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Buckling of Isotropic Clamped-Simple-Clamped-Simple and Clamped-Simple-Simple-Simple Rectangular Plates by Applying 3rd Order Energy Functional.

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ABSTRACT

The work covers the buckling of rectangular Clamped-Simple-Clamped- Simple and Clamped-Simple-Simple-Simple Isotropic plate using odd number Functional. For the derivation of the Energy Functional, 3rd order was adopted. On getting the shape functions, the integral values of the differentiated shape functions of the various boundary conditions were obtained. From these, the stiffness coefficients of the various boundary conditions were derived. The Third order strain energy equation was derived which was further expanded to generate the Third Order Total Potential Energy Functional. The Third Order Total Potential Energy Functional was integrated with respect to the amplitude, giving a result known as the Governing equation. Further minimization of the governing equation gave rise to the critical buckling load equations. The non-dimensional buckling load parameters were obtained by substituting the different aspect ratios, m/n ranging from 1.0 to 2.0, at the interval of 0.1. The graph of non-buckling load parameters against the aspect ratios was plotted, and it was observed that the increase in one axis brought about the decrease in the other axis.

Key words: Total Potential Energy Functional, Buckling Coefficient , Flexural Rigidity 3rd Order Functional

Notation S

§ - Stress,ð- Strain, S- simple support, Cl- clamped support.

n-Length of the primary dimension of the plate, m- Width of the secondary

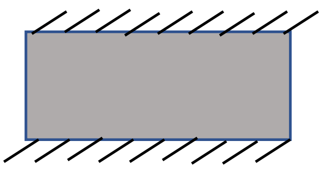
dimension of the plate, t - Tertiary dimension (thickness) of the plate

† - Total Potential Energy Functional, - the aspect ratio,F- the Deflection.

G - the flexural Rigidity, Am - Amplitude, B_x - Buckling Load Equation.

1. Introduction:

From the general research made in the course of this work, a plate can be defined as a structural element with either straight or curves boundaries, having primary, secondary and tertiary dimension (thickness), with the tertiary dimension very small compared to other dimensions. The isotropic rectangularClSClS and ClSSS platehave all their material properties in all directions as the same. These properties includes flexural rigidity, Young elastic modulus of elastic and Poison ratiobut when these materials are not uniform, it's said to be orthotropic plates. Stability analysis which is the same as buckling tendency of rectangular plate has been a subject of study in solid structural mechanics for more than a century. Although the buckling analysis of rectangular plates has received the attention of many researchers for several centuries, its treatment has left much to be done. Other researchers before now have gotten solution using both the Second and Fourth the Order energy functional for Buckling of plate. None of the researchers have any work on buckling of plate using Third order energy functional and so the resolution of the buckling tendency of ClSClS and ClSSS isotropic plate using third order energy functional is the gap the work tends to fill. Diagrammatically, the plates can be shown as



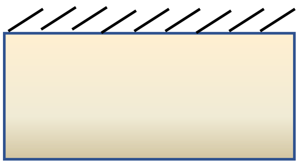


Fig.1aClamped-Simple-Clamped-Simple Plate Fig.1b Clamped-Simple-Simple-Simple Plate

BUCKLING LOAD EQUAUTION;

Total potential energy, $\dagger$ is the summation of strain energy, ϵ and external work, v given as:

$$\dagger = \epsilon + v \qquad 1$$

To derive the strain energy,$\epsilon$the product of normal stress and normal strain in x direction is considered as

$$\S_x \eth_x = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 f}{\partial x^2}\right]^2 + \mu\left[\frac{\partial^2 f}{\partial x \partial y}\right]^2\right) \qquad 2$$

while their product in y direction is considered as

$$\S_y \eth_y = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 f}{\partial y^2}\right]^2 + \mu\left[\frac{\partial^2 f}{\partial x \partial y}\right]^2\right) \qquad 3$$

andfinally the product of the in-plane shear stress and in-plane shear strain is given as:

$$\tau_{xy}\gamma_{xy} = 2\,\frac{Ez^2(1-\mu)}{(1-\mu^2)}\left[\frac{\partial^2 f}{\partial x \partial y}\right]^2 \qquad 4$$

adding all together gives

$$\S_x \eth_x + \S_y \eth_y + \tau_{xy}\gamma_{xy} = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 f}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 f}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 f}{\partial y^2}\right]^2\right) \qquad 5$$

$$\text{but}\,\epsilon = \frac{1}{2}\iint_{xy}\bar{\epsilon}\,\text{dxdy}\,\text{where}\,\bar{\epsilon} = = \frac{Ez^2}{1-\mu^2}\int\left(\left[\frac{\partial^2 f}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 f}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 f}{\partial y^2}\right]^2\right) \qquad 6$$

Upon minimisation of the expressions above, the third order strain energy equation is given as

$$\epsilon = \frac{G}{2}\int_0^n\int_0^m\left(\frac{\partial^3 f}{\partial x^3}\cdot\frac{\partial f}{\partial x} + 2\,\frac{\partial^3 f}{\partial x\,\partial y^2}\cdot\frac{\partial f}{\partial x} + \frac{\partial^3 f}{\partial y^3}\cdot\frac{\partial f}{\partial y}\right)\text{dxdy} \qquad 7$$

with the external load as $v = -\frac{Bx}{2}\int_0^n\int_0^m\left(\frac{\partial h}{\partial x}\right)^2 \text{dxdy}$ 8

The third order total potential energy functional is expressed mathematically as

$$\dagger = \frac{G}{2}\int\int\left(\frac{\partial^3 f}{\partial x^3}\cdot\frac{\partial f}{\partial x} + 2\,\frac{\partial^3 f}{\partial x^2\,\partial y}\cdot\frac{\partial f}{\partial y} + \frac{\partial^3 f}{\partial y^3}\cdot\frac{\partial f}{\partial y}\right)\text{dxdy} - \frac{Bx}{2}\int\int\frac{\partial^2 f}{\partial x^2}\text{dxdy} \qquad 9$$

Rearranging the total potential energy equation, the buckling load equation is gotten as

$$B_x = \frac{\frac{G}{a^2}\int_0^1\int_0^1\left(\left[\frac{\partial^3 h}{\partial R^3}\right]\cdot\frac{\partial h}{\partial R} + 2\frac{1}{p^2}\left[\frac{\partial^3 h}{\partial R \partial Q^2}\right]\cdot\frac{\partial h}{\partial R} + \frac{1}{p^4}\left[\frac{\partial^3 h}{\partial Q^3}\right]\cdot\frac{\partial h}{\partial Q}\right)\text{dRdQ}}{\int_0^1\int_0^1\left(\frac{\partial h}{\partial R}\right)^2\text{dRdQ}} \qquad 10$$

FORMULATION OF SHAPE FUNCTION

For the derivation of the shape function, two major support conditions were considered, namely Simple support which is denoted as S and Clamped support which is denoted as Cl. For Simple support condition, the deflection equation F and the 2nd order derivative of the deflection equation F^{ii}, were equated to zero and simultaneous equations were formed by considering J = 0 at the left hand support for X axis and I = 0 at the top of the support for Y axis while J = 1 at the right hand support X axis and I = 1 at the bottom support for Y axis. For the Clamped support condition, the deflection equation, F and 1st order derivative of the deflection equation, F^i, were equated to zero and simultaneous equations were formed by considering J = 0 at the left hand support for the X axis and I = 0 at the top support for the Y axis, while at the Right hand support, J = 1 for X axis while I= 1 at the bottom support for Y axis. These equations were solved simultaneously to obtain the various values of the primary and secondary dimensions (n_1, m_1, n_2, m_2 n_3, m_3, n_4 andm_4). Where J and I are non-dimensional axis parallel to X and Y axis respectively as earlier explained. For the ClSClS and ClSSS plate their shape functions were derived as explained below.

Shape function For Clamped-Simple - Clamped- Simple Plate

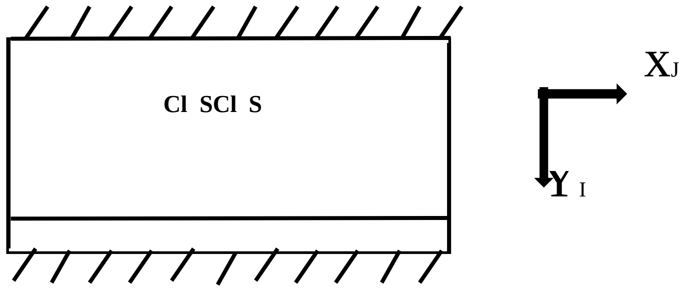


Fig 2a   Isotropic Rectangular ClSClS Plate

The case of horizontal Direction (X- X axis)

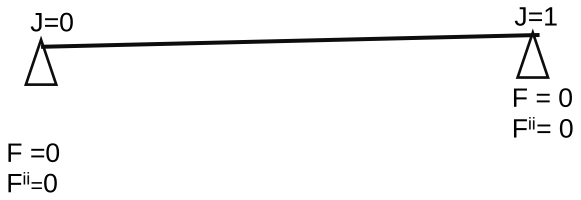


Fig 2bSimple-SimpleSupport on x-x axis

Considering the X- X axis

But $F_x = n_o + n_1R + n_2 R^2 + n_3R^3 + n_4R^4$ 11

$F_x^i = n_1 + 2n_2R + 3n_3R^2 + 4n_4R^3$ 12

$F^{ii} = 2n_2 + 6n_2R + 12n_4R^2$ 13

At the left support, R = 0

When $F_x = 0$

$F_x = 0 = n_o + 0 + 0 + 0 + 0$ 14

$n_o = 0$

Also when $f_x^{ii} = 0$ 15

$F^{ii} = 0 = 2n_2 + 6n_3R + 12n_4R^2$ 16

$2n_2 = 0$ 17

$n_2 = 0$

At the right support, R =1

When $F_x = 0$

$F_x = n_o + n_1R + n_2 R^2 + n_3R^3 + n_4R^4$ 18

$F_x = n_o + n_1 + n_2 + n_3 + n_4$ 19

(where $n_o = n_2 = 0$)

$0 = n_1 + n_3 + n_4$ 20

$n_1 + n_3 = - n_4$ 21

Also when $F_x^{ii} = 0$,

$F^{ii} = 0 = 0 + 6n_3 + 12n_4$ 22

(where $n_2 = 0$)

$6n_3 = -12n_4$ 23

$n_3 = -2n_4$

Substituting $-2n_4$ for a_3 into equation (1.3)

$n_1 + (-2n_4) = -n_4$ 24

$n_1 = 2n_4 - n_4$ 25

$n_1 = n_4$

Substituting back in the general equation $F_x = n_o + n_1R + n_2 R^2 + n_3R^3 + n_4R^4$

$0 = 0 + n_4R + 0 + (-2n_4) R^3 + n_4R^4$. 26

$F_x = n_4 (R-2R^3+R^4)$ 27

The case of vertical direction (Y- Y axis)

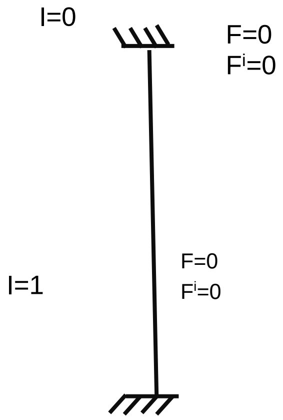


Fig. 2c Clamped-Clamped support on y-y axis

But $F_y = m_o + m_1 I + m_2 I^2 + m_3 I^3 + m_4 I^4$ 28

$F_y^i = m_1 + 2m_2 I + 3m_3 I^2 + 4m_4 I^3$ 29

At the top support, I = 0

When $F_y = 0$

$F_y = 0 = m_o + 0 + 0 + 0 + 0$ 30

$m_o = 0$

Also when $F_y^i = 0$

$F_y^i = 0 = m_1 + 0 + 0 + 0$ 31

$m_1 = 0$

At the bottom support, I = 1

When $F_y = 0$

$F_y = m_o + m_1 I + m_2 I^2 + m_3 I^3 + m_4 I^4$ 32

$F_y = m_o + m_1 + m_2 + m_3 + m_4$ 33

(where $m_o = m_1 = 0$)

$0 = m_2 + m_3 + m_4$ 34

$m_2 + m_3 = - m_4$ 35

Also when $F_y^i = 0$,

$F_y^i = 0 = m_1 + 2m_2 I + 3m_3 I^2 + 4m_3 I^3$ 36

$0 = 2m_2 + 3m_3 + 4m_4$ 37

$2m_2 + 3m_3 = -4m_4$ 38

Solving Equations 1.7 and 1.8 simultaneously, yields

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} * \begin{bmatrix} m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -m_4 \\ -4m_4 \end{bmatrix}$$

$m_2 = m_4,\ m_3 = -2m_4$ 39

Substituting back in the general equation

$F_y = m_o + m_1 I + m_2 I^2 + m_3 I^3 + m_4 I^4$ 40

$0 = 0 + m_4 I^2 + 0 + (-2m_4) I^3 + m_4 I^4.$ 41

$F_y = m_4(I^2 - 2I^3 + I^4)$ 42

But $F = F_X F_y$ 43

$F = n_4(R-2R^3+R^4)m_4(I^2-2I^3+I^4)$ 44

$= n_4m_{,4}(R-2R^3+R^4)(I^2-2I^3+I^4)$ 45

where $n_4m_4 = Am$

$f = Am*h$ 46

Therefore the shape function h for ClSClS panel is $(J-2J^3+J^4)(I^2-2I^3+I^4)$

Shape function for Clamped- Simple – Simple – Simple Plate

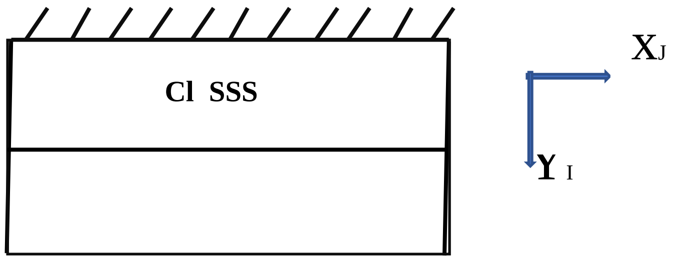


Fig. 3aClSSS RECTANGULAR SHAPE

The case of horizontal direction (X- X axis)

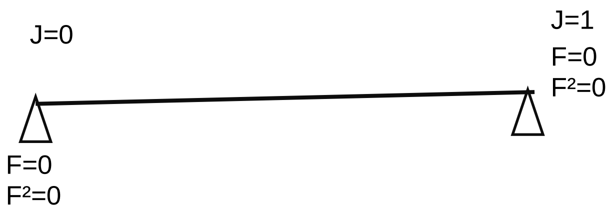


Fig. 3b Simple-Simple support on x-x axis

But $F_x = n_o + n_1J + n_2J^2 + n_3J^3 + n_4J^4$ 47

$F_x^i = n_1 + 2n_2J + 3n_3J^2 + 4n_4J^3$ 48

$F_x^{ii} = 2n_2 + 6n_3J + 12n_4J^2$ 49

At the left support, J = 0

When $F_x = 0$

$F_x = 0 = n_o + 0 + 0 + 0 + 0$

$n_o = 0$

Also when $F_x^{ii} = 0$

$F_x^{ii} = 0 = 2n_2 + 6n_3J + 12n_4J^2$ 50

$2n_2 = 0$

$n_2 = 0$

At the right support, J =1

When $F_x = 0$

$F_x = n_o + n_1 + n_2 + n_3 + n_4$ 51

(where $n_o = n_2 = 0$)

$0 = n_1 + n_3 + n_4$ 52

$n_1 + n_3 = - n_4$ 53

Also when $F_x^{ii} = 0$,

$F_x^{ii} = 0 = 0 + 6n_3 + 12n_4$ 54

(where $n_2 = 0$)

$6n_3 = -12n_4$ 55

$n_3 = -2n_4$

Substituting $-2n_4$ for n_3 into Equation (2.3)

$n_1 + (-2n_4) = -n_4$ 56

$n_1 = 2n_2 - n_4$ 57

$n_1 = n_4$ 58

Substituting back in the general equation

$F_x = n_o + n_1J + n_2 J^2 + n_3J^3 + n_4J^4$ 59

$0 = 0 + n_4J + 0 + (-2n_4) J^3 + n_4J^4.$ 60

$F_x = n_4 (J-2J^3+J^4)$ 61

The case of vertical direction (Y- Y axis)

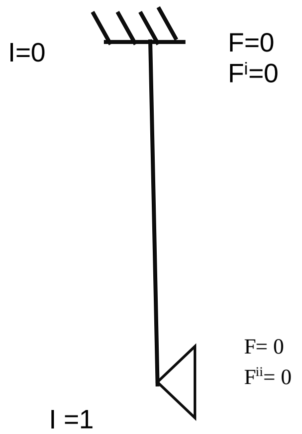


Fig. 3c Clamped-Simple support on y-y axis

But $F_y = m_o + m_1I + m_2I^2 + m_3I^3 + m_4I^4$ 62

$F_y^i = m_1 + 2m_2I + 3m_3I^2 + 4m_4I^3$ 63

At the top support, I = 0

When $F_y = 0$

$F_y = 0 = m_o + 0 + 0 + 0 + 0$ 64

$m_o = 0$

Also when $F_y^i = 0$

$F_y^i = 0 = m_1 + 0 + 0 + 0$

$m_1 = 0$

At the bottom support, I =1

When $F_y = 0$

$F_y = 0 = 0 + 0 + m_2 + m_3 + m_4$ 65

$0 = m_2 + m_3 + m_4$ 66

$m_2 + m_3 = - m_4$ 67

(where $m_o = m_2 = 0$)

$m_2 + m_3 = - m_4$ 68

Also when $F_y^{ii} = 0$,

$F_y^{ii} = 0 = 2 m_2 + 6m_3 + 12m_4$ 69

$= 2m_2 + 6m_3 + 12m_4$ 70

$m_2 + 3m_3 = -6m_4$ 71

Solving the Equations 2.6 and 2.7 simultaneously

$$\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} * \begin{bmatrix} m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -m_4 \\ -6m_4 \end{bmatrix}$$

$m_2 = \frac{3}{2}m_4,\ m_3 = -\frac{5}{2}m_4$.

Substituting back in the general equation $F_y = m_o + m_1 I + m_2 I^2 + m_3 I^3 + m_4 I^4$ 72

$0 = 0 + 0 + \frac{3}{2}m_4 + (-\frac{5}{2}m_4)\ I^3 + I^4$ 73

$0 = m_4 (1.5I^2 - 2.5I^3 + I^4)$ 74

$F_y = m_4 (1.5I^2 - 2.5I^3 + I^4)$ 75

But $F = F_X F_y$ 76

$F = n_4(J - 2J^3 + J^4)\ m_4 (1.5I^2 - 2.5I^3 + I^4)$ 77

$= n_4 m_4 (R - 2R^3 + R^4)\ (1.5I^2 - 2.5I^3 + I^4)$ 78

$F = Am*h$ 79

While the Amplitude, $Am = n_4 m_4$,the shape function h for ClSSS panel is

$(J - 2J^3 + J^4)\ (1.5I^2 - 2.5I^3 + I^4)$ 80

STIFFNESS COEFFICIENTS

Given that the shape functions for ClSClS and ClSSS are $(J-2J^3+J^4)\ (I^2-2I^3+I^4)$ and $(J-2J^3+J^4)\ (1.5I^2 - 2.5I^3+I^4)$ respectively, the differential values known as the C-*values* were integrated to generate the expressions below

For ClSClS plate shape,

$c_1 = \int\int (-12 + 24J + 72J - 192J^3 + 96J^4)$ 81

$x\ (I^4 - 4I^5 + 6I^6 - 4I^7 + I^8)$ 82

$c_1 = (-4\frac{4}{5})\ x\ (\frac{1}{630}) = \frac{-4}{525}$ 83

$c_2 = \int\int (1 - 12J^2 + 8J^3 + 36J^4 - 48J^5 + 16J^6)$

$x\ (2I^2 - 16I^3 + 38I^4 - 36I^5 + 12I^6)$ 84

$c_2 = (\frac{17}{35})\ x\ (\frac{-2}{105}) = \frac{-34}{3675}$ 85

$c_3 = \int\int (J^2 - 4J^4 + 2J^5 + 4J^6 - 4J^7 + J^8)$

$x\ (-24I + 120I^2 - 192I^3 + 96I^4)$

$c_3 = (\frac{31}{630})\ x\ (-\frac{4}{5}) = \frac{-62}{1575}$ 86

$c_6 = \int\int (1 - 12J^2 + 8J^3 + 36J^4 - 48J^5 + 16J^6)$ 87

$x(I^4 - 4I^5 + 6I^6 - 4I^7 + I^8)$

$c_6 = (\frac{17}{35})\ x\ (\frac{19}{630}) = \frac{17}{22050}$ 88

Differentiating the Total potential energy with respect to Am.

$$\frac{d\dagger}{dAm} = 0 = \frac{2AmG}{2}\iint_{xy}\left(\left[\frac{\partial^2 h}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 h}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 h}{\partial y^2}\right]^2\right) dxdy \quad 89$$

$$-qA\iint_{xy} h\ dxdy - \frac{Bx}{2}\int_0^n\int_0^m \left(\frac{\partial h}{\partial x}\right)^2 dxdy - \frac{M\Lambda^2}{2}\int_0^n\int_0^m (f)^2\ dxdy \quad 90$$

and

$$\frac{d\dagger}{dAm} = 0 = \frac{2AmG}{2}\int_0^n\int_0^m\left(\frac{\partial^3 h}{\partial x^3}\cdot\frac{\partial h}{\partial x} + 2\frac{\partial^3 h}{\partial x^2\partial y}\cdot\frac{\partial h}{\partial y} + \frac{\partial^3 h}{\partial y^3}\cdot\frac{\partial h}{\partial y}\right)dxdy - qAm\int_0^n\int_0^m h\ dxdy - \frac{2AmBx}{2}\int_0^a\int_0^b\left(\frac{\partial h}{\partial x}\right)^2 dx - \frac{2M\kappa^2}{2}\int_0^a\int_0^b (f)^2 dxdy \quad 91$$

where Lateral load = $qAm\int_0^n\int_0^m h\ dxdy$, 100

Buckling $= \frac{2AmBx}{2}\int_0^n\int_0^m\left(\frac{\partial h}{\partial x}\right)^2 dx$ 101

Free Vibration $= \frac{2M\kappa^2}{2}\int_0^n\int_0^m (f)^2 dxdy$ 102

For stability analysis of plate, lateral load and free vibration are considered zero.

That means $q = \kappa = 0$ and so substituting the values of q and κ into the equations 3.42b and 3.42c above gives

$$\frac{d\dagger}{dAm} = 0 = \frac{2AmG}{2}\iint_{xy}\left(\left[\frac{\partial^2 h}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 h}{\partial x\partial y}\right]^2 + \left[\frac{\partial^2 h}{\partial y^2}\right]^2\right)dxdy \quad 103$$

$$-\frac{Bx}{2}\int_0^n\int_0^m\left(\frac{\partial h}{\partial x}\right)^2 dxdy \text{ and} \quad 104$$

$$\frac{d\dagger}{dAm} = 0 = \frac{2AmG}{2}\int_0^a\int_0^b\left(\frac{\partial^3 h}{\partial x^3}\cdot\frac{\partial h}{\partial x} + 2\frac{\partial^3 h}{\partial x^2\partial y}\cdot\frac{\partial h}{\partial y} + \frac{\partial^3 h}{\partial y^3}\cdot\frac{\partial h}{\partial y}\right)dxdy - \frac{2AmNx}{2}\int_0^a\int_0^b\left(\frac{\partial h}{\partial x}\right)^2 dxdy \quad 105$$

Making Bx the subject formula

$$Bx = \frac{\frac{2AmG}{2}\int_0^1\int_0^1 \frac{2AmG}{2}\left(\left[\frac{\partial^2 h}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 h}{\partial x\partial y}\right]^2 + \left[\frac{\partial^2 h}{\partial y^2}\right]^2\right)dxdy}{\frac{2Am}{2}\int_0^n\int_0^m\left(\frac{\partial h}{\partial x}\right)^2 dxdy} \quad 106$$

for the 2nd order functional Equation while the 3rd order functional is

$$Bx = \frac{\frac{2AmG}{2}\int_0^1\int_0^1 \left(\left(\left[\frac{\partial^3 h}{\partial x^3}\right]\cdot\frac{\partial h}{\partial y} + 2\left[\frac{\partial^3 h}{\partial x\partial y}\right]\cdot\frac{\partial h}{\partial x} + \left[\frac{\partial^3 h}{\partial y^3}\right]\cdot\frac{\partial h}{\partial y}\right)dxdy}{\frac{2Am}{2}\int_0^n\int_0^m\left(\frac{\partial h}{\partial x}\right)^2 dxdy} \quad 107$$

DETERMINATION OF CRITICAL BUCKLING LOAD COEFFICIENT USING ASPECT RATIO,

$$p = m/n$$

Defining the principal in-plane coordinates(x and y) in terms of non-dimension in-plane coordinates (J and I) as:

$J = \frac{x}{n}$. That is $x = nJ$ 108

$I = \frac{y}{m}$. That is $y = mI$ 109

Where "n" and "m" are plate dimensions in x and y directions. The aspect ratio α (ratio of length in y direction to length in x direction) of $\frac{n}{m}$, where the αranges from 1.0 to 2.0 was considered. Also the aspect ratio of $\frac{n}{m}$, was considered whereα ranges from 0.5 to 1.0. The value of n is less or equal to b. (i. e n≤ m).

While G is flexural rigidity and F is the shape function, † is the total potential energy functional. J and I are non-dimensional axis (quantity) parallel to x and y axis. Substituting nJ and mI for x and y respectively into Equation

$$Bx = \frac{\frac{2AmG}{2}\int_0^1\int_0^1 \left(\left[\frac{\partial^2 h}{\partial n^2 J^2}\right]^2 + 2\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \left[\frac{\partial^2 h}{\partial m^2 I^2}\right]^2\right)nmdJdI}{\frac{2Am}{2}\int_0^n\int_0^m\left(\frac{\partial h}{\partial J}\right)^2 nmdJdI} \quad 110$$

$$Bx = \frac{\frac{2AmG}{2}\int_0^1\int_0^1 \left(\left[\frac{\partial^2 h}{\partial n^2 J^2}\right]^2 + 2\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \left[\frac{\partial^2 h}{\partial m^2 J^2}\right]^2\right)nmdJdI}{\frac{2Am}{2}\int_0^n\int_0^b\left(\frac{\partial h}{\partial J}\right)^2 nmdJdI} \quad 111$$

$$Bx = \frac{\frac{2AmG}{2}\int_0^1\int_0^1 \left(\frac{1}{n^2}\left[\frac{\partial^2 h}{\partial I^2}\right]^2 + 2\frac{1}{n^2 m^2}\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \frac{1}{m^4}\left[\frac{\partial^2 h}{\partial I^2}\right]^2\right)nmdJdI}{\frac{2A}{2n^2}\int_0^n\int_0^m\left(\frac{\partial h}{\partial J}\right)^2 nmdJdI} \quad 112$$

Substituting pn in place of m in the Equation

$$Bx = \frac{G\int_0^1\int_0^1 \left(\frac{1}{n^2}\left[\frac{\partial^2 h}{\partial R^2}\right]^2 + 2\frac{1}{n^4\alpha^2}\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \frac{1}{n^4\alpha^4}\left[\frac{\partial^2 h}{\partial I^2}\right]^2\right)nmdJdI}{\frac{1}{n^2}\int_0^n\int_0^m\left(\frac{\partial h}{\partial J}\right)^2 nmdJdI} \quad 113$$

$$Bx = \frac{\frac{G}{\alpha^4}\int_0^1\int_0^1 \left(\left[\frac{\partial^2 h}{\partial R^2}\right]^2 + 2\frac{1}{\alpha^2}\left[\frac{\partial^2 h}{\partial R\partial Q}\right]^2 + \frac{1}{\alpha^4}\left[\frac{\partial^2 h}{\partial Q^2}\right]^2\right)nmdJdI}{\frac{1}{\alpha^2}\int_0^n\int_0^m\left(\frac{\partial h}{\partial J}\right)^2 nmdJdI} \quad 114$$

$$Bx = \frac{\frac{G}{\alpha^2}\int_0^1\int_0^1 \left(\left[\frac{\partial^2 h}{\partial J^2}\right]^2 + 2\frac{1}{\alpha^2}\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \frac{1}{\alpha^4}\left[\frac{\partial^2 h}{\partial Q^2}\right]^2\right) nmdJdI}{\int_0^n\int_0^m \left(\frac{\partial h}{\partial J}\right)^2 nmdJdI} \quad 115$$

Finally the Buckling Equation Bx with Aspect Ratio, α = m/n

$$Bx = \frac{G\int_0^1\int_0^1 \left(\left[\frac{\partial^2 h}{\partial J^2}\right]^2 + 2\frac{1}{\alpha^2}\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \frac{1}{\alpha^4}\left[\frac{\partial^2 h}{\partial Q^2}\right]^2\right) nmdJdI}{\alpha^2\int_0^n\int_0^m \left(\frac{\partial h}{\partial R}\right)^2 nmdJdI} \quad 116$$

$$Bx = \frac{G\int_0^1\int_0^1 \left(\left[\frac{\partial^3 h}{\partial J^3}\right]\cdot\frac{\partial h}{\partial J} + 2\frac{1}{\alpha^2}\left[\frac{\partial^3 h}{\partial J\partial I}\right]\cdot\frac{\partial h}{\partial J} + \frac{1}{\alpha^4}\left[\frac{\partial^3 h}{\partial I^3}\right]\cdot\frac{\partial h}{\partial I}\right) dJdI}{\alpha^2\int_0^n\int_0^m \left(\frac{\partial h}{\partial J}\right)^2 dJdI} \quad 117$$

$$Bx = \frac{G\int_0^1\int_0^1 \left(\alpha^2\left[\frac{\partial^3 h}{\partial J^3}\right]\cdot\frac{\partial h}{\partial J} + 2\left[\frac{\partial^3 h}{\partial J^2\partial I}\right]\cdot\frac{\partial h}{\partial J} + \frac{1}{\alpha^2}\left[\frac{\partial^3 h}{\partial I^3}\right]\cdot\frac{\partial h}{\partial I}\right) dJdI}{\alpha^2\int_0^n\int_0^m \left(\frac{\partial h}{\partial J}\right)^2 dJdI} \quad 118$$

3.8.2 DETERMINATION OFBUCKLING COEFFICIENT USING ASPECT

RATIO.$p = n/m$

Recall that for y = mI 119

$p = n/m$, that means n = pm

$$Bx = \frac{\frac{2AmG}{2}\int_0^1\int_0^1 \left(\frac{1}{\alpha^4}\left[\frac{\partial^2 h}{\partial J^2}\right]^2 + 2\frac{1}{n^2m^2}\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \frac{1}{m^4}\left[\frac{\partial^2 h}{\partial I^2}\right]^2\right)\alpha m^2 dJdI}{\frac{2Am}{2\alpha^2}\int_0^n\int_0^m \left(\frac{\partial h}{\partial J}\right)^2 \alpha m^2 dJdI} \quad 120$$

Substituting pm in place of n in the Equation

$$Bx = \frac{G\int_0^1\int_0^1 \left(\frac{1}{\alpha^4 m^4}\left[\frac{\partial^2 h}{\partial J^2}\right]^2 + 2\frac{1}{\alpha^2 m^4}\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \frac{1}{m^4}\left[\frac{\partial^2 h}{\partial I^2}\right]^2\right)\alpha m^2 dJdI}{\frac{1}{\alpha^2}\int_0^n\int_0^m \left(\frac{\partial h}{\partial I}\right)^2 \alpha m^2 dJdI} \quad 121$$

$$Bx = \frac{\frac{G}{b^4}\int_0^1\int_0^1 \left(\frac{1}{\alpha^4}\left[\frac{\partial^2 h}{\partial R^2}\right]^2 + 2\frac{1}{\alpha^2}\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \left[\frac{\partial^2 h}{\partial I^2}\right]^2\right)\alpha m^2 dJdI}{\frac{1}{n^2}\int_0^n\int_0^m \left(\frac{\partial h}{\partial R}\right)^2 \alpha m^2 dJdI} \quad 122$$

$$Bx = \frac{\frac{G}{b^2}\int_0^1\int_0^1 \left(\frac{1}{\alpha^4}\left[\frac{\partial^2 h}{\partial J^2}\right]^2 + 2\frac{1}{\alpha^2}\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \left[\frac{\partial^2 h}{\partial I^2}\right]^2\right)\alpha m^2 dJdI}{\int_0^n\int_0^m \left(\frac{\partial h}{\partial J}\right)^2 \alpha m^2 dJdI} \quad 123$$

$$Bx = \frac{G\int_0^1\int_0^1 \left(\frac{1}{\alpha^4}\left[\frac{\partial^2 h}{\partial J^2}\right]^2 + 2\frac{1}{\alpha^2}\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \left[\frac{\partial^2 h}{\partial I^2}\right]^2\right) dJdI}{m^2\int_0^n\int_0^m \left(\frac{\partial h}{\partial J}\right)^2 dJdI} \quad 124$$

$$Bx = \frac{G\int_0^1\int_0^1 \left(\frac{1}{\alpha^4}\left[\frac{\partial^2 h}{\partial J^2}\right]^2 + 2\frac{1}{\alpha^2}\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \left[\frac{\partial^2 h}{\partial I^2}\right]^2\right) dJdI}{m^2\int_0^n\int_0^m \left(\frac{\partial h}{\partial J}\right)^2 dJdI} \quad 125$$

$$Bx = \frac{G\int_0^1\int_0^1 \left(\alpha^2\left[\frac{\partial^2 h}{\partial J^2}\right]^2 + 2\left[\frac{\partial^2 h}{\partial J\partial I}\right]^2 + \frac{1}{\alpha^2}\left[\frac{\partial^2 h}{\partial I^2}\right]^2\right) dJdI}{m^2\int_0^n\int_0^m \left(\frac{\partial h}{\partial J}\right)^2 dJdI} \quad 126$$

This Equations is equivalent to Bx below, which is the Buckling Equation for the Aspect Ratio, p = m/n

$$Bx = \frac{G\int_0^1\int_0^1 \left(\left[\frac{\partial^3 h}{\partial J^3}\right]\cdot\frac{\partial h}{\partial J} + 2\frac{1}{\alpha^2}\left[\frac{\partial^3 h}{\partial J\partial I}\right]\cdot\frac{\partial h}{\partial J} + \frac{1}{\alpha^4}\left[\frac{\partial^3 h}{\partial I}\right]\cdot\frac{\partial h}{\partial I}\right) dJdI}{\alpha^2\int_0^n\int_0^m \left(\frac{\partial h}{\partial J}\right)^2 dJdI} \quad 127$$

$$Bx = \frac{G\int_0^1\int_0^1 \left(\alpha^2\left[\frac{\partial^3 h}{\partial J^3}\right]\cdot\frac{\partial h}{\partial J} + 2\left[\frac{\partial^3 h}{\partial J^2\partial I}\right]\cdot\frac{\partial h}{\partial J} + \frac{1}{\alpha^2}\left[\frac{\partial^3 h}{\partial I^3}\right]\cdot\frac{\partial h}{\partial I}\right) dJdI}{\alpha^2\int_0^n\int_0^m \left(\frac{\partial h}{\partial J}\right)^2 dJdI} \quad 128$$

which can be rewritten

$$Bx = \frac{G\left(\alpha^2 c1 + 2c2 + \frac{1}{\alpha^2}c3\right)}{\alpha^2 c6}$$

where C1 $= \left(\frac{\partial^3 h}{\partial I^3}\right)\cdot\frac{\partial h}{\partial J}$, C2 = $\left(\frac{\partial^2 h}{\partial J^3\,\partial I}\right)\cdot\frac{\partial h}{\partial J}$, C3 = $\left(\frac{\partial^2 h}{\partial I^3}\right)\cdot\frac{\partial h}{\partial J}$andC6 = $\left(\frac{\partial h}{\partial R}\right)\cdot\frac{\partial h}{\partial J}$

Andc_1, c_2, c_3 and c_6 are defined as follows:

$$c_1 = \int_0^1\int_0^1 C1 = \int_0^1\int_0^1 \frac{\partial^3 h}{\partial J^3}\cdot\frac{\partial h}{\partial J}\, dJdI \quad 129$$

$$c_2 = \int_0^1 \int_0^1 C2 = \int_0^1 \int_0^1 \frac{\partial^3 h}{\partial J^2 \partial I} \cdot \frac{\partial h}{\partial I} dJdI \quad 130$$

$$c_3 = \int_0^1 \int_0^1 C3 = \int_0^1 \int_0^1 \frac{\partial^3 h}{\partial J^3} \cdot \frac{\partial h}{\partial I} dJdI \quad 131$$

$$c_6 = \int_0^1 \int_0^1 C6 = \int_0^1 \int_0^1 \left(\frac{\partial h}{\partial J}\right)^2 dJdI \quad 132$$

These c parameters shall be referred to as stiffness components of the rectangular plate.

RESULTS AND DISCUSSION.

The results for the critical buckling load coefficients were gotten for different aspect ratios.For each shape function, the results were presented in two tables. The first table represents the values of the critical buckling coefficients for the aspect ratio of m/n while the second tables contains the critical buckling coefficients for the aspect ratio n/m. Two graphs were plotted for the rectangular plates with Clamped–Simple–Clamped– Simple edge support and also for the plate with Clamped-Simple-Simple- Simple supported edge. The first graph contains the Critical Buckling Load against Aspect Ratio for the aspect ofm/n, while the second graph contains the Critical Buckling Load against Aspect Ratio for the aspect ofn/m. In the first graph, the aspect Ratio is of the range 1.0 to 2.0 while in the second graph it ranges from 0.5 to 1.0 at the same interval in each case, From the graph of critical buckling load against the aspect ratios plotted, it was observed that in the first graph, as the aspect ratio increases from 1.0 to 2.0, the critical buckling load also decreases.

Table 1.1 Non dimensional buckling load parameters for CSSS plate for aspect m/n

m/n		2	1.9	1.8	1.7	1.6
B		17.0778	18.036	19.2033	20.6449	22.4537
Bx	Previous	$17.07771\frac{G}{n^2}$	$18.03599\frac{D}{a^2}$	$19.20324\frac{D}{a^2}$	$20.64487\frac{D}{a^2}$	$22.45361\frac{D}{a^2}$
	Present	$17.07771\frac{D}{a^2}$	$18.03599\frac{D}{a^2}$	$19.20324\frac{D}{a^2}$	$20.64487\frac{D}{a^2}$	$22.45361\frac{D}{a^2}$

m/n		1.5	1.4	1.3	1.2	1.1	1
B		24.76415	27.77746	31.80291	37.33388	45.19029	56.80228
Bx	Previous	$24.7642\frac{G}{n^2}$	$27.7775\frac{G}{n^2}$	$31.803\frac{G}{n^2}$	$37.3339\frac{G}{n^2}$	$45.1903\frac{G}{n^2}$	$56.8023\frac{G}{n^2}$
	Present	$24.76415\frac{G}{n^2}$	$27.77746\frac{G}{n^2}$	$31.80291\frac{G}{n^2}$	$37.33388\frac{G}{n^2}$	$45.19029\frac{G}{a^2}$	$56.80228\frac{G}{n^2}$

Table 1.2 Non dimensional buckling load parameters for CSCS plate for aspect m/n

m/n		2	1.9	1.8	1.7	1.6
B		19.0747	20.44976	22.155	24.30162	27.049
Bx	Previous	$19.1108\frac{G}{n^2}$	$20.4874\frac{G}{n^2}$	$22.1947\frac{G}{n^2}$	$24.34392\frac{G}{n^2}$	$27.0956\frac{G}{n^2}$
	Present	$19.0747\frac{G}{n^2}$	$20.44976\frac{G}{n^2}$	$22.155\frac{G}{n^2}$	$24.30162\frac{G}{n^2}$	$27.049\frac{G}{n^2}$

m/n		1.5	1.4	1.3	1.2	1.1	1
B		30.6365	35.42043	41.9632	51.1755	64.5951	84.9468
Bx	Previous	$30.6866\frac{G}{n^2}$	$35.4763\frac{G}{n^2}$	$42.0272\frac{G}{n^2}$	$51.2509\frac{G}{n^2}$	$64.6872\frac{G}{n^2}$	$85.0645\frac{G}{n^2}$
	Present	$30.6365\frac{G}{n^2}$	$35.42043\frac{G}{n^2}$	$41.9632\frac{G}{n^2}$	$51.1755\frac{G}{n^2}$	$64.5951\frac{G}{n^2}$	$84.9468\frac{G}{n^2}$

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