Comparative Analysis of Two Different but Uniformly Supported Isotropic Plates Using Odd Order Energy Functional


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DOI: https://doi.org/10.55248/gengpi.4.823.50439

ABSTRACT

This research deals on the comparative analysis of two different but uniformly supported plate, using odd energy functional. Two different rectangular isotropic plate with one, simply supported all round and the other Clamped all round, were closely examined. From several minimization of the shape functions, 3rd order strain energy equation was formulated. Further minimization of this gave rise to the Third Order Total Potential Energy Functional. In integrating the Total Energy functional with respect to the amplitude produced the Governing equation. Various coefficients which explains the extent of their stiffnesses were formulated. The Third order strain energy equation was also formulated, from where the Third Order Total Potential Energy Functional was formed. The critical buckling load equations emerged by further minimizing the governing equation. By substituting the different aspect ratios into the equation, the non-dimensional buckling load parameters were obtained. The final outcome in both cases were critically observed as detailed below.

INTRODUCTION

THIRD ORDER ENERGY FUNCTIONAL FOR THE PLATES

The two uniformly supported plate under consideration in the research work are simple-simple-simple simple and clamped-clamped-clamped-clamped rectangular plates. The displacements of a thin rectangular plate include in-plane displacements – u and v and out of plane displacement – w. Considering u and v as the functions of x, y and z, w is only a function of x and y and so x, y and z are the principal coordinates. The implication of this is that w is constant along z direction. This is in consonant with the assumption that “vertical normal strain of a plate is equal to zero”. The vertical shear strains are negligible in classical plate analysis and assumed to be equal to zero. Thus, out of the six engineering strain components, \( \varepsilon_x \), \( \varepsilon_y \), and \( \gamma_{xy} \) were assumed to be zero. Therefore, leaving only three engineering strain components - \( \varepsilon_x \), \( \varepsilon_y \), and \( \gamma_{xy} \).

Upon the minimization of the strain deflection.

METHODOLOGY

Direct integration of the strain energy formed the fundamental for the formulation of the needed functional. The strain, stress, shear stress and shear strain were further introduced by substitution and this gave rise to the flexural rigidity. When the derived strain energy was added to the external work done, the Total potential energy, \( T_p \) was derived. The stages involved were as detailed below,

From the the strain deflection,

\[
\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -z \frac{\partial^2 w}{\partial x \partial y} - z \frac{\partial^2 w}{\partial y \partial x} = -2z \frac{\partial^2 w}{\partial x \partial y}
\]

and stress-strain relationship, the stress-deflection relationship were formulated

\[
\sigma_x = \frac{-Ez}{1-\mu^2} \left( \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right)
\]

Similarly, substituting Equations 3.18 and 3.19 into Equation 3.8 gives:

\[
\sigma_y = \frac{-Ez}{1-\mu^2} \left( \mu \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)
\]

The summation of the product of stress and strain at every point on the plate continuum gives

\[
\sigma \varepsilon = \sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy}
\]
Further substitutions and minimizations give the 3rd Strain energy equation as
\[ E_u = \frac{D}{2} \int_0^1 \int_0^1 \left( \frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial x} + 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2} \right) dxdy \]

or
\[ E_u = \frac{D}{2} \int_0^1 \int_0^1 \left( \frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial x} + 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2} \right) dxdy \]

Upon the addition of the external work done, \( W_e \), the Total energy, \( T_p \) was formulated as
\[ T_p = \frac{E}{2} \int_0^1 \int_0^1 \left( \frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial x} + 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2} \right) dxdy - \frac{E}{2} \int_0^1 \int_0^1 h dxdy \]

or
\[ T_p = \frac{E}{2} \int_0^1 \int_0^1 \left( \frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial x} + 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2} \right) dxdy - \frac{E}{2} \int_0^1 \int_0^1 h dxdy \]

Differentiating the Total Potential energy with respect to the Amplitude and further substitutions gives
\[ B_{cr} = \frac{\frac{\partial^2 w}{\partial y^2}}{\frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial x} + 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2}} dxdy \]

or
\[ B_{cr} = \frac{\frac{\partial^2 w}{\partial y^2}}{\frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial x} + 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2}} dxdy \]

The critical buckling load equation was further reduced to
\[ B_{cr} = \frac{D}{2} \left( c_{r1} + 2 \frac{1}{p^2} c_{r2} + \frac{1}{p^4} c_{r3} \right) \]

where
\[ c_{r1} = \int_0^1 \int_0^1 K \frac{\partial^2 h}{\partial y^2} dxdy \]
\[ c_{r2} = \int_0^1 \int_0^1 K \frac{\partial^2 h}{\partial x \partial y} dxdy \]
\[ c_{r3} = \int_0^1 \int_0^1 K \frac{\partial^2 h}{\partial x^2} dxdy \]

**Critical Buckling Load Equation for SSSS Plate**

The critical buckling load equation for SSSS can be written in terms of stiffness coefficients 
\( (c_{r1}, c_{r2}, c_{r3}, c_{r6}) \) using the \( a^2 = b^2 p^2 \), for the

aspect ratio of \( p = b/a \) as follows
\[ B_{cr} = \frac{D}{2} \left( c_{r1} + \frac{1}{p^2} c_{r2} + \frac{1}{p^4} c_{r3} \right) \]

**Critical Buckling Load Equation for CICICI Plate**

Substituting the stiffness coefficients \( (c_{r1}, c_{r2}, c_{r3}, c_{r6}) \) for the CICICI plate, into the general buckling load equation, the critical buckling load equation the plate can be expressed as
\[ B_{cr} = \frac{D}{2} \left( c_{r1} + \frac{1}{p^2} c_{r2} + \frac{1}{p^4} c_{r3} \right) \]

Considering \( a^2 = b^2 p^2 \), for \( p = b/a \) as the aspect ratio.

**Determination of the Stiffness coefficients of the plates**

From the Polynomial rules, the shape function of the shape function, \( sh \) for SSSS Plate is as \( (R-2R^3+R^5) \ (Q-2Q^3+Q^5) \)

**Differential values for Simple-Simple-Simple-Simple shape**

The various derivatives of the shape functions can be expressed as
\[
\begin{align*}
\text{sh} &= (R - 2R^2 + R^4)(Q - 2Q^3 + Q^4) \\
\frac{\partial \text{sh}}{\partial R} &= (1 - 6R^2 + 4R^4)(Q - 2Q^3 + Q^4) \\
\frac{\partial^2 \text{sh}}{\partial R^2} &= (-12R + 12R^2)(Q - 2Q^3 + Q^4) \\
\frac{\partial^3 \text{sh}}{\partial R^3} &= (-12 + 24R)(Q - 2Q^3 + Q^4) \\
\frac{\partial h}{\partial Q} &= (R - 2R^3 + R^6)(1 - 6Q^2 + 4Q^4) \\
\frac{\partial^2 \text{sh}}{\partial Q^2} &= (R - 2R^3 + R^6)(-12Q + 12Q^2) \\
\frac{\partial^3 \text{sh}}{\partial Q^3} &= (R - 2R^3 + R^6)(-12 + 24Q) \\
\frac{\partial^3 \text{sh}}{\partial R\partial Q^2} &= (1 - 6R^2 + 4R^4)(1 - 6Q^2 + 4Q^4) \\
\frac{\partial^3 \text{sh}}{\partial R\partial Q^3} &= (1 - 6R^2 + 4R^4)(-12Q + 12Q^2) \\
\frac{\partial^3 \text{sh}}{\partial R^3} &= (-12 + 24R)(Q - 2Q^3 + Q^4) \\
\end{align*}
\]

From the expressions above,
\[
\frac{\partial^3 \text{sh}}{\partial R\partial Q} \cdot \frac{\partial h}{\partial R} = (-12 + 24R)(Q - 2Q^3 + Q^4) \cdot (1 - 6R^2 + 4R^4)(Q - 2Q^3 + Q^4)
\]

Collecting the like terms together yields
\[
(-12 + 24R)(1 - 6R^2 + 4R^4) x (Q - 2Q^3 + Q^4)(Q - 2Q^3 + Q^4)
\]

and multiplying out each bracket gives
\[
-12(1 - 6R^2 + 4R^4) + 24R(1 - 6R^2 + 4R^4) x Q(Q - 2Q^3 + Q^4) - 2Q^3(Q - 2Q^3 + Q^4) + Q^4(Q - 2Q^3 + Q^4)
\]

which finally changes to
\[
(-12 + 72R^2 - 40R^4 + 24R - 144R^2 + 96R^4) x (Q^2 - 2Q^4 + Q^6 - 2Q^7 + 4Q^5 - 2Q^7 - Q^5 - Q^6)
\]

Therefore
\[
\frac{\partial^3 \text{sh}}{\partial R\partial Q} \cdot \frac{\partial h}{\partial Q} = (-12 + 24R + 72R^2 - 192R^3 + 96R^4)
\]
\[
x(Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^6)
\]

But
\[
crt = \int \int \int [\frac{\partial^3 \text{sh}}{\partial R\partial Q} \cdot \frac{\partial h}{\partial R} dRdQ]
\]

That implies that
\[
crt = \int \int \int (-12 + 24R + 72R^2 - 192R^3 + 96R^4) x (Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^6) dRdQ
\]

\[
= \left[ \left[ \left[ \left[ \frac{-12R}{2} + \frac{24R^2}{3} - \frac{12R^3}{4} - \frac{192R^4}{5} + \frac{96R^5}{6} \right] \cdot \left( \frac{Q^2}{2} - \frac{4Q^4}{5} + \frac{2Q^5}{6} + \frac{4Q^6}{7} - \frac{4Q^7}{8} + \frac{Q^8}{9} \right) \right] \right] \right] dRdQ
\]

\[
= \left[ \left[ \left[ \left[ \frac{-12R}{2} + \frac{24R^2}{3} - \frac{12R^3}{4} - \frac{192R^4}{5} + \frac{96R^5}{6} \right] \cdot \left( \frac{Q^2}{2} - \frac{4Q^4}{5} + \frac{2Q^5}{6} + \frac{4Q^6}{7} - \frac{4Q^7}{8} + \frac{Q^8}{9} \right) \right] \right] \right] dRdQ
\]

Therefore
\[
crt = (-4 \frac{6}{5}) \times \frac{31}{630} = \frac{-124}{525}
\]

also,
\[
\frac{\partial^3 \text{sh}}{\partial R\partial Q^2} \cdot \frac{\partial h}{\partial R} = (1 - 6R^2 + 4R^4)(-12Q + 12Q^2) x (1 - 6R^2 + 4R^4)(Q - 2Q^3 + Q^4)
\]

Collecting the like terms together yields
= (1 − 6R^2 + 4R^3)(1 − 6R^2 + 4R^3) x (−12Q + 12Q^2) (Q − 2Q^2 + Q^4)

and multiplying out each bracket gives

= 1(1 − 6R^2 + 4R^3)(1 − 6R^2 + 4R^3) + 4R^3(1 − 6R^2 + 4R^3) x

−12Q − Q^2 + Q^4 + 12Q^2 (Q − 2Q^2 + Q^4)

which finally changes to

(1 − 6R^2 + 4R^3 − 6R^2 + 36R^4 − 24R^3 + 4R^3 − 24R^5 + 16R^6) x (−12Q^2 + 24Q^4 − 12Q^5 + 12Q^3 − 24Q^5 + 12Q^6)

= (1 − 12R^2 + 8R^3 + 36R^4 − 48R^5 + 16R^6) x (−12Q^2 + 24Q^4 − 36Q^3 + 12Q^6)

Therefore $\frac{\partial h}{\partial Q} \frac{dR}{dQ} = (1 − 12R^2 + 8R^3 + 36R^4 − 48R^5 + 16R^6) x$

(−12Q^2 + 24Q^4 − 36Q^3 + 12Q^6)

= 4

But $\text{crt}_2 = \int \frac{\partial h}{\partial Q} \frac{dR}{dQ} dQ = \int \frac{\partial h}{\partial Q} \frac{dR}{dQ} dQ$

That implies that $\text{crt}_2 = \int \int_b (1 − 12R^2 + 8R^3 + 36R^4 − 48R^5 + 16R^6) x$

(−12Q^2 + 24Q^4 − 36Q^3 + 12Q^6) dQ

= \left( \int \left[ \frac{1}{3} \left( \frac{12R^2}{5} + \frac{9R^4}{5} − \frac{48R^5}{6} + \frac{16R^6}{7} \right) x \left( \frac{-12Q^2}{3} + \frac{12Q^4}{4} + \frac{36Q^3}{5} + \frac{12Q^6}{7} \right) \right] \right)

= \left( \frac{1}{3} \left( \frac{12R^2}{5} + \frac{9R^4}{5} − \frac{48R^5}{6} + \frac{16R^6}{7} \right) \right) x \left( \frac{-12}{3} + \frac{12}{4} + \frac{36}{5} + \frac{12}{7} \right)

Therefore $k_2 = \left( \frac{17}{35} \right) x \left( \frac{-11}{35} \right) = \frac{-289}{1225}$

Also,

$\frac{\partial h}{\partial Q} \frac{dQ}{dQ} = (R − 2R^3 + R^4) x (−12 + 24Q) (R − 2R^2 + R^4)$

Collecting the like terms together yields

(R − 2R^3 + R^4)(R − 2R^2 + R^4) x (−12 + 24Q)(1 − 6Q^2 + 4Q^3)

and multiplying out each bracket, gives

= R(R − 2R^3 + R^4) − 2R^2 (R − 2R^3 + R^4) + R^4 (R − 2R^2 + R^4) x

−12(1 − 6Q^2 + 4Q^3) + 24Q(1 − 6Q^2 + 4Q^3)

which finally changes to

(R^2 − 2R^4 + 4R^6 − 2R^7 + R^8) x (−12 + 24Q^2 − 48Q^3 + 24Q − 144Q^3 + 96Q^6)

= (R^2 − 4R^4 + 2R^3 − 4R^6 − 4R^7 + 4R^8) x (−12 + 24Q + 72Q^2 − 192Q^3 + 96Q^6)

Therefore $\frac{\partial h}{\partial Q} \frac{dQ}{dQ} = (R^2 − 4R^4 + 2R^3 − 4R^6 − 4R^7 + 4R^8) x (−12 + 24Q + 72Q^2 − 192Q^3 + 96Q^6)$

But $\text{crt}_3 = \int \frac{\partial h}{\partial Q} \frac{dQ}{dQ} dQ = \int \frac{\partial h}{\partial Q} \frac{dQ}{dQ} dQ$

That implies that

$\text{crt}_3 = \int \int_b (R^2 − 4R^4 + 2R^3 − 4R^6 − 4R^7 + 4R^8)

x (−12 + 24Q + 72Q^2 − 192Q^3 + 96Q^6) dQ$

= \left( \int \left[ \frac{4R^5}{5} + \frac{2R^6}{6} + \frac{4R^7}{7} + \frac{3R^8}{8} + \frac{R^9}{9} x \left( \frac{-12Q^2}{3} + \frac{24Q^4}{2} + \frac{72Q^3}{3} \right) \right] \right)

= \left( \frac{1}{3} \left( \frac{4}{5} + \frac{2}{6} + \frac{4}{7} \right) + \frac{1}{9} x \left( \frac{-12}{3} + \frac{24}{2} + \frac{72}{3} \right) \right)$

Therefore $k_3 = \left( \frac{31}{630} \right) x \left( \frac{-4}{35} \right) = \frac{-1124}{225}$

Also,

$\frac{\partial h}{\partial R} = (1 − 6R^2 + 4R^3)(Q − 2Q^2 + Q^4) x (1 − 6R^2 + 4R^3)(Q − 2Q^2 + Q^4)$

Collecting the like terms together yields

(1 − 6R^2 + 4R^3)(Q − 2Q^2 + Q^4)(Q − 2Q^2 + Q^4)

and multiplying out each bracket gives
\[(1 - 6R^2 + 4R^3) - 6R^2(1 - 6R^2 + 4R^3) + 4R^3(1 - 6R^2 + 4R^3)\times Q \times (Q^2 - 2Q^3 + Q^4) - 2Q^2(Q - 2Q^2 + Q^3) + Q^4(Q - 2Q^3 + Q^4)
\]
\[= (1 - 6R^2 + 4R^3 - 6R^2 + 36R^4 - 24R^5 + 4R^3 - 24R^5 + 16R^6) \times (Q^2 - 2Q^3 + Q^4 - 2Q^4 + 4Q^5 - 2Q^7 + Q^8 - 2Q^9 + Q^{10})\]

which finally changes to

\[(1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6) \times (Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^8)\]

Therefore, \( \frac{\partial \text{cts}}{R^3} = (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6) \times (Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^8)\)

But \( \text{cts} = \int \text{Cl1} \, dR \, dQ \)

That implies that \( \text{cts} = \int_{0}^{1} \int_{0}^{1} (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6) \times (Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^8) \, dR \, dQ \)

\[= \left[ \int_{0}^{1} \left( \frac{B}{12R^5} + \frac{\partial R^5}{4} + \frac{36R^4}{5} \right) \times \left( \frac{Q^2}{3} - \frac{4Q^5}{5} + \frac{Q^6}{6} \right) \right] \right] \]

\[= \left[ \left( \frac{1}{12} \cdot \frac{1}{8} + \frac{9}{2} + \frac{9}{2} \cdot \frac{1}{3} - \frac{4}{5} + \frac{2}{7} + \frac{1}{3} \right) \right] \]

Therefore, \( \text{cts} = \frac{17}{53} \times \frac{31}{230} = \frac{26}{2260} \)

Similarly from Polynomial rules, the shape function, \( sh \) for CICICI panel

\( is (R^2) - (2R^4 + R^4) (Q^2 - 4Q^4 + Q^8) \)

\[\text{Differential values for Clamped-Clamped-Clamped shape}\]

Also the various differential values for CICICI shape functions are as detail

\[sh = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)\]

\[\frac{\partial sh}{\partial R} = (2R - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4)\]

\[\frac{\partial^2 sh}{\partial R^2} = (2 - 12R + 12R^2)(Q^2 - 2Q^3 + Q^4)\]

\[\frac{\partial^3 sh}{\partial R^3} = (-12 + 24R)(Q^2 - 2Q^3 + Q^4)\]

\[\frac{\partial sh}{\partial Q} = (R^2 - 2R^3 + R^4)(2Q^2 - 6Q^2 + 4Q^3)\]

\[\frac{\partial^2 sh}{\partial Q^2} = (R^2 - 2R^3 + R^4)(2 - 12Q + 12Q^2)\]

\[\frac{\partial^3 sh}{\partial Q^3} = (R^2 - 2R^3 + R^4)(-12 + 24Q)\]

\[\frac{\partial^3 sh}{\partial R \partial Q^2} = (2R - 6R^2 + 4R^3)(Q^2 - 6Q^2 + 4Q^4)\]

\[\frac{\partial^3 sh}{\partial R^3 \partial Q} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)\]

\[\frac{\partial^3 sh}{\partial R \partial Q^3} = (2R - 6R^2 + 4R^3)(-12 + 24Q)\]

\[\frac{\partial^3 sh}{\partial R^2 \partial Q^2} = (2R - 6R^2 + 4R^3)(Q^2 - 6Q^2 + 4Q^4)\]

\[\frac{\partial^3 sh}{\partial R^3 \partial Q^2} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)\]

\[\frac{\partial^3 sh}{\partial R^2 \partial Q} = (2R - 6R^2 + 4R^3)(Q^2 - 6Q^2 + 4Q^4)\]

\[\frac{\partial^3 sh}{\partial R \partial Q^3} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)\]

\[\frac{\partial^3 sh}{\partial R^3 \partial Q^3} = (2R - 6R^2 + 4R^3)(-12 + 24Q)\]

\[\frac{\partial^3 sh}{\partial R^2 \partial Q^2} = (2R - 6R^2 + 4R^3)(Q^2 - 6Q^2 + 4Q^4)\]

\[\frac{\partial^3 sh}{\partial R^3 \partial Q^2} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)\]

\[\frac{\partial^3 sh}{\partial R^2 \partial Q} = (2R - 6R^2 + 4R^3)(Q^2 - 6Q^2 + 4Q^4)\]

\[\frac{\partial^3 sh}{\partial R \partial Q^3} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)\]

\[\frac{\partial^3 sh}{\partial R^3 \partial Q^3} = (2R - 6R^2 + 4R^3)(-12 + 24Q)\]

\[\frac{\partial^3 sh}{\partial R^2 \partial Q^2} = (2R - 6R^2 + 4R^3)(Q^2 - 6Q^2 + 4Q^4)\]

\[\frac{\partial^3 sh}{\partial R^3 \partial Q^2} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)\]

\[\frac{\partial^3 sh}{\partial R^2 \partial Q} = (2R - 6R^2 + 4R^3)(Q^2 - 6Q^2 + 4Q^4)\]

\[\frac{\partial^3 sh}{\partial R \partial Q^3} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)\]

\[\frac{\partial^3 sh}{\partial R^3 \partial Q^3} = (2R - 6R^2 + 4R^3)(-12 + 24Q)\]

\[\frac{\partial^3 sh}{\partial R^2 \partial Q^2} = (2R - 6R^2 + 4R^3)(Q^2 - 6Q^2 + 4Q^4)\]

\[\frac{\partial^3 sh}{\partial R^3 \partial Q^2} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)\]

\[\frac{\partial^3 sh}{\partial R^2 \partial Q} = (2R - 6R^2 + 4R^3)(Q^2 - 6Q^2 + 4Q^4)\]

\[\frac{\partial^3 sh}{\partial R \partial Q^3} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)\]

\[\frac{\partial^3 sh}{\partial R^3 \partial Q^3} = (2R - 6R^2 + 4R^3)(-12 + 24Q)\]

From the expressions above,

\[\frac{\partial^3 sh}{\partial R^2} \times \frac{\partial sh}{\partial R} = (-12 + 24R)(Q^2 - 2Q^3 + Q^4) \times (2R - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4)\]

Collecting the like terms together yields

\[(-12 + 24R)(2R - 6R^2 + 4R^3) \times (Q^2 - 2Q^3 + Q^4)(Q^2 - 2Q^3 + Q^4)\]
and multiplying out each bracket gives
\[-12(2R - 6R^2 + 4R^3) + 24R(2R - 6R^2 + 4R^3) \times Q^4(Q^2 - 2Q^3 + Q^4)\]
\[-2Q^3(Q^2 - 2Q^3 + Q^4) + Q^3(Q^2 - 2Q^3 + Q^4)\]
\[= (-24R + 72R^2 - 48R^3 + 48R^4 - 144R^3 + 96R^4)\]
\[x(Q^4 - 2Q^3 + Q^2 - 2Q^3 + Q^4)\]

which finally changes to
\[(-24R + 120R^2 - 192R^3 + 96R^4) x (Q^4 - 4Q^3 + 6Q^2 - 4Q^2 + Q^4)\]

Therefore \(\frac{\partial^3 R}{\partial x \partial y \partial z} \cdot \frac{\partial h}{\partial R} = (-24R + 120R^2 - 192R^3 + 96R^4) x (Q^4 - 4Q^3 + 6Q^2 - 4Q^2 + Q^4)\)

But \(crt_1 = \iint \partial^3 R \partial x \partial y \partial z dRdQ\)

That implies that
\[crt_1 = \int \int \int (-24R + 120R^2 - 192R^3 + 96R^4) x (Q^4 - 4Q^3 + 6Q^2 - 4Q^2 + Q^4) dRdQ\]

\[= \left[ (-\frac{24}{4} + \frac{120}{5} - \frac{192}{6} + \frac{96}{5}) x \left( \frac{Q^4}{5} - 4Q^3 + 6Q^2 - 4Q^2 + Q^4 \right) \right] \]

Therefrom
\[crt_1 = \left( \frac{4}{5} \times \frac{1}{5} \right) x \left( \frac{3}{5} \right) = \frac{4}{5} \]

Also,
\[\frac{\partial^3 h}{\partial x \partial y \partial z} \cdot \frac{\partial h}{\partial R} = (2R - 6R^2 + 4R^3)(2 - 12Q + 12Q^2)x(2R - 6R^2 + 4R^3)(Q^2 - 2Q^2 + Q^4)\]

Collecting the like terms together yields
\[(2R - 6R^2 + 4R^3)(2R - 6R^2 + 4R^3) x (2 - 12Q + 12Q^2)(Q^2 - 2Q^2 + Q^4)\]

and multiplying out each bracket gives
\[2R x 2(2Q^2 - 2Q^3 + Q^4) - 12Q(Q^2 - 2Q^3 + Q^4) + 12Q^2(Q^2 - 2Q^2 + Q^4)\]
\[(4R^2 - 12R^2 + 8R^3 + 12R^2 + 36R^3 - 24R^2 + 8R^3 - 24R^2 + 16R^4)\]
\[(2Q^2 - 4Q^3 + 12Q^4 + 12Q^2 - 12Q^5 + 12Q^5 + 12Q^4 + 12Q^0)\]

which finally changes to
\[(4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6)x(2Q^2 - 16Q^3 + 38Q^4 - 36Q^5 + 12Q^6)\]

Therefore
\[\frac{\partial^3 h}{\partial x \partial y \partial z} \cdot \frac{\partial h}{\partial R} = (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6)x(2Q^2 - 16Q^3 + 38Q^4 - 36Q^5 + 12Q^6)\]

But \(crt_2 = \iint \partial^3 R \partial y \partial z dRdQ\)

That implies that
\[crt_2 = \int \int \int (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6) x (2Q^2 - 16Q^3 + 38Q^4 - 36Q^5 + 12Q^6) dRdQ\]

\[= \left[ \left( \frac{4}{3} \times \frac{24}{5} + \frac{52}{5} - \frac{48}{5} \right) x \left( \frac{24}{5} - \frac{16}{5} \right) \right] \]

Therefore \(crt_2 = \left( \frac{2}{5} \times \frac{2}{5} \right) = \frac{4}{25}\)

Also,
\[\frac{\partial^3 h}{\partial y \partial z} \cdot \frac{\partial h}{\partial R} = (R^2 - 2R^3 + R^4)(-12 + 24Q)x(R^2 - 2R^3 + R^4)(2Q - 6Q^2 + 4Q^3)\]

Collecting the like terms together yields
\[(R^2 - 2R^3 + R^4)(R^2 - 2R^3 + R^4) x (-12 + 24Q)(2Q - 6Q^2 + 4Q^3)\]

and multiplying out each bracket gives
\[R^2(R^2 - 2R^3 + R^4) - 2R^3(R^2 - 2R^3 + R^4) + R^4(R^2 - 2R^3 + R^4)\]
\[-12(2Q - 6Q^2 + 4Q^3) + 24Q(2Q - 6Q^2 + 4Q^3)\]
\( (R^4 - 2R^3 + R^2 - 2R^3 + 4R^6 - 2R^7 + R^6 - 2R^7 + R^6) \)
\( x (-24Q + 72Q^2 - 48Q^3 + 48Q^2 - 144Q^3 + 96Q^4) \)

which finally changes to
\( (R^4 - 4R^3 + 6R^6 - 4R^7 + R^6) x (-24Q + 120Q^2 - 192Q^3 + 96Q^4) \)

Therefore
\[ \frac{\partial^3 n}{\partial Q^3} + \frac{\partial h}{\partial Q} = (R^4 - 4R^3 + 6R^6 - 4R^7 + R^6) x (-24Q + 120Q^2 - 192Q^3 + 96Q^4) \]

But \( c_{rt} = \int \int \frac{\partial^3 n}{\partial Q^3} + \frac{\partial h}{\partial Q} dQdR \)

That implies that
\[ c_{rt} = \int_0^1 \int_0^1 (R^4 - 4R^3 + 6R^6 - 4R^7 + R^6) x (-24Q + 120Q^2 - 192Q^3 + 96Q^4)dQdR \]
\[ = 3.118 \]
\[ = \left( \frac{4}{5} - \frac{24}{5} + 6 \frac{R^6}{7} - \frac{48}{5} + \frac{R^7}{9} \right) x \left( \frac{-24}{2} + \frac{120}{3} - \frac{192}{4} + \frac{96}{5} \right) \]

Therefore \( c_{rt} = \frac{4}{155} \) \( x (-\frac{2}{9}) = \frac{4}{1385} \)

Also
\[ (\frac{\partial h}{\partial R})^2 = (2R - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4) \times (2R - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4) \]

Collecting the like terms together yields
\( (2R - 6R^2 + 4R^3)(2R - 6R^2 + 4R^3) \times (Q^2 - 2Q^3 + Q^4)(Q^2 - 2Q^3 + Q^4) \)

and multiplying out each bracket gives
\[ 2R(2R - 6R^2 + 4R^3) - 6R^2(2R - 6R^2 + 4R^3) + 4R^3(2R - 6R^2 + 4R^3) \]
\[ x Q^2(Q^2 - 2Q^3 + Q^4) - 2Q^3(Q^2 - 2Q^3 + Q^4) + Q^4(Q^2 - 2Q^3 + Q^4) \]
\[ = (4R^2 - 12R^3 + 8R^4 - 12R^3 + 36R^4 + 24R^5 + 8R^4 - 24R^5 + 16R^6) \]
\[ x(Q^4 - 2Q^5 - Q^6 - 2Q^7 - Q^8) \]

which finally changes to
\( (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6) \times (Q^4 - 4Q^5 - 6Q^6 - 4Q^7 + Q^8) \)

Therefore
\[ (\frac{\partial h}{\partial R})^2 = (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6) \times (Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) \]

But \( c_{rt} = \int \int \frac{\partial^3 n}{\partial Q^3} dQdR \)

That implies that
\[ c_{rt} = \int_0^1 \int_0^1 (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6) \times (Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)dQdR \]
\[ = 64 \]
\[ = \left( \frac{4}{5} - \frac{24}{5} + \frac{52}{6} - \frac{48}{7} + \frac{16}{9} \right) x \left( \frac{4}{5} + \frac{6}{7} - \frac{4}{9} + \frac{1}{9} \right) \]

Therefore \( c_{rt} = \frac{2}{66350} \)

**Results**

The substitution of the stiffness coefficients into the critical buckling load equations gave the results as detailed below
The non dimensional buckling load parameters for SiSiSiSi and ClClClCl plates were presented on Table 2a and 2b and the behavior of the critical buckling load against Aspect ratio is shown in Figure 1.

Table 2a  Non dimensional buckling load parameters for SSSS and ClClClCl plates for aspect ratio of b/a

<table>
<thead>
<tr>
<th>b/a</th>
<th>2</th>
<th>1.9</th>
<th>1.8</th>
<th>1.7</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>15.4371</td>
<td>16.111</td>
<td>16.9186</td>
<td>17.8983</td>
<td>19.1036</td>
</tr>
<tr>
<td>B_{crt}</td>
<td>15.43632 \frac{D}{a^2}</td>
<td>16.11018 \frac{D}{a^2}</td>
<td>16.91777 \frac{D}{a^2}</td>
<td>17.89753 \frac{D}{a^2}</td>
<td>19.10282 \frac{D}{a^2}</td>
</tr>
</tbody>
</table>

Table 2b

<table>
<thead>
<tr>
<th>b/a</th>
<th>1.5</th>
<th>1.4</th>
<th>1.3</th>
<th>1.2</th>
<th>1.1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>B_{crt}</td>
<td>20.6098 \frac{D}{a^2}</td>
<td>22.52811 \frac{D}{a^2}</td>
<td>25.02498 \frac{D}{a^2}</td>
<td>28.35882 \frac{D}{a^2}</td>
<td>32.94889 \frac{D}{a^2}</td>
<td>39.50795 \frac{D}{a^2}</td>
</tr>
</tbody>
</table>

Table 1: Shape functions and stiffness coefficients

<table>
<thead>
<tr>
<th>Shape Functions, sh</th>
<th>Stiffness Coefficients, crt</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiSiSiSi shssss = (R-2R^3+R^4) x (Q-2Q^3+Q^4)</td>
<td>crt_1 = -124 \frac{525}{236219}, \quad \text{crt}_2 = -289 \frac{1225}{3591}, \quad \text{crt}_3 = -124 \frac{525}{236219}, \quad \text{crt}_4 = 527 \frac{22050}{02390}</td>
</tr>
<tr>
<td>ClClClCl shcccc = (R^2-2R^2+R^4) \times (Q^2-2Q^2+Q^4)</td>
<td>\text{crt}_1 = -2 \frac{1575}{00127}, \quad \text{crt}_2 = -4 \frac{11025}{00036}, \quad \text{crt}_3 = -2 \frac{1575}{00127}, \quad \text{crt}_4 = 2 \frac{66150}{00003}</td>
</tr>
</tbody>
</table>

Result for Simple-Simple-Simple-Simple and Clamped-Clamped-Clamped-Clamped Plates

The non-dimensional buckling load parameters for SiSiSiSi and ClClClCl plates were presented on Table 2a and 2b and the behavior of the critical buckling load against Aspect ratio is shown in Figure 1.

Table 2a  Non dimensional buckling load parameters for SSSS and ClClClCl plates for aspect ratio of b/a

<table>
<thead>
<tr>
<th>b/a</th>
<th>2</th>
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<th>1.8</th>
<th>1.7</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
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<td>16.111</td>
<td>16.9186</td>
<td>17.8983</td>
<td>19.1036</td>
</tr>
<tr>
<td>B_{crt}</td>
<td>15.43632 \frac{D}{a^2}</td>
<td>16.11018 \frac{D}{a^2}</td>
<td>16.91777 \frac{D}{a^2}</td>
<td>17.89753 \frac{D}{a^2}</td>
<td>19.10282 \frac{D}{a^2}</td>
</tr>
</tbody>
</table>

Table 2b

<table>
<thead>
<tr>
<th>b/a</th>
<th>1.5</th>
<th>1.4</th>
<th>1.3</th>
<th>1.2</th>
<th>1.1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>B_{crt}</td>
<td>20.6098 \frac{D}{a^2}</td>
<td>22.52811 \frac{D}{a^2}</td>
<td>25.02498 \frac{D}{a^2}</td>
<td>28.35882 \frac{D}{a^2}</td>
<td>32.94889 \frac{D}{a^2}</td>
<td>39.50795 \frac{D}{a^2}</td>
</tr>
</tbody>
</table>
Table 2b: \( B \)-values from present study compared with previous works for SiSiSiSi rectangular plate buckling.

<table>
<thead>
<tr>
<th>Aspect Ratios ((p = b/a))</th>
<th>( B_{crt} )-Values from Present Study ((i))</th>
<th>( B_{crt} )-Values from Ibearugbulem et al. (2014) ((ii))</th>
<th>( B_{crt} )-Values from Ventsel &amp; Krauthemmer (2001), Megson (2010), Chajes (1974) ((iii))</th>
<th>Percentage Difference Between ((i)) and ((ii))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>39.508</td>
<td>39.508</td>
<td>39.488</td>
<td>0</td>
</tr>
<tr>
<td>1.1</td>
<td>32.9489</td>
<td>32.9492</td>
<td>32.932</td>
<td>-0.00091</td>
</tr>
<tr>
<td>1.2</td>
<td>28.3588</td>
<td>28.3593</td>
<td>28.344</td>
<td>-0.00176</td>
</tr>
<tr>
<td>1.3</td>
<td>25.025</td>
<td>25.0256</td>
<td>25.011</td>
<td>-0.0024</td>
</tr>
<tr>
<td>1.4</td>
<td>22.5281</td>
<td>22.5288</td>
<td>22.515</td>
<td>-0.00311</td>
</tr>
<tr>
<td>1.5</td>
<td>20.6095</td>
<td>20.6102</td>
<td>20.597</td>
<td>-0.0034</td>
</tr>
<tr>
<td>1.6</td>
<td>19.1028</td>
<td>19.1036</td>
<td>19.091</td>
<td>-0.00419</td>
</tr>
<tr>
<td>1.7</td>
<td>17.8975</td>
<td>17.8983</td>
<td>17.886</td>
<td>-0.00447</td>
</tr>
</tbody>
</table>
Table 3a continued

<table>
<thead>
<tr>
<th>Aspect Ratio, $p$</th>
<th>$B_{crit}$, Present</th>
<th>$B_{crit}$, Previous</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>16.9178</td>
<td>16.9186</td>
<td>-0.0008</td>
</tr>
<tr>
<td>1.9</td>
<td>16.1102</td>
<td>16.111</td>
<td>-0.0009</td>
</tr>
<tr>
<td>2</td>
<td>15.4363</td>
<td>15.4371</td>
<td>-0.0008</td>
</tr>
</tbody>
</table>

Figure 2: Graph of critical buckling load, $B_{crit}$ against aspect ratio, $p = b/a$ for CICICICI (Present)

Figure 2b: Graph of critical buckling load, $B_{crit}$ against aspect ratio, $p = b/a$ for CICICICI (Previous)
Table 3b: B-values from present study compared with previous works for different aspect ratio for ClClClCl rectangular plate buckling.

<table>
<thead>
<tr>
<th>Aspect Ratios (p = b/a)</th>
<th>B-Values from Present Study (i)</th>
<th>B-Values from Ibearugbulem et al. (2014) (ii)</th>
<th>B-Values from Ventsel&amp; Krauthammer (2001), Megson(2010), Chajes (1974) (iii)</th>
<th>Percentage Difference Between (i) and (ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>108.6667</td>
<td>108.667</td>
<td>108.654</td>
<td>-0.00028</td>
</tr>
<tr>
<td>1.1</td>
<td>91.0823</td>
<td>91.082</td>
<td>91.068</td>
<td>0.000329</td>
</tr>
<tr>
<td>1.2</td>
<td>79.41538</td>
<td>79.415</td>
<td>79.402</td>
<td>0.000478</td>
</tr>
<tr>
<td>1.3</td>
<td>71.3566</td>
<td>71.3565</td>
<td>71.3425</td>
<td>0.00014</td>
</tr>
<tr>
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<td>65.5979</td>
<td>65.5829</td>
<td>0.000152</td>
</tr>
<tr>
<td>1.5</td>
<td>61.3621</td>
<td>61.3621</td>
<td>61.3501</td>
<td>0</td>
</tr>
<tr>
<td>1.6</td>
<td>58.1679</td>
<td>58.167</td>
<td>58.151</td>
<td>0.001547</td>
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<tr>
<td>1.7</td>
<td>55.7064</td>
<td>55.706</td>
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<td>53.773</td>
<td>53.761</td>
<td>0.000744</td>
</tr>
<tr>
<td>1.9</td>
<td>52.2299</td>
<td>52.229</td>
<td>52.212</td>
<td>0.001723</td>
</tr>
<tr>
<td>2</td>
<td>50.9792</td>
<td>50.979</td>
<td>50.965</td>
<td>0.000392</td>
</tr>
</tbody>
</table>

Conclusion

From the conducted research, one can conclude that:

i. Non dimensional parameters along x and y respectively formed a differential product of \((R-2R^2+R^4)\times(Q^2-2Q^3+Q^4)\) for the case of Simple-Simple -Simple-Simple plate while \((R^2-2R^3+R^4)\times(Q^2-2Q^3+Q^4)\) Clamped-Clamped-Clamped-Clamped and so giving different Differential values

ii. In both cases, as the aspect ratio increases, the corresponding critical buckling load reduces. That means under the same stress, they behave alike.

iii. At every stiffness, the ClClClCl Plate shows high value of stiffness coefficient compared to its corresponding value of SiSiSiSi Plate. That means that the slightest load to cause buckling is more in SiSiSiSi Plate than in ClClClCl Plate.

References


