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On Non-homogeneous Cubic Equation with Four Unknowns $xy + 2z^2 = 10w^3$

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ABSTRACT:

This paper is devoted to obtain non-zero distinct integer solutions to non-homogeneous cubic equation with four unknowns given by $xy + 2z^2 = 10w^3$.

Keywords: Non-homogeneous cubic, Cubic with four unknowns, Integer solutions.

Introduction:

Diophantine equations theory have been working by numerous mathematicians since ancient time of Diophantus. There is a big gap in the general theory of homogeneous/non homogeneous cubic equations with three or more variables. Also, the cubic diophantine equations are rich in variety and offer an unlimited field for research [1, 2]. For an extensive review of various problems, one may refer [3-30]. This paper concerns with another interesting non-homogeneous cubic diophantine equation with four unknowns $xy + 2z^2 = 10w^3$ for determining its infinitely many non-zero integral solutions.

Method of Analysis:

The non-homogeneous cubic equation with four unknowns to be solved is

$$xy + 2z^2 = 10w^3 \tag{1}$$

The process of obtaining different sets of non-zero distinct integer solutions to (1) is

illustrated below:

Illustration 1:

The introduction of the linear transformations

$$x = 2p, y = 2q, z = p - q, (p \neq q) \tag{2}$$

in (1) leads to

$$p^2 + q^2 = 5w^3 \tag{3}$$

Assume

$$w = a^2 + b^2 \tag{4}$$

Express the integer 5 on the R.H.S. of (3) as the product of complex conjugates as

$$5 = (2 + i)(2 - i) \tag{5}$$

Using (4) and (5) in (3) and employing the method of factorization ,define

$$p+iq=(2+i)\,(a+ib)^3 \tag{6}$$

On equating the real and imaginary parts in (6) ,it is seen that

$$p=2f(a,b)-g(a,b)\ ,q=f(a,b)+2g(a,b) \tag{7}$$

where

$$f(a,b)=a^3-3ab^2\ ,g(a,b)=3a^2b-b^3 \tag{8}$$

In view of (2) ,the integer solutions to (1) are given by

$$x=4f(a,b)-2g(a,b)\ ,y=2f(a,b)+4g(a,b)\ ,z=f(a,b)-3g(a,b)$$

along with (4).

Note 1:

Apart from (5) ,the integer 5 on the R.H.S. of (3) is expressed as below:

$$5=(1+2i)(1-2i)\ ,$$

$$5=\frac{(2+11i)\,(2-11i)}{25}\ ,$$

$$5=\frac{(2+29i)\,(2-29i)}{169}$$

The repetition of the above process leads to three more sets of integer solutions to (1).

Illustration 2:

Introducing the linear transformations

$$x=u+z\ ,y=u-z \tag{9}$$

in (1) gives

$$u^2+z^2=10w^3 \tag{10}$$

which is satisfied by

$$u=10^2\,m(m^2+n^2) \tag{11}$$

$$z=10^2\,n(m^2+n^2)\ ,w=10(m^2+n^2) \tag{12}$$

In view of (9) , it is seen that

$$x=10^2\,(m+n)\,(m^2+n^2)\ ,y=10^2\,(m-n)\,(m^2+n^2) \tag{13}$$

Thus , (12) and (13) give the integer solutions to (1).

Illustration 3:

Express the integer 10 on the R.H.S. of (10) as the product of complex conjugates as

$$10=(3+i)(3-i) \tag{14}$$

Using (4) & (14) in (10) and following the procedure as in Illustration 1 ,the integer

solutions to (1) are found to be

$$x = 4f(a,b) + 2g(a,b) \ , y = 2f(a,b) - 4g(a,b) \ , z = f(a,b) + 3g(a,b) \quad (15)$$

Thus , (4) and (15) represent the integer solutions to (1).

Note 2:

It is to be noted that the integer 10 on the R.H.S. of (10) is also considered as

$$10 = (1+3i)(1-3i)$$

This choice leads to a different set of integer solutions to (1).

Illustration 4:

Introducing the transformations

$$x = 2p, y = 2w, z = p - w, p \neq w \quad (16)$$

in (1) ,it is written as

$$p^2 = w^2(5w-1) \quad (17)$$

There are two sets of integer solutions to (17) as shown below:

$$w = \begin{cases} 5k^2 - 6k + 2 \ , \\ 5k^2 - 4k + 1 \end{cases}$$
$$p = \begin{cases} (5k^2 - 6k + 2)(5k-3) \ , \\ (5k^2 - 4k + 1)(5k-2) \end{cases} \quad (18)$$

In view of (16) , the corresponding two sets of values of x, y, z satisfying (1) are

respectively given by

$$x = 2(5k-3)(5k^2-6k+2) \ , y = 2(5k^2-6k+2) \ , z = (5k-4)(5k^2-6k+2)$$
$$x = 2(5k-2)(5k^2-4k+1) \ , y = 2(5k^2-4k+1) \ , z = (5k-3)(5k^2-4k+1)$$

It is worth to mention that , apart from the above integer solutions , there are other choices of integer solutions to (1) which are exhibited below in Table 1:

Table 1 – Integer solutions

S. No	x	y	z	w
1	$2a^{3s}$	$4a^{3s}$	a^{3s}	a^{2s}
2	$2a^{6s}$	4	a^{3s}	a^{2s}
3	$4a^{6s}$	2	a^{3s}	a^{2s}

4	$5000\,a^{6s}$	1	$50a^{3s}$	$10a^{2s}$
5	$10\,(5a^2+1)^2$	1	$5a(5a^2+1)$	$(5a^2+1)$
6	$5\,(5a^2+1)^2$	2	$5a(5a^2+1)$	$(5a^2+1)$
7	$2\,(5a^2+1)^2$	5	$5a(5a^2+1)$	$(5a^2+1)$
8	$5\,(5a^2+1)$	$2\,(5a^2+1)$	$5a(5a^2+1)$	$(5a^2+1)$
9	$2k^3s^2$	$5s-ks^2$	k^2s^2	ks
10	$10k^3s^2$	$s-5ks^2$	$5k^3s^2$	ks
11	$10k^2s$	ks^2-5s	$5ks$	ks

Conclusion:

In this paper, we have made an attempt to determine different patterns of non-zero distinct integer solutions to the non- homogeneous cubic equation with four unknowns given by $xy + 2z^2 = 10w^3$ As the cubic equations are rich in variety, one may search for other forms of cubic equations with multi-variables to obtain their corresponding solutions .

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