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Comparative Analysis of Two Different but Uniformly Supported Isotropic Plates Using Odd Order Energy Functional

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ABSTRACT

This research deals on the comparative analysis of two different but uniformly supported plate, using odd energy functional. Two different rectangular isotropic plate with one, simply supported all round and the other Clamped all round, were closely examined. From several minimization of the shape functions, 3rd order strain energy equation was formulated. Further minimization of this gave rise to the Third Order Total Potential Energy Functional. Integrating the t Total Energy functional with respect to the amplitude produced the Governing equation. Various coefficients which explains the extent of their stiffnesses were formulated. The Third order strain energy equation was also formulated, from where the Third Order Total Potential Energy Functional was formed. The critical buckling load equations emerged by further minimizing the governing equation. By substituting the different aspect ratios into the equation, the non-dimensional buckling load parameters were obtained. The final outcome in both cases were critically observed as detailed below.

INTRODUCTION

THIRD ORDER ENERGY FUNCTIONAL FOR THE PLATES

The two uniformly supported plate under consideration in the research work are simple-simple-simple simple and clamped-clamped-clamped-clamped rectangular plates. The displacements of a thin rectangular plate include in-plane displacements – u and v and out of plane displacement – w. Considering u and v as the functions of x, y and z, w is only a function of x and y and so x, y and z are the principal coordinates. The implication of this is that w is constant along z direction. This is in consonant with the assumption that– "vertical normal strain of a plate is equal to zero". The vertical shear strains are negligible in classical plate analysis and assumed to be equal to zero. Thus, out of the six engineering strain components, ε_z , γ_{xz} and γ_{yz} were assumed to be zero. Therefore, leaving only three engineering strain components - ε_x, -ε_y, and γ_{xy}.

Upon the minimization of the strain deflection.

METHODOLOGY

Direct integration of the strain energy formed the fundamental for the formulation of the needed functional. The strain, stress, shear stress and shear strain were further introduced by substitution and this gave rise to the flexural rigidity. When the derived strain energy was added to the external work done, the Total potential energy, T_p was derived. The stages involved were as detailed below,

From the f the strain deflection,

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -z\frac{\partial^2 w}{\partial x \partial y} - z\frac{\partial^2 w}{\partial x \partial y} = -2z\frac{\partial^2 w}{\partial x \partial y} \qquad 1$$

and stress strain relationship, the stress deflection relationship were formulated

$$\sigma_x = \frac{-Ez}{1-\mu^2}\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) \qquad 2$$

Similarly, substituting Equations 3.18 and 3.19 into Equation 3.8 gives:

$$\sigma_y = \frac{-Ez}{1-\mu^2}\left(\mu\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \qquad 3$$

The summation of the product of stress and strain at every point on the plate continuum gives

$$\sigma\varepsilon = \sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} \qquad 4$$

Further substitutions and minimizations gives the 3rd Strain energy equation as

$$E_u = \frac{D}{2}\int_0^a\int_0^b\left(\frac{\partial^3 w}{\partial x^3}.\frac{\partial w}{\partial x} + 2\frac{\partial^3 w}{\partial x\,\partial y^2}.\frac{\partial w}{\partial x} + \frac{\partial^3 w}{\partial y^3}.\frac{\partial w}{\partial y}\right)dxdy \quad 5$$

or

$$E_u = \frac{D}{2}\int_0^a\int_0^b\left(\frac{\partial^3 w}{\partial x^3}.\frac{\partial w}{\partial x} + 2\frac{\partial^3 w}{\partial x^2\,\partial y}.\frac{\partial w}{\partial y} + \frac{\partial^3 w}{\partial y^3}.\frac{\partial w}{\partial y}\right)dxdy \quad 6$$

Upon the addition of the external work done, W_{rk} the Total potential energy, T_p was formulated as

$$T_p = \frac{A^2D}{2}\int_0^a\int_0^b \left(\frac{\partial^3 h}{\partial x^3}.\frac{\partial h}{\partial x} + 2\frac{\partial^3 h}{\partial x\,\partial y^2}.\frac{\partial h}{\partial x} + \frac{\partial^3 h}{\partial y^3}.\frac{\partial h}{\partial y}\right)dxdy - \frac{A^2Nx}{2}\int\int\frac{\partial^2 h}{\partial x^2}.h dxdy \quad 7$$

or

$$T_p = \frac{A^2D}{2}\int_0^a\int_0^b \left(\frac{\partial^3 h}{\partial x^3}.\frac{\partial h}{\partial x} + 2\frac{\partial^3 h}{\partial x^2\,\partial y}.\frac{\partial h}{\partial y} + \frac{\partial^3 h}{\partial y^3}.\frac{\partial h}{\partial y} - \frac{Nx}{2}\int\int\frac{\partial^2 h}{\partial x^2}.h\right)dxdy \quad 8$$

Differentiating the Total Potential energy with respect to the Amplitude and further substitutions gives

$$B_{crt} = \frac{\frac{2AD}{2}\int_0^1\int_0^1\left(\left(\left[\frac{\partial^3 h}{\partial x^3}\right]\frac{\partial h}{\partial x} + 2\left[\frac{\partial^3 h}{\partial x^2\,\partial y}\right]\frac{\partial h}{\partial y} + \left[\frac{\partial^3 h}{\partial y^3}\right]\frac{\partial h}{\partial y}\right)dxdy\right.}{\frac{2A}{2}\int_0^a\int_0^b\left(\frac{\partial h}{\partial x}\right)^2 dxdy} \quad 9$$

or

$$B_{crt} = \frac{\frac{2AD}{2}\int_0^1\int_0^1\left(\left(\left[\frac{\partial^3 h}{\partial x^3}\right]\frac{\partial h}{\partial x} + 2\left[\frac{\partial^3 h}{\partial x\,\partial y^2}\right]\frac{\partial h}{\partial x} + \left[\frac{\partial^3 h}{\partial y^3}\right]\frac{\partial h}{\partial y}\right)dxdy\right.}{\frac{2A}{2}\int_0^a\int_0^b\left(\frac{\partial h}{\partial x}\right)^2 dxdy} \quad 10$$

The critical buckling load equation was further reduced to

$$B_{crt} = \frac{\frac{D}{a^2}\left(cr_1 + 2\frac{1}{p^2}\,cr_2 + \frac{1}{p^4}cr_3\right)}{cr_6} \quad 11$$

where

$$cr_1 = \int_0^1\int_0^1 \overline{K1}\,dRdQ = \int_0^1\int_0^1 \frac{\partial^3 h}{\partial R^3}.\frac{\partial h}{\partial R}dRdQ \quad 12$$

$$cr_2 = \int_0^1\int_0^1 \overline{K2}\,dRdQ = \int_0^1\int_0^1 \left[\frac{\partial^3 h}{\partial R\partial Q^2}\right].\frac{\partial h}{\partial R}\,dRdQ \quad 13$$

$$cr_3 = \int_0^1\int_0^1 \overline{K3}\,dRdQ = \int_0^1\int_0^1 \frac{\partial^3 h}{\partial Q^3}.\frac{\partial h}{\partial Q}dRdQ \quad 14$$

$$cr_4 = \int_0^1\int_0^1 \overline{K6}\,dRdQ = \int_0^1\int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 dRdQ \quad 15$$

Critical Buckling Load Equation for SiSiSiSi plates

The critical buckling load equation for SSSS can be written in terms of stiffness coefficients

(cr_1, cr_2, cr_3 and cr_6) using the $a^2 = b^2/p^2$, for the .

aspect ratio of p = b/a as follows

$$B_{crt} = \frac{D\left(cr_1 + \frac{2}{p^2}cr_2 + \frac{1}{p^4}cr_3\right)}{cr_6 a^2} \quad 16$$

Critical Buckling Load Equation for ClClClCl Plates

Substituting the stiffness coefficients (cr_1, cr_2, cr_3 and cr_6) for the ClClClCl plate, into the general buckling load equation, the critical buckling load equation the plate can be expressed as

$$B_{crt} = \frac{D\left(cr_1 + \frac{2}{p^2}cr_2 + \frac{1}{p^4}cr_3\right)}{k_6 a^2} \quad 17$$

Considering $a^2 = b^2/p^2$, for p = b/a as the aspect ratio.

Determination of the Stiffness coefficients of the plates

From the Polynomial rules, the shape function of the shape function, sh for SiSiSiSi Plate

is as $(R-2R^3+R^4)\,(Q-2Q^3+Q^4)$ 18

Differential values for Simple-Simple-Simple-Simple shape

The various derivatives of the shape functions can be expressed as

$sh = (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$ 19

$\frac{\partial sh}{\partial R} = (1 - 6R^2 + 4R^3)(Q - 2Q^3 + Q^4)$ 20

$\frac{\partial^2 sh}{\partial R^2} = (-12R + 12R^2)(Q - 2Q^3 + Q^4)$ 21

$\frac{\partial^3 sh}{\partial R^3} = (-12 + 24R)(Q - 2Q^3 + Q^4)$ 22

$\frac{\partial h}{\partial Q} = (R - 2R^3 + R^4)(1 - 6Q^2 + 4Q^3)$ 23

$\frac{\partial^2 sh}{\partial Q^2} = (R - 2R^3 + R^4)(-12Q + 12Q^2)$ 24

$\frac{\partial^3 sh}{\partial Q^3} = (R - 2R^3 + R^4)(-12 + 24Q)$ 25

$\frac{\partial^2 sh}{\partial R\,\partial Q} = (1 - 6R^2 + 4R^3)(1 - 6Q^2 + 4Q^3)$ 26

$\frac{\partial^3 sh}{\partial R\,\partial Q^2} = (1 - 6R^2 + 4R^3)(-12Q + 12Q^2)$ 27

$\frac{\partial^3 sh}{\partial R^3} = (-12 + 24R)(Q - 2Q^3 + 4Q^4)$ 28

$\overline{Crt1} = \frac{\partial^3 sh}{\partial R^3} * \frac{\partial sh}{\partial R}$ 29

$\overline{Crt2} = \frac{\partial^3 sh}{\partial R\,\partial Q^2} * \frac{\partial sh}{\partial R}$ 30

$\overline{Crt3} = \frac{\partial^3 sh}{\partial Q^3} * \frac{\partial sh}{\partial Q}$ 31

$\overline{Crt6} = \frac{\partial^2 sh}{\partial R^2}$ 41

From the expressions above,

$\frac{\partial^3 sh}{\partial R^3} * \frac{\partial sh}{\partial R} = (-12 + 24R)(Q - 2Q^3 + Q^4)x(1 - 6R^2 + 4R^3)(Q - 2Q^3 + Q^4)$

Collecting the like terms together yields

$(-12 + 24R)\ (1 - 6R^2 + 4R^3)\ x\ (Q - 2Q^3 + Q^4)(Q - 2Q^3 + Q^4)$

and multiplying out each bracket gives

$-12(1 - 6R^2 + 4R^3) + 24R\ (1 - 6R^2 + 4R^3)$

$x\ Q(Q - 2Q^3 + Q^4) - 2Q^3(Q - 2Q^3 + Q^4) + Q^4(Q - 2Q^3 + Q^4)$

which finally changes to

$(-12 + 72R^2 - 48R^3 + 24R - 144R^3 + 96R^4)\ x\ (Q^2 - 2Q^4 + Q^5 - 2Q^4 + 4Q^6 - 2Q^7 + Q^5 - 2Q^7 + Q^8)$

Therefore $\frac{\partial^3 sh}{\partial R^3} * \frac{\partial sh}{\partial R} = (-12 + 24R + 72R^2 - 192R^3 + 96R^4)$

$x(Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^8)$ 42

But $crt_1 = \iint \overline{Crt1}\, dRdQ = \iint \frac{\partial^3 sh}{\partial R^3} * \frac{\partial sh}{\partial R}\, dRdQ$

That implies that $cr_1 = \int_0^1 \int_0^1 (-12 + 24R + 72R^2 - 192R^3 + 96R^4)$ x

$(Q^2 - 4Q^4 + 2Q^5 + 4Q^6 - 4Q^7 + Q^8)dRdQ$ 43

$= [[(\frac{-12R}{1} + \frac{24R^2}{2} + \frac{72R^3}{3} - \frac{192R^4}{4} + \frac{96R^5}{5})\ x\ (\frac{Q^3}{3} - \frac{4Q^5}{5} + \frac{2Q^3}{6} + \frac{4Q^7}{7} - \frac{4Q^8}{8} + \frac{Q^9}{9})]^1]^1$

$= (-12 + \frac{24}{2} + \frac{72}{3} - \frac{192}{4} + \frac{96}{5})\ x\ (\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9})$

Therefore $crt_1 = (-4\frac{4}{5})\ x\ (\frac{31}{630}) = \frac{-124}{525}$

also,

$\frac{\partial^3 sh}{\partial R\,\partial Q^2} * \frac{\partial sh}{\partial R} = (1 - 6R^2 + 4R^3)(-12Q + 12Q^2)\ x\ (1 - 6R^2 + 4R^3)(Q - 2Q^3 + Q^4)$

Collecting the like terms together yields

$= (1 - 6R^2 + 4R^3)\,(1 - 6R^2 + 4R^3)\ x\ (-12Q + 12Q^2)\ \ (Q - 2Q^3 + Q^4)$

and multiplying out each bracket gives

$= 1(1 - 6R^2 + 4R^3) - 6R^2\,(1 - 6R^2 + 4R^3) + 4R^3(1 - 6R^2 + 4R^3)\ x$

$-12Q(Q - 2Q^3 + Q^4) + 12Q^2\,(Q - 2Q^3 + Q^4)$

which finally changes to

$(1 - 6R^2 + 4R^3 - 6R^2 + 36R^4 - 24R^5 + 4R^3 - 24R^5 + 16R^6)x\,(-12Q^2 + 24Q^4 - 12Q^5 + 12Q^3 - 24Q^5 + 12Q^6)$

$= (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6)\ x\ (-12Q^2 + 12Q^3 + 24Q^4 - 36Q^5 + 12Q^6)$

Therefore $\frac{\partial^3 h}{\partial R\,\partial Q^2} * \frac{\partial h}{\partial R} = (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6)x$

$(-12Q^2 + 12Q^3 + 24Q^4 - 36Q^5 + 12Q^6)$ 44

But $crt_2 = \iint \overline{Crt2}\,dRdQ = \iint \frac{\partial^3 h}{\partial R^3} * \frac{\partial h}{\partial R}\,dRdQ$

That implies that $crt_2 = \int_0^1 \int_0^1 (1 - 12R^2 + 8R^3 + 36R^4 - 48R^5 + 16R^6)\ x$

$(-12Q^2 + 12Q^3 + 24Q^4 - 36Q^5 + 12Q^6)\ dRdQ$ 45

$= [[(\frac{R}{1} + \frac{12R^3}{3} + \frac{8R^4}{4} + \frac{36R^5}{5} - \frac{48R^6}{6} + \frac{16R^7}{7})\ x\ (\frac{-12Q^3}{3} - \frac{12Q^4}{4} +$

$\frac{24Q^5}{5} - \frac{36Q^6}{6} + \frac{12^7}{7})]^1]^1$

$= (\frac{1}{1} + \frac{12}{3} - \frac{8}{4} + \frac{36}{5} - \frac{48}{6} + \frac{16}{7})\ x\ (\frac{-12}{3} + \frac{12}{4} + \frac{24}{5} + \frac{36}{6} - \frac{12}{7})$

Therefore $k_2 = (\frac{17}{35})\ x\ (\frac{-17}{35}) = \frac{-289}{1225}$

also,

$\frac{\partial^3 sh}{\partial Q^3} * \frac{\partial sh}{\partial Q} = (R - 2R^3 + R^4)(-12 + 24Q)\ x\ (R - 2R^3 + R^4)(1 - 6Q^2 + 4Q^3)$

Collecting the like terms together yields

$(R - 2R^3 + R^4)(R - 2R^3 + R^4)\ x\ (-12 + 24Q)\,(1 - 6Q^2 + 4Q^3)$

and multiplying out each bracket, gives

$= R(R - 2R^3 + R^4) - 2R^3\,(R - 2R^3 + R^4) + R^4(R - 2R^3 + R^4)\ x$

$-12(1 - 6Q^2 + 4Q^3) + 24Q(1 - 6Q^2 + 4Q^3)$

which finally changes to

$(R^2 - 2R^4 + R^5 - 2R^4 + 4R^6 - 2R^7 + R^5 - 2R^7 + R^8)\quad x\,(-12 + 72Q^2 - 48Q^3 + 24Q - 144Q^3 + 96Q^4)$

$= (R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8)\ x\,(-12 + 24Q + 72Q^2 - 192Q^3 + 96Q^4)$

Therefore $\frac{\partial^3 sh}{\partial Q^3} * \frac{\partial sh}{\partial Q} = (R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8)\ x\,(-12 + 24Q + 72Q^2 - 192Q^3 + 96Q^4)$

But $crt_3 = \iint \overline{Crt3}\,dRdQ = \iint \frac{\partial^3 sh}{\partial R^3} * \frac{\partial sh}{\partial R}\,dRdQ$

That implies that

$crt_3 = \int_0^1 \int_0^1 (R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8)$

$x\,(-12 + 24Q + 72Q^2 - 192Q^3 + 96Q^4)dRdQ$ 46

$= [[(\frac{R^3}{3} + \frac{4R^5}{5} + \frac{2R^6}{6} + \frac{4R^7}{7} - \frac{4R^8}{8} + \frac{R^9}{9})\ x\ (\frac{-12Q}{1} + \frac{24Q^2}{2} + \frac{72Q^3}{3}$

$+ \frac{192Q^4}{4} + \frac{96Q^5}{5})]^1]^1$

$= (\frac{1}{3} + \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9})\ x\ (\frac{-12}{1} + \frac{24}{2} + \frac{72}{3} - \frac{192}{4} + \frac{96}{5})$

Therefore $k_3 = (\frac{31}{630})\ x\ (-4\frac{4}{35}) = \frac{-124}{525}$ also

$(\frac{\partial sh}{\partial R})^2 = (1 - 6R^2 + 4R^3)(Q - 2Q^3 + Q^4)\ x\ (1 - 6R^2 + 4R^3)(Q - 2Q^3 + Q^4)$

Collecting the like terms together yields

$(1 - 6R^2 + 4R^3)(1 - 6R^2 + 4R^3)\ x\ (Q - 2Q^3 + Q^4)(Q - 2Q^3 + Q^4)$

and multiplying out each bracket gives

$1(1-6R^2+4R^3)-6R^2(1-6R^2+4R^3)+4R^3(1-6R^2+4R^3) x\, Q(Q- 2Q^3+Q^4)-2Q^3(Q-2Q^3+Q^4)+Q^4(Q-2Q^3+Q^4)$

$= (1-6R^2+4R^3 -6R^2+36R^4-24R^5+4R^3 -24R^5+16R^6)\ x\ (Q^2-2Q^4+Q^5- 2Q^4+4Q^6-2Q^7+Q^5-2Q^7+Q^8)$

which finally changes to

$(1-12R^2+8R^3 +36R^4-48R^5+16R^6)\ x(Q^2-4Q^4+2Q^5+4Q^6- 4Q^7+Q^8)$

Therefore$\left(\frac{\partial sh}{\partial R}\right)^2 = (1-12R^2+8R^3 +36R^4-48R^5+16R^6)$ x

$(Q^2-4Q^4+ 2Q^5+4Q^6-4Q^7+Q^8)$

But $crt_6 = \iint \overline{Crt6} dRdQ = \iint (\frac{\partial sh}{\partial R})^2 dRdQ$

That implies that $crt_6 = \int_0^1 \int_0^1 (1-12R^2+8R^3 +36R^4-48R^5+16R^6)$ x

$(Q^2-4Q^4+ 2Q^5+4Q^6-4Q^7+Q^8)dRdQ$ 47

$= [[(\frac{R}{1} - \frac{12R^3}{3} + \frac{8R^4}{4} + \frac{36R^5}{5} - \frac{48R^6}{6} + \frac{16R^7}{7})\ x\ (\frac{Q^3}{3} - \frac{4Q^5}{5} + \frac{2Q^6}{6}$
$+ \frac{4Q^7}{7} - \frac{4Q^8}{8} + \frac{Q^9}{9})]^1]^1$

$= [[(\frac{1}{1} - \frac{12}{3} + \frac{8}{4} + \frac{36}{5} - \frac{48}{6} + \frac{16}{7})\ x\ (\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9})]^1]^1$

Therefore $crt_6 = (\frac{17}{35})\ x\ (\frac{31}{630}) = \frac{527}{22050}$

Similarly from Polynomial rules, the shape function, sh for ClClClCl panel

is $(R^2-2R^3+R^4)(Q^2-2Q^3+Q^4)$ 48

Differential values for Clamped-Clamped-Clamped-Clamped shape

Also the various differential values for ClClClCl shape functions are as detail

$sh = (R^2-2R^3+R^4)(Q^2-2Q^3+Q^4)$ 49

$\frac{\partial sh}{\partial R} = (2R-6R^2+4R^3)(Q^2-2Q^3+Q^4)$ 50

$\frac{\partial^2 sh}{\partial R^2} = (2-12R+12R^2)(Q^2-2Q^3+Q^4)$ 51

$\frac{\partial^3 sh}{\partial R^3} = (-12+24R)(Q^2-2Q^3+Q^4)$ 52

$\frac{\partial sh}{\partial Q} = (R^2-2R^3+R^4)(2Q-6Q^2+4Q^3)$ 53

$\frac{\partial^2 sh}{\partial Q^2} = (R^2-2R^3+R^4)(2-12Q+12Q^2)$ 54

$\frac{\partial^3 sh}{\partial Q^3} = (R^2-2R^3+R^4)(-12+24Q)$ 55

$\frac{\partial^2 sh}{\partial R\, \partial Q} = (2R-6R^2+4R^3)(2Q-6Q^2+4Q^3)$ 56

$\frac{\partial^3 sh}{\partial R\, \partial Q^2} = (2R-6R^2+4R^3)(2-12Q+12Q^2)$ 57

$\overline{Crt1} = \frac{\partial^3 sh}{\partial R^3} * \frac{\partial sh}{\partial R}$ 58

$\overline{Crt2} = \frac{\partial^3 sh}{\partial R\, \partial Q^2} * \frac{\partial sh}{\partial R}$ 59

$\overline{Crt3} = \frac{\partial^{3s} h}{\partial Q^3} * \frac{\partial sh}{\partial Q}$ 60

$\overline{Crt6} = \frac{\partial^2 sh}{\partial R^2}$ or $(\frac{\partial sh}{\partial R})^2$ 61

From the expressions above,

$\frac{\partial^3 sh}{\partial R^3} * \frac{\partial sh}{\partial R} = (-12+24R)(Q^2-2Q^3+Q^4)\ x\ (2R-6R^2+4R^3)(Q^2-2Q^3+Q^4)$

Collecting the like terms together yields

$(-12+24R)\ (2R-6R^2+4R^3)\ x\ (Q^2-2Q^3+Q^4)(Q^2-2Q^3+Q^4)$

and multiplying out each bracket gives

$-12(2R-6R^2+4R^3)\ +24R\,(2R-6R^2+4R^3)$ x $Q^2(Q^2-2Q^3+Q^4)$

$-2Q^3(Q^2-2Q^3+Q^4)+Q^4(Q^2-2Q^3+Q^4)$

$=(-24R+72R^2-48R^3+48R^2-144R^3+96R^4)$

$x(Q^4-2Q^5+Q^6-2Q^5+4Q^6-2Q^7+Q^6-2Q^7+Q^8)$

which finally changes to

$(-24R+120R^2-192R^3+96R^4)$ x $(Q^4-4Q^5+6Q^6-4Q^7+Q^8)$

Therefore $\frac{\partial^3 sh}{\partial R^3}*\frac{\partial sh}{\partial R}=(-24R+120R^2-192R^3+96R^4)$ x $(Q^4-4Q^5+6Q^6-4Q^7+Q^8)$

But $crt_1=\iint\overline{Crt1}\,dRdQ=\iint\frac{\partial^3 h}{\partial R^3}*\frac{\partial h}{\partial R}\,dRdQ$

That implies that

$crt_1=\int_0^1\int_0^1(-24R+120R^2-192R^3+96R^4)$ x $(Q^4-4Q^5+6Q^6-4Q^7+Q^8)dRdQ$ 62

$=[[(\frac{-24R^2}{2}+\frac{120R^3}{3}-\frac{192R^4}{4}+\frac{96R^5}{5})$ x $(\frac{Q^5}{5}-\frac{4Q^6}{6}+\frac{6Q^7}{7}-\frac{4Q^8}{8}+\frac{Q^9}{9})]^1]^1$

$=[[(\frac{-24}{2}+\frac{120}{3}-\frac{192}{4}+\frac{96}{5})$ x $(\frac{1}{5}-\frac{4}{6}+\frac{6}{7}-\frac{4}{8}+\frac{1}{9})$

Therefore

$crt_1=(-\frac{4}{5})$ x $(\frac{1}{630})=\frac{-4}{3150}$ also**,**

$\frac{\partial^3 sh}{\partial R\,\partial Q^2}*\frac{\partial sh}{\partial R}=(2R-6R^2+4R^3)(2-12Q+12Q^2)x(2R-6R^2+4R^3)(Q^2-2Q^3+Q^4)$

Collecting the like terms together yields

$=(2R-6R^2+4R^3)\,(2R-6R^2+4R^3)$ x $(2-12Q+12Q^2)\ (Q^2-2Q^3+Q^4)$

and multiplying out each bracket gives

$2R$ x $2(Q^2-2Q^3+Q^4)-12Q(Q^2-2Q^3+Q^4)+12Q^2(Q^2-2Q^3+Q^4)$

$=(4R^2-12R^3+8R^4-12R^3+36R^4-24R^5+8R^4-24R^5+16R^6)$

x $(2Q^2-4Q^3 2Q^4-12Q^3+12Q^4-12Q^5+12Q^4+24Q^5+12Q^6)$

which finally changes to

$(4R^2-24R^3+52R^4-48R^5+16R^6)x\,(2Q^2-16Q^3+38Q^4-36Q^5+12Q^6)$ `

Therefore

$\frac{\partial^3 sh}{\partial R^3}*\frac{\partial sh}{\partial R}=(4R^2-24R^3+52R^4-48R^5+16R^6)x(2Q^2-16Q^3+38Q^4-36Q^5+12Q^6)$

But $crt_2=\iint\overline{Crt2}\,dRdQ=\iint\frac{\partial^3 h}{\partial R\,\partial Q^2}*\frac{\partial h}{\partial R}\,dRdQ$

That implies that

$crt_2=\int_0^1\int_0^1(4R^2-24R^3+52R^4-48R^5+16R^6)$ x $(2Q^2-16Q^3+38Q^4-36Q^5+12Q^6)$

$dRdQ$ 63

$=[[(\frac{4R^3}{3}+\frac{24R^4}{4}+\frac{52R^5}{5}-\frac{48R^7}{6}+\frac{16R^8}{7})$ x $(\frac{2Q^3}{3}-\frac{16Q^4}{4}+\frac{38Q^5}{5}-\frac{36Q^6}{6}+\frac{12^7}{7})]^1]^1$

$=(\frac{4}{3}-\frac{24}{4}+\frac{52}{5}-\frac{48}{6}+\frac{16}{7})$ x $(\frac{2}{3}-\frac{16}{4}+\frac{38}{5}-\frac{36}{6}+\frac{12}{7})$

Therefore $crt_2=(\frac{2}{105})$ x $(\frac{-2}{105})=\frac{-4}{11025}$ also,

$\frac{\partial^3 sh}{\partial Q^3}*\frac{\partial sh}{\partial Q}=(R^2-2R^3+R^4)(-12+24Q)x\,(R^2-2R^3+R^4)(2Q-6Q^2+4Q^3)$

Collecting the like terms together yields

$(R^2-2R^3+R^4)(R^2-2R^3+R^4)$ x $(-12+24Q)\,(2Q-6Q^2+4Q^3)$

and multiplying out each bracket gives

$R^2(R^2-2R^3+R^4)-2R^3(R^2-2R^3+R^4)+R^4(R^2-2R^3+R^4)$

$x-12(2Q-6Q^2+4Q^3)+24Q(2Q-6Q^2+4Q^3)$

$= (R^4 - 2R^5 + R^6 - 2R^5 + 4R^6 - 2R^7 + R^6 - 2R^7 + R^8)$

$x\ (-24Q + 72Q^2 - 48Q^3 + 48Q^2 - 144Q^3 + 96Q^4)$

which finally changes to

$(R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)\ x\ (-24Q + 120Q^2 - 192Q^3 + 96Q^4)$

Therefore

$\frac{\partial^3 sh}{\partial Q^3} * \frac{\partial sh}{\partial Q} = (R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)\ x\ (-24Q + 120Q^2 - 192Q^3 + 96Q^4)$

But $crt_3 = \iint \overline{Crt3} dRdQ = \iint \frac{\partial^3 sh}{\partial Q^3} * \frac{\partial sh}{\partial Q} dRdQ$

That implies that

$crt_3 = \int_0^1 \int_0^1 (R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)\ x\ \ (-24Q + 120Q^2 - 192Q^3 + 96Q^4) dRdQ$ 3.118

$= [[(\frac{R^5}{5} - \frac{4R^6}{6} + \frac{6R^7}{7} - \frac{4R^8}{8} + \frac{R^9}{9})\ x\ (\frac{-24Q^2}{2} + \frac{120Q^3}{3} - \frac{192Q^4}{4} + \frac{96Q^5}{5})]^1]^1$

$= (\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9})\ x\ (-\frac{24}{2} + \frac{120}{3} - \frac{192}{4} + \frac{96}{5})$

Therefore $crt_3 = (\frac{1}{630})\ x\ (-\frac{4}{5}) = \frac{-4}{3150} = \frac{-2}{1575}$

Also

$(\frac{\partial sh}{\partial R})^2 = (2R - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4)\ \ x\ (2R - 6R^2 + 4R^3)(Q^2 - 2Q^3 + Q^4)$

Collecting the like terms together yields

$(2R - 6R^2 + 4R^3)(2R - 6R^2 + 4R^3)\ x\ (Q^2 - 2Q^3 + Q^4)(Q^2 - 2Q^3 + Q^4)$

and multiplying out each bracket gives

$2R(2R - 6R^2 + 4R^3) - 6R^2(2R - 6R^2 + 4R^3) + 4R^3(2R - 6R^2 + 4R^3)$

$x\ Q^2(Q^2 - 2Q^3 + Q^4) - 2Q^3(Q^2 - 2Q^3 + Q^4) + Q^4(Q^2 - 2Q^3 + Q^4)$

$= (4R^2 - 12R^3 + 8R^4 - 12R^3 + 36R^4 - 24R^5 + 8R^4 - 24R^5 + 16R^6)$

$x(Q^4 - 2Q^5 + Q^6 - 2Q^5 + 4Q^6 - 2Q^7 - Q^6 - 2Q^7 + Q^8)$

which finally changes to

$(4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6)\ x\ (Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)$

Therefore

$(\frac{\partial sh}{\partial R})^2 = (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6)\ x\ (Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)$

But $crt_6 = \iint \overline{Crt6} dRdQ = \iint (\frac{\partial sh}{\partial R})^2\ dRdQ$

That implies that

$crt_6 = \int_0^1 \int_0^1 (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^6)\ \ x\ (Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8) dRdQ$ 64

$= [[(\frac{4R^3}{3} - \frac{24R^4}{4} + \frac{52R^5}{5} - \frac{48R^6}{6} + \frac{16R^7}{7})\ x\ (\frac{Q^5}{5} - \frac{4Q^6}{6} + \frac{6Q^7}{7} - \frac{4Q^8}{8} + \frac{Q^9}{9})]^1]^1$

$= [[(\frac{4}{3} - \frac{24}{4} + \frac{52}{5} - \frac{48}{6} + \frac{16}{7})\ x\ (\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9})]^1]^1$

Therefore $crt_6 = (\frac{2}{105})\ x\ (\frac{1}{630}) = \frac{2}{66150}$

Results

The substitution of the stiffness coefficients into the critical buckling load equations gave the results as detailed below

Table 1: Shape functions and stiffness coefficients

Shape Functions, sh	Stiffness Coefficients, crt			
	crt_1	crt_2	crt_3	crt_6
SiSiSiSi shssss = (R-2R3+R4) x (Q-2Q3+Q4)	$\frac{-124}{525}$ = 0.236219	$\frac{-289}{1225}$ = 0.23591	$\frac{-124}{525}$ = 0.236219	$\frac{527}{22050}$ = 0.02390
ClClClCl shcccc = $(R^2 - 2R^3 + R^4)$ x$(Q^2 - 2Q^3 + Q^4)$	$\frac{-2}{1575}$ = 0.00127	$\frac{-4}{11025}$ = 0.00036	$\frac{-2}{1575}$ = 0.00127	$\frac{2}{66150}$ = 0.00003

Result for Simple-Simple-Simple-Simple and Clamped-Clamped-Clamped-Clamped Plates

The non- dimensional buckling load parameters for SiSiSiSi and ClClClCl plates were presented on Table 2a and 2b and the behavior of the critical buckling load against Aspect ratio is shown in Figure1.

Table.2a Non dimensional buckling load parameters for SSSS and ClClClCl plates for aspect ratio of b/a

b/a		2	1.9	1.8	1.7	1.6
B		15.4371	16.111	16.9186	17.8983	19.1036
B_{crt}	SiSiSiSi	$15.43632\frac{D}{a^2}$	$16.11018\frac{D}{a^2}$	$16.91777\frac{D}{a^2}$	$17.89753\frac{D}{a^2}$	$19.10282\frac{D}{a^2}$
	ClClClCl	$50.9792\frac{D}{a^2}$	$52.2299\frac{D}{a^2}$	$53.7734\frac{D}{a^2}$	$55.7064\frac{D}{a^2}$	$58.1679\frac{D}{a^2}$

Table 2b

b/a		1.5	1.4	1.3	1.2	1.1	1
B		20.6102	22.5288	25.0256	28.3593	32.9492	39.508
B_{crt}	SiSiSiSi	$20.6098\frac{D}{a^2}$	$22.52811\frac{D}{a^2}$	$25.02498\frac{D}{a^2}$	$28.35882\frac{D}{a^2}$	$32.94889\frac{D}{a^2}$	$39.50795\frac{D}{a^2}$
	ClClClCl	$61.3621\frac{D}{a^2}$	$65.59795\frac{D}{a^2}$	$71.35659\frac{D}{a^2}$	$79.41538\frac{D}{a^2}$	$91.08228\frac{D}{a^2}$	$108.6667\frac{D}{a^2}$

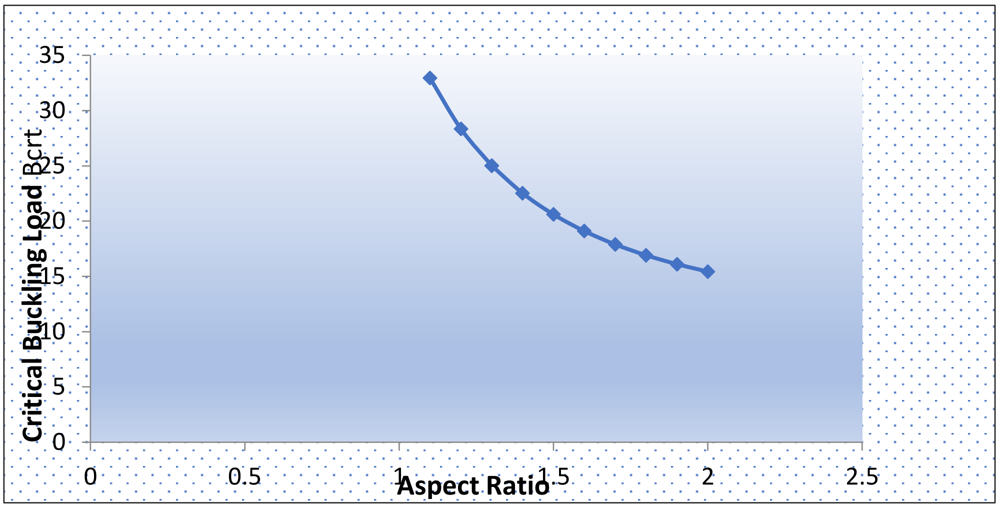

Figure 1a: Graph of critical buckling load against aspect ratio, p = b/a for SiSiSiSi (Present)

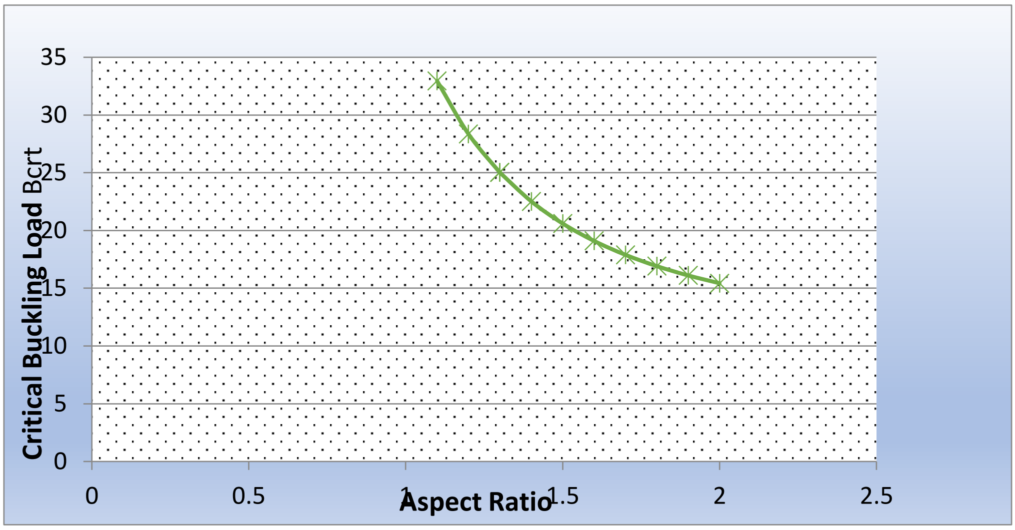

Figure 1b: Graph of critical buckling load against aspect ratio, p = b/a for SiSiSiSi (Previous)

Table 2b: B-values from present study compared with previous works for SiSiSiSi rectangular plate buckling.

Aspect Ratios (p = b/a)	B_{crt}-Values from Present Study (i)	B_{crt}-Values from Ibearugbulem et al. (2014) (ii)	B_{crt}-Values from Ventsel& Krauthermmer (2001),Megson (2010), Chajes (1974) (iii)	Percentage Difference Between (i) and (ii)
1	39.508	39.508	39.488	0
1.1	32.9489	32.9492	32.932	-0.00091
1.2	28.3588	28.3593	28.344	-0.00176
1.3	25.025	25.0256	25.011	-0.0024
1.4	22.5281	22.5288	22.515	-0.00311
1.5	20.6095	20.6102	20.597	-0.0034
1.6	19.1028	19.1036	19.091	-0.00419
1.7	17.8975	17.8983	17.886	-0.00447

1.8	16.9178	16.9186	16.906	-0.00473
1.9	16.1102	16.111	16.099	-0.00497
2	15.4363	15.4371	15.425	-0.00518

Table 3a continued

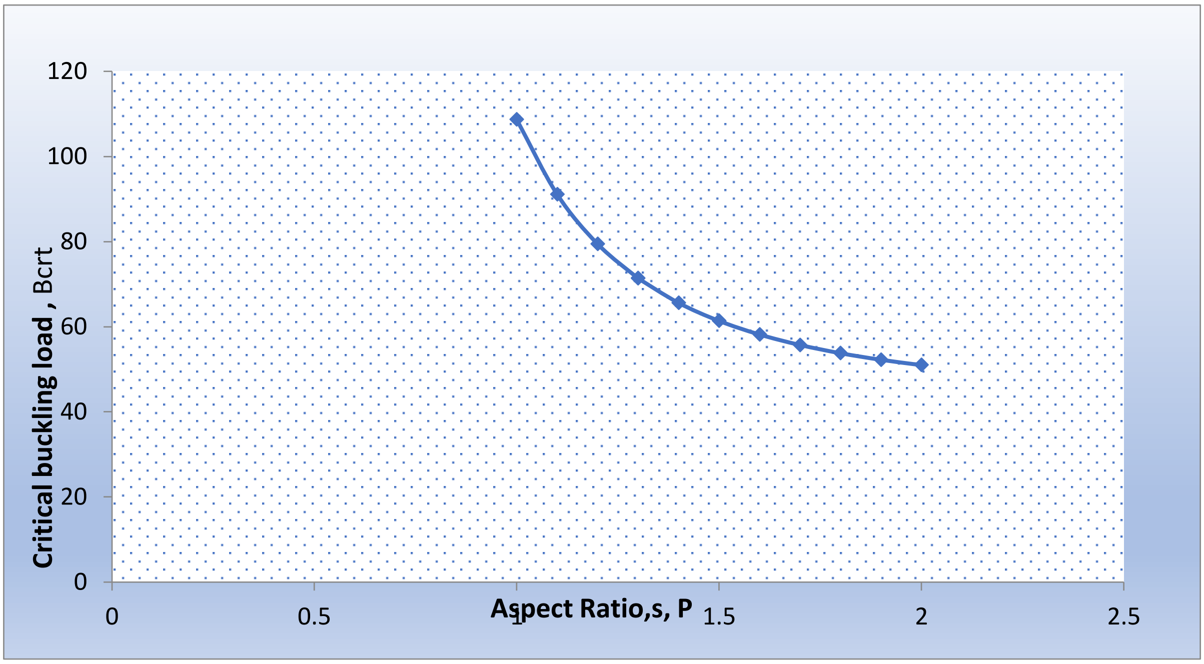

Figure 2: Graph of critical buckling load, B_{crt} against aspect ratio, p = b/a for ClClClCl (Present)

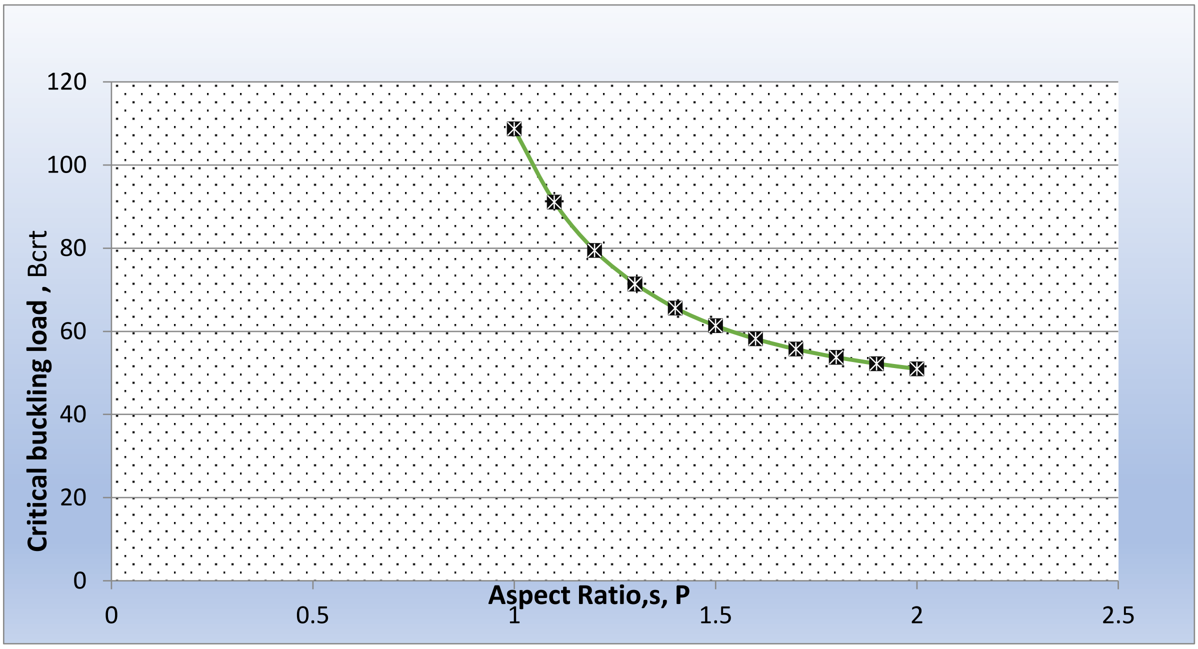

Figure 2b: Graph of critical buckling load, B_{crt} against aspect ratio, p = b/a for ClClClCl (Previous)

Table 3b: B-values from present study compared with previous works for different aspect ratio for ClClClCl rectangular plate buckling.

Aspect Ratios (p = b/a)	B-Values from Present Study (i)	B-Values from Ibearugbulem et al. (2014) (ii)	B-Values from Ventsel& Krauthermmer (2001), Megson(2010), Chajes (1974) (iii	Percentage Difference Between (i) and (ii)
1	108.6667	108.667	108.654	-0.00028
1.1	91.0823	91.082	91.068	0.000329
1.2	79.41538	79.415	79.402	0.000478
1.3	71.3566	71.3565	71.3425	0.00014
1.4	65.598	65.5979	65.5829	0.000152
1.5	61.3621	61.3621	61.3501	0
1.6	58.1679	58.167	58.151	0.001547
1.7	55.7064	55.706	55.694	0.000718
1.8	53.7734	53.773	53.761	0.000744
1.9	52.2299	52.229	52.212	0.001723
2	50.9792	50.979	50.965	0.000392

Conclusion

From the conducted research, one can conclude that:

i. Non dimensional parameters along x and y respectively formed a differential product of ($R-2R^3+R^4$) x ($Q^2-2Q^3+Q^4$) for the case of Simple-Simple -Simple-Simple plate while ($R^2-2R^3+R^4$) x ($Q^2-2Q^3+Q^4$ Clamped- Clamped- Clamped- Clamped and so giving different Differential values

ii. In both cases, as the aspect ratio increases, the corresponding critical buckling load Reduces. That means under the same stress, they behave alike.

iii. At every Stiffness, crt the ClClClCl Plate shows high value of stiffness coefficient compared to its corresponding value of SiSiSi Plate. That means that the slightest load to cause buckling is more in SiSiSiSi Plate than in ClClClCl Plate.

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