pentagon is very close to the golden ratio.

Similarity Dimension $\sum_{k=1}^{5} r^{d} = 1 \quad \Rightarrow \quad d = \frac{\log(1/5)}{\log(r)} = \frac{\log(5)}{\log(2/(3-\sqrt{5}))} = 1.67228$

 $(Image\ Source:\ https://larryriddle.agnesscott.org/ifs/pentagon/pentagon.htm\)\ (Image\ Source:\ https://www.mathsisfun.com/geometry/pentagram.html)$

a/b = 1.618...

b/c = 1.618...

c/d = 1.618...

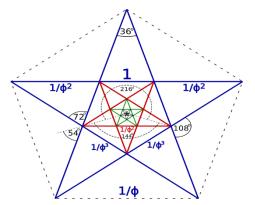
108 is an auspicious number in Vedic, i.e., Hindu culture; each interior angle of a pentagon is 108 degrees. This might seem like a coincidence, but in the grand scheme of things, the chances are very low.

 $108 = 1^{1} * 2^{2} * 3^{3}$, the first 3 Fibonacci numbers excluding 0.

Another connection between 108 and the golden ratio is:

 $2 \sin (108^{\circ}/2) = \text{GOLDEN RATIO}$

The Interdisciplinary Journal of Yagya Research say that a pentagonal yagya kund is used for rituals to ward off diseases.



(Image Source: <u>https://www.researchgate.net/publication/321682622 The celestial inspirations for Giza Stonehenge and Washington DC</u>?Chann el= doi&linkId=5a31a1db0f7e9b2a2873ae35&showFulltext=true)

Line and line segment lengths inside a pentagram are all connected by integer powers of 1. Furthermore, the golden ratio has accurate mathematical correlations to pi and e at the different pentagram angles. Traditional references to the number of degrees in these angles have been commonly misinterpreted as allusions to the Earth's processional cycle rather than its orbital dance with Venus and the one cubit AU difference between Mars and Earth's orbital radii. [16]

This creates a connection between pentagrams and Venus, indicating a link between Shiva and Shukra. Fortunately, our theory makes this link often and accurately.

Shiva has a secret relationship with Venus, or Shukra, which he can manage since Venus emerges as a morning or evening star not far from the Sun and absorbs it back into himself. Venus and Solar Shiva are associated with ecstasy, metamorphosis, death, and rebirth.

One of the Navagraha goddesses is Venus, also known as Shukra.

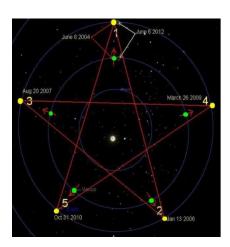
According to the Puranas, Shiva, the destroyer, has three eyes, but Shukra has only one. Shukracharya was Sage Bhrigu's son and the primary instructor of the Asuras (Demons). He was a devotee of Lord Shiva and learned Sanjivani Vidya (the ability to bring someone back from the dead) or MritSanjeevani Vidya (the power of resurrection) from him. He rules over areas of one's life such as beauty, physical appearance, sexuality, riches, and luxury.

He has a white complexion and drives a chariot driven by eight horses while brandishing an Aayudh-weapon, according to Matsya Purana. He wears a crown and sits on a white lotus with his four hands. Shukracharya holds in one hand Dunda, Rudraxmala, and Patr (a vessel), and Varadmudra in the other.

According to the Mahabharata, he is the maestri of Rasa, Mantra, and medicine. He dedicated his whole life to penance, which gained him the goodwill of Lord Shiva. He gave his whole riches to his demonic pupils. He is also the author of the political science book Shukra Niti.

Shukra Planet is associated with Shukranu (sperm), which is associated with life. As a planet, he contributes to the overall well-being of the universe. He is one of the planets in the court of Brahma. He also appeases other planets that prevent rain. As a consequence, he helps to replenish life on Earth via rain.

Overall, we learn that Shukra is almost synonymous with rejuvenation and life. As suggested, the MritSanjeevani Vidya that Shiva gifted Shukra contained the Maha Mrutyunjaya mantra. This brings our discovery full circle.





(Image Source : https://agent-jl36.livejournal.com/103853.html) (Image source : https://ancientskiesbook.com/2021/04/the-yantra-and-the-magic-square-of-shukra.html#:~:text=They%20made%20the%20Pentagon%20an,the%20five%20petals%20of%20Venus.)

There are astonishing relationships between pentagrams and shukra.

When plotting the visible regions of Venus, one comes upon a surprising shape: a pentagon. Vedic astronomers excelled in making observations at twilight and daybreak. They included the Pentagon into the Venusian Yantra of Shukra.

The Maya worshipped Venus and even developed their own Venus calendars.

The Maya inhabit a region that is far from the Indian subcontinent, including portions of Mexico and South America. Is it possible that the Maya worshipped Venus as their "god" and were the asuras that Shukracharya supervised? Since "Mayan" or "Maya" means "illusion" in Sanskrit, it seems to reason that Chak'ek or Kukulcan, the alleged Venus deity, would have certain characteristics with the planet.

Archaeoastronomer Anthony Aveni from Colgate University claims that the "House of the Governor" royal mansion was built with Venus observation in mind. The building is adorned with Venus emblems, and from the main entrance, visitors can still view the planet aligned with the statues that stand at the site's northern and southern boundaries, denoting the northern and southern limits of Venus' yearly horizon journey.

Eight years equals five Venus cycles. As an example, if Venus first became visible in the east as a morning star on Independence Day, then it would do so again eight years later. One Venus Round is equivalent to 13 of these Sun/Venus cycles. Because of this 8:5 ratio, Venus follows the path of a five-pointed star around the ecliptic every 8 years, which is a fascinating fact in its own right.

The science of cymatics examines the visual manifestations of acoustic and vibratory phenomena on a flat surface such as a plate, diaphragm, or membrane.

Cymatic patterns form when the water in a spinning bucket moves at a different speed than its perimeter. The created frequency proportion once again dictates the geometry.

Similar to how weather patterns spiral about a centre and eventually close into a circle, this is how cyclones form. In recent times, the most intense storms have sometimes left behind cymatic cloud formations, such as a pentagram within a pentagonal eye.

Around a planet, gravity creates a spherical bubble that functions as a damping well, allowing waves to reflect and organise themselves.

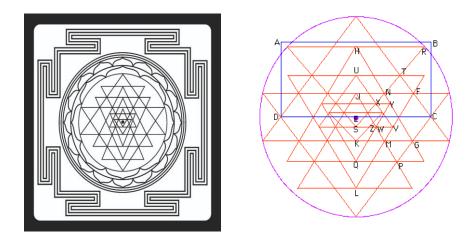
A similar phenomenon occurred in the solar system's plasma disc aeons ago, when the planets formed in roughly perpendicular orbits along a Golden Spiral. As a result, the planets and their satellites show off a wide range of resonant behaviours. The most blatant example is the harmonic resonance of Venus's orbit around the Earth. [17]

In Vedic tradition, the Sri Yantra (also known as the Sri chakra) plays a crucial role. The "Theory of Everything" is supposedly included.

The Sri Yantra is a fractal-like design with deep ties to the golden ratio; it may even precede Pingala by millennia, making it the earliest proof of the golden ratio's existence.

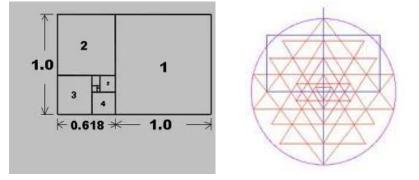
Most Sri Yantra depictions use a base angle of roughly 52 degrees for their biggest triangles, which is quite similar to the base angle of the Great Pyramid of Cheops (51 deg 50'). The Golden Ratio, phi, is achieved when the hypotenuse is equal to 50 percent of the base.

Using this angle as the foundation, the design was tested using the two biggest triangles. Using a strategy very similar to the 7x7 grid, building proceeds. The form is extremely close to instances in the literature, and the result is noteworthy since the error is negligible (at Z and W). The discrepancy is only.3616% of the whole diameter. The inaccuracy for a base angle of 53 degrees is 9.125%, therefore even a slight adjustment in the base angle may have a significant impact. [18]



(Image Source: http://alumni.cse.ucsc.edu/~mikel/sriyantra/golden.html)

The distance from the lower corners of the 2×1 rectangle to the top and bottom vertices of the Sri Yantra is PHI = (1/2)(diagonal of rectangle + 1) = (1/2)(sqrt(5) + 1). Since PHI^A2 = PHI + 1, the radius of the circle is sqrt (PHI^A2 - 1^A2) = sqrt ((PHI + 1) - 1) = sqrt(PHI). Allow D to represent the line length connecting the hypotenuses of the two greatest triangles. Then, D units above and below the bases of the two greatest triangles, respectively, mark the highest and lowest baselines, respectively. From those parts, the rest of the structure mirrors itself[19].



(Image Source: https://captajitvadakayil.in/2022/02/24/brain-massage-with-sri-yantra-capt-ajit-vadakayil/)

Besides the ratio, the Egyptian pyramids seem to have an interesting link with Sri Yantra[20].

At an angle of 51 degrees, 49 minutes, and 38.25 seconds (about 51.5 degrees), the Egyptian pyramids and the Sri Yantra pyramid share the same measurement. The pyramids are symmetrical, following the Fibonacci sequence, and their proportions are in terms of PHI[21].

Saptarshi Mandala's rotation around Dhruva nakshatra constitutes the occult practice known as Omkaaram. The rotation will follow a Fibonacci sequence. Laya is the term for a slow or leisurely pace. It takes the shape of a Veena. The swastik represents this turn. Shree represents Laya. The Hamsa Dhwani is another term for the Veena's sound.

Conclusions

It is safe to say that nothing is a coincidence in the Vedic world, metaphorically put, if one catches a string, one would discover a whole web of interconnected and complex relations with it. Sacred geometry is sometimes beyond the understanding of an average person, it is mind-baffling to realise that our ancestors knew all these several millennia ago. To conclude, Musical notes, shapes of galaxies and even the proportions of a bee are related to the "Fibonacci" numbers". Apart from recognizing that they are of Indian origin, it is more important to focus on how, when, where and who of the matter. The Maha Mrutyunjaya and Sri Yantras are two of the many yantras which have sophisticated mechanisms and meaning attached to them. They have obvious connections with the Virahanka numbers and are most likely predating them. The link between Venus and Shiva which ultimately leads to the connection with the Golden ratio shows that the series and the ratio are an embodiment of the universe, the micro and macrocosm. It is beautiful to recognise how almost everything pleasant to the eye is connected somehow to the ratio, evidently so as described by S Kak in his paper "Physics of Aesthetics". We may think that we are at a peak point in human civilization, but as we saw above, we clearly lost touch with ancient knowledge and disregard it as primitive and mythological. The ones who do not learn from history, are doomed to repeat it.

Acknowledgements

I would like to express my full gratitude to my mentor, Dr. G. Srinivasa Rao, for his valuable insights and guidance. His motivation and enthusiasm have fueled the progress of this paper.

Additionally, my biggest inspiration behind this topic is the findings of Shree Praveen Mohan; his incredible information and thought process have urged me to write a formal paper.

References

[1] V Pletser 2017 "Fibonacci Numbers and the Golden Ratio in Biology, Physics, Astrophysics, Chemistry and Technology: A Non-Exhaustive Review"

[2] Štefan Porubský: Leonardo Pisani. Retrieved 2023/8/11 from Interactive Information Portal for Algorithmic Mathematics, Institute of Computer Science of the Czech Academy of Sciences, Prague, Czech Republic, web-page https://www.cs.cas.cz/portal/AlgoMath/Biographies/Fibonacci.htm

[3] Dr. Srinivasa Rao Gundu, Dr. Panem Charanarur"A Scientific Study on Nava Padma Koulinee Yoga Sadhana", International Journal of Research Publication and Reviews (IJRPR), Volume.2,Issue. 12,December.2021, pp 1424-1435.(ISSN 2582-7421)

[4] Motilal Banarsi Das 2002 "Panchatantram of Visnusarma" (edited with Hindi commentary by Shyamacharana Pandeya)

[5] J Shah 1991 "A HISTORY OF PINGALA'S COMBINATORICS" https://web.northeastern.edu/shah/papers/Pingala.pdf

[6] P Singh 1985 "The So-called Fibonacci Numbers in Ancient and Medieval India "https://www.cs.umd.edu/~gasarch/BLOGPAPERS/fibfibs.pdf

[7] Madhusudhana, V.(ed.)1981.Pingala Naga Viracitam Chandahsastram (with commentary by Halayudha).ParimalaPublications,33/17. Delhi-110007:Shakti Nagar.

[8] Paramananda Singh, Acharya Hemachandra and the Fibonacci numbers, math.Ed.siwan, 20(10):28-30, 1986, IISN0047-6269.

[9] Panchatantram of Visnusarma (edited with Hindi commentary by Shyamacharana Pandeya), Motilal Banarsi Das, Delhi, 2002.

[10] Binomial Theorem in Ancient India. Amulya Kumar Bag, History of Science, Ancient Period Unit II, No.1, Park Street, Calcutta-16(1966).

[11] S Kak 2004 "Recursionism and Reality: Representing and Understanding the World"

[12] E Cervantes, José J M Gómez 17.02.2018 "Seed shape quantification in the order Cucurbitales"

[13]Fredrick W Bunce "The Yantras of Deities and their Numerological Foundations: An Iconographic Consideration "

[15] YN Chaudhari 2023 "The Comparative Study of Dark Matter -Dark Energy and Mahamrityunjay Yantra"

[16] Srinivasa Rao Gundu "Observed Mathematical Mysticism and Allegorical Phenomena in Rudraadhyaaya ", International Journal of Advanced Scientific Technologies in Engineering and Management Sciences (IJASTEMS), Volume.6, Issue.12, July.2020, pp.16 - 22. (ISSN: 2454-356X)

[17] R Spivey 2017 "The celestial inspirations for Giza, Stonehenge and Washington D.C."

[18] R Merrick 2017 "Harmonic Patterning in Nature"

[19] Srinivasa Rao Gundu "Glimpses of decoded scientific, computational and technological prophecies of Bhagavadgeetha", International Journal of Advanced Scientific Technologies in Engineering and Management Sciences (IJASTEMS), Volume.7, Issue. 1, January.2021, pp.1 - 12. (ISSN: 2454-356X)

[20] Pisano, L. (2003). Fibonacci's Liber abaci. A translation into modern English of Leonardo Pisano's Book of Calculation. Transl. from Latin and with an introduction, notes and bibliography by L. E. Sigler. Paperback ed. (English) Sources and Studies in the History of Mathematics and Physical Sciences. New York: Springer.

[21] Persaud-Sharma D and O'Leary JP. Fibonacci Series, 2015; 2(5): 1066 Golden Proportions, and the Human Biology. Austin J Surg.

[22] T Bhatt 2020 "THE COMPARATIVE STUDY OF QUANTUM THEORY AND SHREE YANTRA"

[23] Srinivasa Rao Gundu "Sri Chakra: The Manifestation of neo-platonic cosmic allegorical mysticism-A Scientific Study ", International Journal of Advanced Scientific Technologies in Engineering and Management Sciences (IJASTEMS), Volume.6, Issue.12, December.2020, pp.6-15.(ISSN: 2454-356X)