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Rayleigh-Exponential-Gamma Distribution: Theory and Properties

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ABSTRACT:

In practice, many components' lifetime distributions have a bathtub shape for their failure rate function. However, only a few distributions have a bathtub-shaped failure rate function. Models with bathtub-shaped failure rate functions are useful in reliability analysis, particularly in decision-making related to reliability, cost analysis, and burn-in analysis. When considering a failure mechanism, the failure of units in a system may be caused by random failure caused by changes in temperature, voltage, and so on, or it may be caused by ageing. In this paper, we propose a new probability distribution called Rayleigh-Exponential-Gamma distribution, provide a comprehensive study of its theory and derive appropriate expressions for its statistical properties. The method of maximum likelihood was employed to estimate its parameter.

KEYWORDS: Exponential-Gamma, Rayleigh Distribution, Survival function, Hazard function, Maximum Likelihood Estimate, Rayleigh-Exponential-Gamma Distribution

I. INTRODUCTION

It is well understood that there are numerous distributions in the statistical field and the development and complexity that occur in the information extracted from statistical data. Particularly in the areas concerned with survival and reliability analysis, new distributions are required to address problems in the data that existing statistical distributions could not handle. The modelling and analysis of lifetime data are essential in many applied sciences, including medicine, engineering, and finance. Exponential, Gamma, and Rayleigh distributions and their generalizations have been used to model various lifetime data. Researchers in many fields of study have contributed to developing and applying new probability distribution models. The following is a brief kind of literature review on the development of new probability distribution and their applications by different scholars such as [1], [2],[3],[4],[5],[6], and some contributions in term of applications of probability into various areas of life such as [7],[8],[9],[10],[11],[13],[14]

II. METHODS

2.1 The Derivation of the new model Rayleigh-Exponential- Gamma Distribution (REGD)

The probability density function of the REGD is derived in this section

Theorem 1: Let x be continuous independent random variables such that; $x \sim \mathrm{RD}(x,\sigma)$ follows an Rayleigh distribution and, let $f(x)$ and $F(x)$ be the probability density function and CDF of gamma distribution given as;

$$f(x) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}, \quad x > 0, \lambda > 0 \tag{1}$$

and $$F(x) = 1 - e^{-x^2/2\sigma^2}, \quad x > 0, \lambda > 0 \tag{2}$$

From Alzeetreh et al (2013), let,

$$g(x) = \frac{f(x)(-\log(1-\mathrm{F(x)})}{\sigma^2(1-F(x)} \exp(-\log(1-F(x))^2 / 2\sigma^2, \quad x > 0 \tag{3}$$

be the pdf of the ED-X, where $f(x)$ and $1-F(x)$ is the pdf and the survival function of the baseline distribution

Similarly, Ogunwale et al. (2019) developed Exponential Gamma Distribution and defined the pdf and cdf as

$$\mathrm{f}(x)=\frac{\lambda^{\alpha+1}X^{\alpha-1}}{\Gamma(\alpha)}e^{-2/x}, \quad x>0,\lambda>0 \tag{4}$$

$$\mathrm{F}(x)=\frac{\lambda\gamma(\alpha,\mathrm{x})}{2^{\alpha}\Gamma(\alpha)}, \quad x>0,\lambda>0 \tag{5}$$

Inserting (4) and (5) into (3) above, then pdf of the Rayleigh-Exponential-Gamma distribution is given as

$$\mathrm{f}(x)=\frac{\lambda^{\alpha+1}x^{\alpha-1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}.\exp\left(-\frac{4\lambda x\sigma^{2}-\theta^{2}}{2\sigma^{2}}\right), \quad x>0,\lambda,\alpha,\sigma,\theta>0 \tag{6}$$

2.2 Statistical Properties of the Rayleigh-Exponential-Gamma Distribution (REGD)

The statistical properties of REGD, especially the first four moments, the variance, coefficient of variation, moment generating function, characteristic function, skewness, and kurtosis are obtained in this section as follows

(i) Moments

Theorem 2: If X is a random variable distributed as an REGD $(x;\theta,\alpha,\sigma,\lambda)$, then the rth non-central moment is given by

$$\mu_{r}^{'}=\frac{\lambda^{\alpha+r}2^{\alpha}(\theta 2^{\alpha+r}\sigma^{2\alpha+r}+\theta^{3\alpha+r})}{(4\lambda)^{\alpha+r}(\sigma^{2\alpha+2}2^{\alpha}\Gamma(\alpha)-\lambda\sigma^{2\alpha+2r}\Upsilon(\alpha,x)} \tag{7}$$

Proof:

$$\mu_{r}^{'}=\int_{0}^{\infty}x^{r}f(x;\lambda)dx$$

$$=\int_{0}^{\infty}x^{r}.\frac{\lambda^{\alpha+1}x^{\alpha-1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}.\exp\left(-\frac{4\lambda x\sigma^{2}-\theta^{2}}{2\sigma^{2}}\right)dx \tag{8}$$

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}\int_{0}^{\infty}x^{\alpha+r-1}.\exp\left(-\frac{4\lambda x\sigma^{2}-\theta^{2}}{2\sigma^{2}}\right)dx \tag{9}$$

Let $u=\frac{4\lambda x\sigma^{2}-\theta^{2}}{2\sigma^{2}},\mathrm{u}\,2\sigma^{2}+\theta^{2}=4\lambda x\sigma^{2} \quad x=\frac{u2\sigma^{2}+\theta^{2}}{4\lambda\sigma^{2}},\mathrm{dx}=\frac{2\sigma^{2}du+\theta^{2}}{4\lambda\sigma^{2}}$, then, so that (9) reduces to:

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}\left(\frac{2\sigma^{2}+\theta^{2}}{4\lambda\sigma^{2}}\right)^{\alpha+r}\int_{0}^{\infty}(U)^{\alpha+r-1}e^{-u}du \tag{10}$$

But $\int_{0}^{\infty}u^{r}e^{-u}du=\Gamma(r+1)$, then

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta(2\sigma^{2})^{\alpha+r}+\lambda^{\alpha+1}2^{\alpha}\theta(\theta^{2})^{\alpha+r}}{(4\lambda\sigma^{2})^{\alpha+r}(\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x}))}\Gamma(\alpha+r) \quad (11)$$

$$\mu_{r}^{'}=\frac{\lambda^{\alpha+1}2^{\alpha}(\theta 2^{\alpha+r}\sigma^{2\alpha+r}+\theta^{3\alpha+r})}{(4\lambda)^{\alpha+r}(\sigma^{2\alpha+2}2^{\alpha}\Gamma(\alpha)-\lambda\sigma^{2\alpha+2r}\gamma(\alpha,\mathrm{x}))}\Gamma(\alpha+r) \quad (12)$$

the first(mean), second, third, and fourth moment is obtained by substituting r=1,2,3 and 4 respectively as follows;

Substituting $\mathrm{r}=1,2,3,4$, in (12) we obtain the mean, the second moment, third, and the fourth moment for REGD. We also obtained the Variance from the association

$$V(x)=\mu_{2}^{'}-\left(\mu_{1}^{'}\right)^{2}$$

The mean is given as

$$mean=\mu_{1}^{'}=\mu_{r}^{'}=\frac{2^{\alpha}(\theta 2^{\alpha+1}\sigma^{2\alpha+1}+\theta^{3\alpha+1})}{4^{\alpha+1}(2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x}))}\Gamma(\alpha+1) \quad (13)$$

$$\mu_{2}^{'}=\frac{2^{\alpha}(\theta 2^{\alpha+2}\sigma^{2\alpha+2}+\theta^{3\alpha+2})}{4^{\alpha+2}(2^{\alpha}\Gamma(\alpha)-\lambda\sigma^{2}\gamma(\alpha,\mathrm{x}))}\Gamma(\alpha+2)$$

$$Variance=V(x)=\frac{8^{2\alpha}2\theta-\theta\sigma^{4\alpha+2}+32^{3\alpha}\theta^{6\alpha+4}-\theta 2^{2\alpha+2}+\sigma^{6\alpha+2}}{4^{2\alpha+2}(4^{2\alpha}\Gamma(\alpha)-\lambda^{2}\sigma^{2}\gamma(\alpha,\mathrm{x})^{2})}\Gamma(\alpha+1) \quad (14)$$

(ii) Coefficient of variation (C.V) is a standardized measure of the dispersion of a probability distribution and is given as;

$$C.V=\frac{\sigma}{\mu}$$

$$=\frac{(8^{2\alpha}2\theta-\theta\sigma^{4\alpha+2}+32^{3\alpha}\theta^{6\alpha+4}-\theta 2^{2\alpha+2}+\sigma^{6\alpha+2})^{\frac{1}{2}}}{(4^{2\alpha+2}(4^{2\alpha}\Gamma(\alpha)-\lambda^{2}\sigma^{2}\gamma(\alpha,x))^{\frac{1}{2}}}\cdot\frac{4^{\alpha+1}(2^{\alpha}\Gamma(\alpha)-\lambda\Upsilon(\alpha,x)}{2^{\alpha}(\theta 2^{\alpha+1}\sigma^{2\alpha+1}+\theta^{3\alpha+1})} \quad (15)$$

(iii) Moment generating function (M.G.F)

Theorem 3: If X is a continuous random variable distributed as an REGD $(x;\theta,\alpha,\sigma,\lambda)$, then the moment generating function is defined as

$$M_{x}(t)=\frac{\lambda^{\alpha+1}4^{2\alpha}\theta\sigma^{2\alpha}}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)(4^{\alpha}\lambda^{\alpha}\sigma^{2\alpha}-t^{\alpha}-\theta^{2\alpha})}\Gamma(\alpha+1)$$

Proof:

$$\mathrm{M}_{x}(t)=E\left(e^{tx}\right)=\int_{0}^{\infty}e^{tx}f(x;\lambda)dx$$

$$M_{x}(t)=\int_{0}^{\infty}\frac{\lambda^{\alpha+1}x^{\alpha-1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}.\exp\left(-\frac{4\lambda x\sigma^{2}-\theta^{2}}{2\sigma^{2}}\right)e^{tx}dx$$

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}\int_{0}^{\infty}x^{\alpha-1}.\exp\left(-\frac{x(4\lambda\sigma^{2}-t)-\theta^{2}}{2\sigma^{2}}\right)dx \quad (16)$$

(12)

Let $u=\dfrac{x(4\lambda\sigma^2-t)-\theta^2}{2\sigma^2}$, $x=\dfrac{u2\sigma^2}{(4\lambda\sigma^2-\mathrm{t})-\theta^2}$, then $dx=\dfrac{du2\sigma^2}{(4\lambda\sigma^2-\mathrm{t})-\theta^2}$ so that (16) is reduced to

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta}{\sigma^2 2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)}\int_0^{\infty}\left(\frac{u2\sigma^2}{(4\lambda\sigma^2-t)-\theta^2}\right)^{\alpha-1}e^{-u}\frac{du2\sigma^2}{(4\lambda\sigma^2-t-\theta^2)}$$

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta}{\sigma^2 2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)}\left(\frac{(2\sigma^2)^{\alpha}}{(4\lambda\sigma^2-t)-\theta^2}\right)^{\alpha}\Gamma(\alpha+1)$$

$$M_x(t)=\frac{\lambda^{\alpha+1}4^{2\alpha}\theta\sigma^{2\alpha}}{\sigma^2 2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)(4^{\alpha}\lambda^{\alpha}\sigma^{2\alpha}-t^{\alpha}-\theta^{2\alpha})}\Gamma(\alpha+1) \tag{17}$$

(iv) Characteristic function (C.F)

Theorem 5: If X is a random variable distributed as a REGD $(x;\theta,\alpha,\sigma,\lambda)$ then the characteristics function $\phi_x(it)$ is defined as

$$\phi_x(it)=\frac{\lambda^{\alpha+1}4^{2\alpha}\theta\sigma^{2\alpha}}{\sigma^2 2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)(4^{\alpha}\lambda^{\alpha}\sigma^{2\alpha}-it^{\alpha}-\theta^{2\alpha})}\Gamma(\alpha+1)$$

Proof:

$$\phi_x(it)=E\left(e^{itx}\right)=\int_0^{\infty}e^{itx}f(x;\lambda)dx$$

$$\phi_x(it)=\int_0^{\infty}\frac{\lambda^{\alpha+1}x^{\alpha-1}2^{\alpha}\theta}{\sigma^2 2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}.\exp\left(-\frac{4\lambda x\sigma^2-\theta^2}{2\sigma^2}\right)e^{itx}dx$$

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta}{\sigma^2 2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}\int_0^{\infty}x^{\alpha-1}.\exp\left(-\frac{x(4\lambda\sigma^2-it)-\theta^2}{2\sigma^2}\right)dx \tag{18}$$

(14)

Let $u=\dfrac{x(4\lambda\sigma^2-it)-\theta^2}{2\sigma^2}$, $x=\dfrac{u2\sigma^2}{(4\lambda\sigma^2-i\,\mathrm{t})-\theta^2}$, then $dx=\dfrac{du2\sigma^2}{(4\lambda\sigma^2-i\,\mathrm{t})-\theta^2}$ so that (18) is reduced to

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta}{\sigma^2 2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)}\left(\frac{(2\sigma^2)^{\alpha}}{(4\lambda\sigma^2-it)-\theta^2}\right)^{\alpha}\Gamma(\alpha+1)$$

$$\phi_x(it)=\frac{\lambda^{\alpha+1}4^{2\alpha}\theta\sigma^{2\alpha}}{\sigma^2 2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)(4^{\alpha}\lambda^{\alpha}\sigma^{2\alpha}-it^{\alpha}-\theta^{2\alpha})}\Gamma(\alpha+1) \tag{19}$$

(vi) Cumulative distribution function (CDF)

The cumulative distribution function of a random variable *X* evaluated at *x* is the probability that *X* will take a value less than or equal to *x* and is defined as;

$$F(x)=P\left(X\le x\right)=\int_0^{x}f(x)dx$$

Theorem 6: If X is a continuous random variable from the Rayleigh-Exponential-Gamma distribution, the cumulative density function (CDF) is defined by

$$F(x)=\frac{\lambda^{\alpha+1}\theta 2^{2\alpha}\sigma^{2\alpha}\Upsilon(\alpha,\mathrm{x})+\lambda^{\alpha+1}2^{\alpha}\theta^{2\alpha+1}\Upsilon(\alpha,x)}{4^{\alpha}\lambda^{\alpha}\sigma^{2\alpha}(\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x}))} \quad x,\lambda>0$$

Proof:

$$\mathrm{f}(x)=\frac{\lambda^{\alpha+1}x^{\alpha-1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}.\exp\left(-\frac{4\lambda x\sigma^{2}-\theta^{2}}{2\sigma^{2}}\right), \quad x,\alpha,\lambda>0$$

$$F(x)=\int_{0}^{x}\frac{\lambda^{\alpha+1}x^{\alpha-1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}.\exp\left(-\frac{4\lambda x\sigma^{2}-\theta^{2}}{2\sigma^{2}}\right)\mathrm{dx},$$

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}\int_{0}^{x}x^{\alpha-1}.\exp\left(-\frac{4\lambda x\sigma^{2}-\theta^{2}}{2\sigma^{2}}\right)\mathrm{dx}, \tag{20}$$

let $u=\frac{4\lambda x\sigma^{2}-\theta^{2}}{2\sigma^{2}}$, $x=\frac{u2\sigma^{2}+\theta^{2}}{4\lambda\sigma^{2}}$, then $dx=\frac{2\sigma^{2}du+\theta^{2}}{4\lambda\sigma^{2}}$ so that (20) is reduced to:

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\Upsilon(\alpha,\mathrm{x})}\int_{0}^{x}\left(\frac{u2\sigma^{2}+\theta^{2}}{4\lambda\sigma^{2}}\right)e^{-u}\,\frac{2\sigma^{2}du+\theta^{2}}{4\lambda\sigma^{2}}$$

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\Upsilon(\alpha,\mathrm{x})}\left(\frac{2\sigma^{2}+\theta^{2}}{4\lambda\sigma^{2}}\right)^{\alpha-1}\frac{2\sigma^{2}+\theta^{2}}{4\lambda\sigma^{2}}\int_{0}^{x}u^{\alpha-1}e^{-u}du$$

Note that $\int_{0}^{x}u^{\alpha-1}e^{-u}du$ is $\Upsilon(\alpha+1)$ then,

$$=\frac{\lambda^{\alpha+1}2^{\alpha}\theta}{\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\Upsilon(\alpha,\mathrm{x})}\left(\frac{2\sigma^{2}+\theta^{2}}{4\lambda\sigma^{2}}\right)^{\alpha}\Upsilon(\alpha+1)$$

applying the limits, then we have

$$=\frac{\lambda^{\alpha+1}\theta 2^{2\alpha}\sigma^{2\alpha}\Upsilon(\alpha,\mathrm{x})+\lambda^{\alpha+1}2^{\alpha}\theta^{2\alpha+1}\Upsilon(\alpha,x)}{4^{\alpha}\lambda^{\alpha}\sigma^{2\alpha}(\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x}))} \quad \lambda>0,\ \ x>0 \tag{21}$$

(vii) Survival function

The survival function is also known as the reliability function is a function that gives the probability that a patient, device, or other objects of interest will survive beyond any given specified time and is defined as; $S(x)=1-F(x)$ where; $F(x)$ is the cumulative distribution function of *x* then,

$$=1-\left(\frac{\lambda^{\alpha+1}\theta 2^{2\alpha}\sigma^{2\alpha}\Upsilon(\alpha,\mathrm{x})+\lambda^{\alpha+1}2^{\alpha}\theta^{2\alpha+1}\Upsilon(\alpha,x)}{4^{\alpha}\lambda^{\alpha}\sigma^{2\alpha}(\sigma^{2}2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x}))}\right) \tag{22}$$

(viii) Hazard function

The hazard function also called the force of mortality, instantaneous failure rate, instantaneous death rate, or age-specific failure rate is the instantaneous risk that the event of interest happens, within a very narrow time frame and is defined as;

$$h(x)=\frac{f(x)}{S(x)}$$

Where $f(x)$ and $S(x)$ are pdf and survival function of REGD then,

$$= \frac{\dfrac{\lambda^{\alpha+1}x^{\alpha-1}2^{\alpha}\theta}{\sigma^2 2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}.\exp\left(-\dfrac{4\lambda x\sigma^2-\theta^2}{2\sigma^2}\right)}{=1-\left(\dfrac{\lambda^{\alpha+1}\theta 2^{2\alpha}\sigma^{2\alpha}\Upsilon(\alpha,\mathrm{x})+\lambda^{\alpha+1}2^{\alpha}\theta^{2\alpha+1}\Upsilon(\alpha,x)}{4^{\alpha}\lambda^{\alpha}\sigma^{2\alpha}(\sigma^2 2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x}))}\right)} \quad (23)$$

2.3 The Maximum Likelihood Estimator for Rayleigh-Exponential-Gamma Distribution (REGD)

Theorem 7: Let $\mathrm{X}_1,\mathrm{X}_2,...,\mathrm{X}_n$ be a random sample of size n from Exponential-Exponential distribution(REGD). Then the likelihood function is given by

$$\mathrm{L}(\alpha,\lambda;x)=\left(\frac{\lambda^{\alpha+1}x^{\alpha-1}2^{\alpha}\theta}{\sigma^2 2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,\mathrm{x})}\right)^n.\prod_{i=1}^{n}x_i^{\alpha-1}\exp\left(-\frac{4\lambda x\sigma^2-\theta^2}{2\sigma^2}\right) \quad (24)$$

By taking natural logarithm of (24), the log likelihood function is obtained as;

$$\log_e(L)=\alpha n\log_e\lambda+n\log_e{}^2+n\log_e\theta-2n\log_e\sigma^2-n\log_e\Gamma(\alpha)+n\log_e\lambda+n\log_e\Upsilon(\alpha,\mathrm{x})+$$
$$(\alpha-1)\log_e\sum_{i=1}^{n}X_i-\frac{4\lambda\sigma^2\sum_{i=1}^{n}X_i-\theta^2}{2\sigma^2} \quad (25)$$

differentiating with respect to $(\theta,\alpha,\sigma,\lambda)$ (25) and equate to zero then,

$$\frac{d\log_e L}{d\alpha}=n\log_e\lambda-n\log_e\Gamma(\alpha)^{'}+n\log_e\Upsilon(\alpha,\mathrm{x})^{'}-\sum\log xi=0 \quad (26)$$

$$\frac{d\log_e L}{d\lambda}=\frac{\alpha n}{\lambda}+\frac{2n}{\lambda}-\frac{4\sigma^2\sum_{i=1}^{n}X_i-\theta^2}{2\sigma^2}=0 \quad (27)$$

$$\frac{d\log_e L}{d\theta}=\frac{n}{\theta}-\frac{2\theta}{2\sigma^2}=0 \quad (28)$$

$$\frac{d\log_e L}{d\sigma}=-\frac{2n}{\sigma^2}+\frac{1}{\sigma^2}+\frac{4\lambda\sigma^2\sum_{i=1}^{n}X_i^2-\theta^2}{4\sigma^3}=0 \quad (29)$$

III. CONCLUSION

In practice, many components' lifetime distributions have a bathtub shape for their failure rate function. However, only a few distributions have a bathtub-shaped failure rate function. Models with bathtub-shaped failure rate functions are useful in reliability analysis, particularly in decision-making related to reliability, cost analysis, and burn-in analysis. When considering a failure mechanism, the failure of units in a system may be caused by random failure caused by changes in temperature, voltage, and so on, or it may be caused by ageing. In this paper, we propose a new probability distribution called Rayleigh-Exponential-Gamma distribution, provide a comprehensive study of its theory and derive appropriate expressions for its statistical properties. The method of maximum likelihood was employed to estimate its parameter.

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