WWW.IJRPR.COM

International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Stability Analysis of Thin Plates Using Euler-Bernoulli Approach with Trigonometric Shape Functions

Uzoukwu C. S., Ibearugbulem O. M., Ogbonna N. S., Okoro M. U., Chike K. O., Ikpa P. N.

Civil Engineering Department, Federal University of Technology Owerri, Imo State, Nigeria

Email: sunny.uzoukwu@futo.edu.ng

DOI: https://doi.org/10.55248/gengpi.4.723.49289

ABSTRACT

The work deals with the application of the Euler-Bernoulli Approach in the analysis of the thin rectangular plate with trigonometric functions. The Total Potential energy was reduced to an equilibrium equation. Unknown values of coefficients of deflection were generated after the integration of the derived equation. The exact stiffness coefficients of plates for various boundary conditions were derived when the Exact shape functions were substituted in the strong equation. From the outcome, critical buckling loads were formulated for the plates under consideration. The values of the stiffness coefficients and that of the buckling load were also calculated using the Ritz approach. Comparing the two methods, it was observed that for this analysis, this method is not only simpler but more flexible.

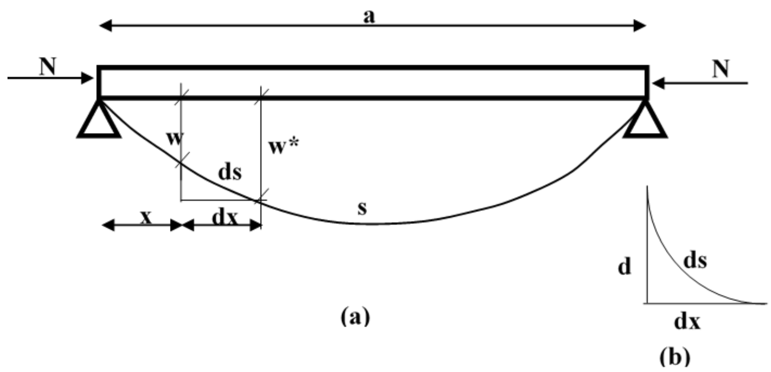
Introduction

This is the study of a rectangular flat plate clamped on two adjacent supports and simply supported on the other two supports and also a rectangular flat plate simply supported on two opposite supports and clamped on the other two. The total potential equation was differentiated with respect to the deflection. Originally the plate is assumed to be a straight configuration. It is also assumed that prior to buckling, the plate is flat. During the buckling activities, a neutral equilibrium develops. That is the situation were the plate condition is now between that of straight configuration and bent configuration. Considering the original length and new length as a and S respectively.

The out of plane deflection of the plate at a point x meters from left support is w.

At a point x+dx meters from the left support;

Deflection is w^*=w+dw



(a) A section of plate element that buckled under in plane load, N_x
(b) a finite portion of the buckled part

Figure 1: Section of A Plate Subject to in plane Forces

Strains and Displacements

Strain in a plate can either occur as decrease/increase in size or as a change in shape. **Error! Reference source not found.**1 is a typical plate that has strained as a result of increase/decrease in size, while **Error! Reference source not found.** is a typical plate that has strained as a result of distortion in shape.

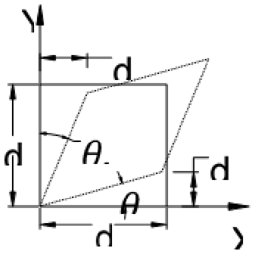


Figure 2.1: Typical Plate Strained as a result of Distortion

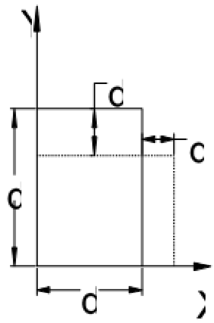


Figure 2.2: Typical Plate Strained as a result of increase or decrease in size

Considering U, V, and W as the displacements, dx, dy, and dz be elemental lengths, and du, dv and dw be elemental displacements, in the X, Y, and Z directions, respectively.

The strain;

$$\text{Stain}, \varepsilon = \frac{\text{Change in length}}{\text{original length}} \tag{1.1}$$

From figure

$$\varepsilon_x = \frac{du}{dx} \tag{1.2}$$

$$\varepsilon_y = \frac{dv}{dy} \tag{1.3}$$

Applying the Pythagoras theorem,

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} \tag{1.4}$$

But for small values of $\theta, \tan\theta = \theta$;

$$\therefore \theta = \frac{\text{Opposite}}{\text{Adjacent}} \tag{1.5}$$

Relating the Pythagoras theorem to Figure 3.2;

$$\theta_x = \frac{dv}{dx} \tag{1.6}$$

$$\theta_y = \frac{du}{dy} \tag{1.7}$$

For classical Plate:

$$\text{Total strain in the xy plane, } \gamma_{xy} = \theta_x + \theta_y \tag{1.8}$$

$$\gamma_{xy} = \frac{dv}{dx} + \frac{du}{dy} \tag{1.9}$$

In the same way, although z-plane is not visible in the diagram;

$$\gamma_{xz} = \frac{dw}{dx} + \frac{du}{dz} \tag{1.10}$$

$$\gamma_{yz} = \frac{dw}{dy} + \frac{dv}{dz} \tag{1.11}$$

But based on Kirchoff's assumptions,

$$\gamma_{xz} = \gamma_{yz} = 0 \tag{1.12}$$

$$\therefore \frac{dw}{dx} + \frac{du}{dz} = 0 \tag{1.13}$$

$$\frac{dw}{dy} + \frac{dv}{dz} = 0 \tag{1.14}$$

Making du and dv subjects of formular in equation 3.13 and equation 3.14 respectively;

$$du = -\frac{dw}{dx} * dz \qquad 1.15$$

$$dv = -\frac{dw}{dy} * dz \qquad 1.16$$

Integrating both sides of equation 3.15 and equation 3.16, with respect to z

$$\int du = \int \left(-\frac{dw}{dx}\right) dz, \therefore u = -z\frac{dw}{dx} + u_0 \qquad 1.17$$

$$\int dv = \int \left(-\frac{dw}{dy}\right) dz, \therefore v = -z\frac{dw}{dy} + v_0 \qquad 1.18$$

Differentiating equation 3.17 and 3.18 with respect to x and y, we obtain;

$$\frac{du}{dx} = -z\frac{d^2w}{dx^2} \qquad 1.19$$

$$\frac{du}{dy} = -z\frac{d^2w}{dxdy} \qquad 1.20$$

$$\frac{dv}{dy} = -z\frac{d^2w}{dy^2} \qquad 1.21$$

$$\frac{dv}{dx} = -z\frac{d^2w}{dxdy} \qquad 1.22$$

Substituting equation 3.19 and 3.21 into equation 3.2 and 3.3, we obtain;

$$\varepsilon_x = -z\frac{d^2w}{dx^2} \qquad 1.23$$

$$\varepsilon_y = -z\frac{d^2w}{dy^2} \qquad 1.24$$

Substituting equation 3.20 and 3.22 into equation 3.9, we obtain;

$$\gamma_{xy} = -z\frac{d^2w}{dxdy} + -z\frac{d^2w}{dxdy} \qquad = -2z\frac{d^2w}{dxdy} \qquad 1.25$$

Hooke's and Poisson's relations

The constitutive equations relate the stress components to strain components.

$$According\ to\ Hooke, \qquad \sigma \propto \varepsilon, \therefore \sigma = E\varepsilon \qquad 1.26$$

$$According\ to\ Poisson, \sigma = \sigma_A - \mu\sigma_L \qquad 1.27$$

$$\tau_{xy} = \frac{E(1-\mu)}{2(1-\mu^2)}\gamma_{xy} \qquad 1.28$$

Also from Stress displacement relation comes

$$\tau_{xy} = \frac{-Ez(1-\mu)}{2(1-\mu^2)}.2\frac{d^2w}{dxdy} \qquad 1.29$$

Given that the Net Total potential energy as

$$T_p = E_u + M_{wk} \qquad 1.30$$

where

$$E_u = \frac{D}{2}\int_0^a\int_0^b \left(\left[\frac{\partial^2 w}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 w}{\partial x\partial y}\right]^2 + \left[\frac{\partial^2 w}{\partial y^2}\right]^2\right)\partial x\,\partial y \qquad 1.31$$

and average work done as

$$M_{wk} = \frac{1}{2}\int_0^a\int_0^a \left[Nx\left(\frac{\partial w}{\partial x}\right)^2 + 2Nxy\frac{\partial w}{\partial x}.\frac{\partial w}{\partial y} + Ny\left(\frac{\partial w}{\partial y}\right)^2\right]\partial x\partial y \qquad 1.32$$

Therefore Equation 1.30 can be expressed as,

$$T_p = \frac{D}{2}\int_0^a\int_0^b \left(\left[\frac{\partial^2 w}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 w}{\partial x\partial y}\right]^2 + \left[\frac{\partial^2 w}{\partial y^2}\right]^2\right)\partial x\,\partial y + \frac{1}{2}\int_0^a\int_0^a \left[Nx\left(\frac{\partial w}{\partial x}\right)^2 + 2Nxy\frac{\partial w}{\partial x}.\frac{\partial w}{\partial y} + Ny\left(\frac{\partial w}{\partial y}\right)^2\right]\partial x\partial y \qquad 1.33$$

After decoupling the plate governing Equation to 2 separate 4th differential equations gives Equation 1.37,

$$\int_{d_1}^{d_2} \left(\frac{d^4 w_Q}{Q^4} + \frac{\beta^2 \cdot 2\ddot{w}_R}{\overline{w_R}} \frac{d^2 w_Q}{dQ^2} \right) \partial \mathrm{Q} = 0 \qquad 1.34$$

If the integral is zero, then the integrand is also equal to zero.

$$\frac{d^4 w_Q}{Q^4} + \frac{\beta^2 \cdot 2\ddot{w}_R}{\overline{w_R}} \frac{d^2 w_Q}{dQ^2} = 0 \qquad 1.35$$

$$Let\ m^2 = \frac{\beta^2 \cdot 2\ddot{w}_R}{\overline{w_R}} \qquad 1.36$$

$$Therefore, \frac{d^4 w_Q}{dQ^4} + m^2 \frac{d^2 w_Q}{dQ^2} = 0 \qquad 1.37$$

$$Recall, \quad \frac{d^4 w_R}{dR^4} + k^2 \frac{d^2 w_R}{dR^2} = 0 \qquad 1.38$$

Let the solution to of Equation 1.38 be expressed in exponential form as:

$$w_R = c_0 + c_1 R + c_2 e^{gR} \qquad 1.39$$

The fourth derivative of equation 3.123a is:

$$\frac{d^4 w_R}{dR^4} = g^4 c_2 e^{gR} \qquad 1.40$$

While the second derivative of Equation 3.123a gives:

$$\frac{d^2 w_R}{dR^2} = g^2 c_2 e^{gR} \qquad 1.41$$

After series of resolutions, the exact deflection equation of thin rectangular plates under buckling load in the form of Trigonometric series was reduced to

$$\mathrm{w} = \mathrm{w_R} \times \mathrm{w_Q} \qquad 1.42$$

where

$$w_R = c_0 + c_1 R + c_2 coskR + c_3 sinkR \qquad 1.43$$

$$w_Q = l_0 + l_1 Q + l_2 cosmQ + l_3 sinmQ \qquad 1.44$$

$$w = (c_0 + c_1 R + c_2 coskR + c_3 sinkR)(l_0 + l_1 Q + l_2 cosmQ + l_3 sinmQ) \qquad 1.45$$

For the plate of orientations Clamped-Simple-Simple-Simple and Clamped-Simple-Clamped-Simple

CSSS Rectangular Plate

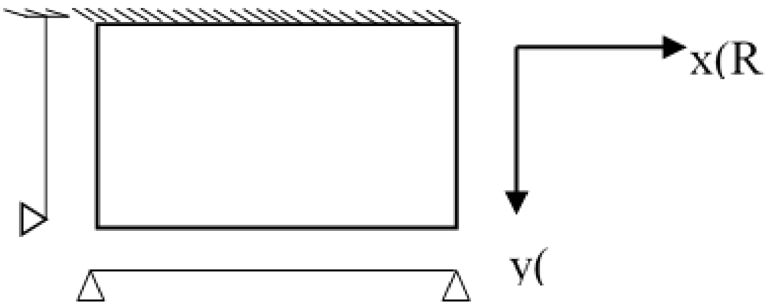


Figure 3: CSSS Rectangular Plate

That is

$$w_R = c_3(\sin n\pi R) \qquad 1.46$$

$$w_Q = l_3(k - kQ - kCoskQ + SinkQ) \qquad 1.47$$

$$w = A(\sin n\pi R)(k - kQ - kCoskQ + SinkQ) \qquad 1.48$$

$$where, A = c_3 * l_3 \qquad 1.49$$

Similarly,

For CSCS Rectangular Plate

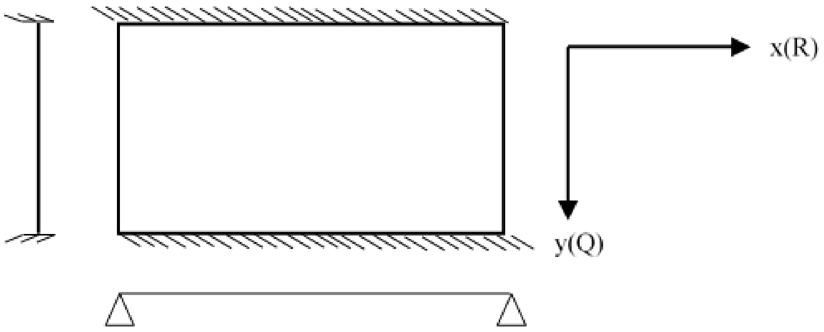


Figure 4: CSCS Rectangular Plate

$$Similarly, w_R = c_0\,(\sin\pi R) \quad 1.51$$

$$w_Q = l_3(1 - Cos2\pi Q\,) \quad 1.52$$

$$w = A\,(\sin\pi R)(1 - Cos2\pi Q) \quad 1.53$$

$$where, A = c_0 * l_3 \quad 1.54$$

The Integrands of the CSSS Deflection Equation

$$\text{For CS, } d_1 = 0.3, \quad d_2 = 1 \quad 1.55$$

$$\text{For SS, } d_1 = 0, \quad d_2 = 1 \quad 1.56$$

$$k = 4.49340946$$

$$k_2 = -\pi^2 \int_0^1 \int_{0.3}^1 (\text{Sin}\pi\text{R})(k - kQ - k\text{Cos}kQ + \text{Sin}kQ)dRdQ = -\pi^2 \left(\frac{2}{\pi}\right)(3.14980711)$$
$$= -6.29961422\pi \quad 1.57$$

$$k_4 = \pi^4 \int_0^1 \int_{0.3}^1 (\text{Sin}\pi\text{R})(k - kQ - k\text{Cos}kQ + \text{Sin}kQ)\,dRdQ = \pi^4 \left(\frac{2}{\pi}\right)(3.14980711)$$
$$= 6.29961422\,\pi^3 \quad 1.58$$

$$k_{23} = -\pi^2 \int_0^1 \int_{0.3}^1 (\text{Sin}\pi\text{R})(k^3\text{Cos}kQ - k^2\text{Sin}kQ)\,dRdQ = -\pi^2 \left(\frac{2}{\pi}\right)(-41.36922381) =$$
$$= 82.73844762\pi \quad 1.59$$

$$k_3 = \int_0^1 \int_{0.3}^1 (\text{Sin}\pi\text{R})\,(-k^5\text{Cos}kQ + k^4\text{Sin}kQ)dRdQ = \left(\frac{2}{\pi}\right)(835.27476932)$$
$$= \frac{1670.54953864}{\pi} \quad 1.60$$

Bringing Equations 1.57 to 3.309 together gives:

$$k_t = 6.29961422\,\pi^3 + \frac{2}{\beta^2}82.73844762\pi + \frac{1}{\beta^4}\frac{1670.54953864}{\pi} \quad 1.61$$

and then

$$\frac{N_x a^2}{D} = \frac{6.29961422\pi^3 + \frac{164.7689524\pi}{\beta^2} + \frac{1670.54953864}{\pi\beta^4}}{6.29961422\pi} \quad 1.62$$

The Integrands of the CSCS Deflection Equation

$$\text{For CC, } d_1 = 0.25, \quad d_2 = 0.75 \quad 1.63$$

For SS, $d_1 = 0, \quad d_2 = 1$ 1.64

$$k_2 = -\pi^2 \int_0^1 \int_{0.25}^{0.75} (\mathrm{Sin}\pi R)(1 - \mathrm{Cos}2\pi Q)\, dRdQ = -\pi^2 \left(\frac{2}{\pi}\right)\left(0.5 + \frac{1}{\pi}\right) = -\pi^2 \left(\frac{1}{\pi} + \frac{2}{\pi^2}\right)$$

$$= -\pi - 2 \quad 1.65$$

$$k_4 = \pi^4 \int_0^1 \int_{0.25}^{0.75} (\mathrm{Sin}\pi R)(1 - \mathrm{Cos}2\pi Q) dRdQ = \pi^4 \left(\frac{2}{\pi}\right)\left(0.5 + \frac{1}{\pi}\right) = \pi^4 \left(\frac{1}{\pi} + \frac{2}{\pi^2}\right)$$

$$= \pi^3 + 2\pi^2 \quad 1.66$$

$$k_{23} = -4\pi^4 \int_0^1 \int_{0.25}^{0.75} (\mathrm{Sin}\pi R)(\mathrm{Cos}2\pi Q) \quad dRdQ = -4\pi^4 \left(\frac{2}{\pi}\right)\left(\frac{-1}{\pi}\right) = 8\pi^2 \quad 1.67$$

$$k_3 = -16\pi^4 \int_0^1 \int_{0.25}^{0.75} (\sin \pi R)(\mathrm{Cos}2\pi Q)\, dRdQ = -16\pi^4 \left(\frac{2}{\pi}\right)\left(\frac{-1}{\pi}\right) = 32\pi^2 \quad 1.68$$

Bringing Equations 1.65 to 1.68 together gives:

$$k_t = \pi^3 + 2\pi^2 + \frac{2}{\beta^2} 8\pi^2 + \frac{1}{\beta^4} 32\pi^2 \quad 1.69$$

and finally

$$\frac{N_x a^2}{D} = \frac{\pi^3 + 2\pi^2 + \frac{16\pi^2}{\beta^2} + \frac{32\pi^2}{\beta^4}}{\pi + 2} \quad 1.70$$

Results

Substituting the derived variables into the equations gives the following values

For the Stiffness Coefficients of the plate types

	k_2	k_4	k_{23}	k_3
CSCS	-5.14159265	50.7454855	78.9568352	315.82734
CSSS	-19.79082	195.3275815	259.93050	531.7524335

Table 1: Stiffness Coefficients for different Aspect ratios for CSSS Plates

b/a	k_4	k_{23}	k_3	k_2	K_t	$\bar{N}$
1	195.32758	259.93050	531.75243	-19.79082	1246.94101	63.00603
1.1	195.32758	259.93050	531.75243	-19.79082	988.15884	49.93016
1.2	195.32758	259.93050	531.75243	-19.79082	812.78142	41.06860
1.3	195.32758	259.93050	531.75243	-19.79082	689.11895	34.82013
1.4	195.32758	259.93050	531.75243	-19.79082	598.98231	30.26566
1.5	195.32758	259.93050	531.75243	-19.79082	531.41443	26.85156
1.6	195.32758	259.93050	531.75243	-19.79082	479.53727	24.23029
1.7	195.32758	259.93050	531.75243	-19.79082	438.87719	22.17579
1.8	195.32758	259.93050	531.75243	-19.79082	406.43317	20.53645
1.9	195.32758	259.93050	531.75243	-19.79082	380.13668	19.20773
2	195.32758	259.93050	531.75243	-19.79082	358.52736	18.11584

Table 2: Stiffness Coefficients for different Aspect ratios for CSCS Plates

b/a	k_4	k_{23}	k_3	k_2	k_t	$\bar{N}$
1	50.74549	78.95684	315.82734	-5.14159	524.48650	102.00857
1.1	50.74549	78.95684	315.82734	-5.14159	396.96697	77.20701
1.2	50.74549	78.95684	315.82734	-5.14159	312.71647	60.82093
1.3	50.74549	78.95684	315.82734	-5.14159	254.76546	49.54991
1.4	50.74549	78.95684	315.82734	-5.14159	213.52613	41.52918
1.5	50.74549	78.95684	315.82734	-5.14159	183.31499	35.65335
1.6	50.74549	78.95684	315.82734	-5.14159	160.62194	31.23973
1.7	50.74549	78.95684	315.82734	-5.14159	143.20102	27.85149
1.8	50.74549	78.95684	315.82734	-5.14159	129.56994	25.20035
1.9	50.74549	78.95684	315.82734	-5.14159	118.72345	23.09079
2	50.74549	78.95684	315.82734	-5.14159	109.96311	21.38697

Ritz Stiffness Co-Efficient and Buckling load for CSSS Plate

$$k_N = \int_0^1 \int_0^1 \left[\frac{\partial h}{\partial R}\right]^2 \partial R \partial Q$$

$$\begin{aligned}
&= [151.9232788056477QR - 99.63725180358355Q^2R + 33.21241726786119Q^3R \\
&- 46.54463606534013R\mathrm{Sin}[4.49340946Q] + 44.34817378231254QR\mathrm{Sin}[4.49340946Q] \\
&+ 5.268963937410621R\mathrm{Sin}[8.98681892Q] + \mathrm{Cos}[8.98681892Q](2.4674011002723395R \\
&+ 0.39269908169872414\mathrm{Sin}[6.283185307179586R]) + 24.17934079264701Q\mathrm{Sin}[6.283185307179586R] \\
&- 15.857761140632187Q^2\mathrm{Sin}[6.283185307179586R] + 5.28592038021073Q^3\mathrm{Sin}[6.283185307179586R] \\
&- 7.407808904212188\mathrm{Sin}[4.49340946Q]\mathrm{Sin}[6.283185307179586R] \\
&+ 7.058231074553438Q\mathrm{Sin}[4.49340946Q]\mathrm{Sin}[6.283185307179586R] \\
&+ 0.8385816556118363\mathrm{Sin}[8.98681892Q]\mathrm{Sin}[6.283185307179586R] + \mathrm{Cos}[4.49340946Q](9.869604401089358QR \\
&+ 1.5707963267948966Q\mathrm{Sin}[6.283185307179586R])]_0^1 \\
&= 83.03104317 \qquad 1.71
\end{aligned}$$

$$k_2 = \int_0^1 \int_0^1 \left[\frac{\partial^2 h}{\partial R^2}\right]^2 \partial R \partial Q$$

$$\begin{aligned}
&= [1499.4226611281463QR - 983.380258913097Q^2R + 327.79341963769906Q^3R \\
&- 459.3771449575835R\mathrm{Sin}[4.49340946Q] + 437.69893114218763QR\mathrm{Sin}[4.49340946Q] \\
&+ 52.00258966584899R\mathrm{Sin}[8.98681892Q] + \mathrm{Cos}[8.98681892Q](24.352272758500607R \\
&- 3.8757845850374775\mathrm{Sin}[6.283185307179586R]) - 238.6405283025484Q\mathrm{Sin}[6.283185307179586R] \\
&+ 156.50982914500727Q^2\mathrm{Sin}[6.283185307179586R] - 52.16994304833577Q^3\mathrm{Sin}[6.283185307179586R] \\
&+ 73.11214336344156\mathrm{Sin}[4.49340946Q]\mathrm{Sin}[6.283185307179586R] \\
&- 69.66194847731829Q\mathrm{Sin}[4.49340946Q]\mathrm{Sin}[6.283185307179586R] \\
&- 8.276469198899381\mathrm{Sin}[8.98681892Q]\mathrm{Sin}[6.283185307179586R] + \mathrm{Cos}[4.49340946Q](97.40909103400243QR \\
&- 15.50313834014991Q\mathrm{Sin}[6.283185307179586R])]_0^1 \\
&= 819.48354906 \qquad 1.72
\end{aligned}$$

$$k_{12} = \int_0^1 \int_0^1 \left[\frac{\partial^2 h}{\partial R \partial Q}\right]^2 \partial R \partial Q$$

$$\begin{aligned}
&= [1155.3302312787532QR - 44.34817378231255R\mathrm{Sin}[4.49340946Q] - 106.38422073286706R\mathrm{Sin}[8.98681892Q] \\
&+ \mathrm{Cos}[8.98681892Q](-49.818625901791805R - 7.928880570316098\mathrm{Sin}[6.283185307179586R]) \\
&+ 183.87651721151624Q\mathrm{Sin}[6.283185307179586R] \\
&- 7.058231074553439\mathrm{Sin}[4.49340946Q]\mathrm{Sin}[6.283185307179586R] \\
&- 16.93157459661509\mathrm{Sin}[8.98681892Q]\mathrm{Sin}[6.283185307179586R] + \mathrm{Cos}[4.49340946Q](199.27450360716722R \\
&+ 31.71552228126439\mathrm{Sin}[6.283185307179586R])]_0^1 \\
&= 1005.87435445 \qquad 1.73
\end{aligned}$$

$$k_3 = \int_0^1 \int_0^1 \left[\frac{\partial^2 h}{\partial Q^2}\right]^2 \partial R \partial Q$$

$$\begin{aligned}
&= [2159.6823480577527QR + \mathrm{Sin}[8.98681892Q](217.63536188606108R \\
&- 34.6377436357601\mathrm{Sin}[6.283185307179586R]) + \mathrm{Cos}[8.98681892Q](101.916380099526R \\
&- 16.220495674872033\mathrm{Sin}[6.283185307179586R]) - 343.72412120170253Q\mathrm{Sin}[6.283185307179586R]]_0^1 \\
&= 2057.76596700 \qquad 1.74
\end{aligned}$$

$$\bar{N} = \frac{\left[819.48354906 + 2\frac{1}{\beta^2}1005.87435445 \quad + \frac{1}{\beta^4}2057.76596700\right]}{83.03104317} \qquad 1.75$$

Ritz Stiffness Co-Efficient and Buckling load for CSCS Plate

$$k_N = \int_0^1 \int_0^1 \left[\frac{\partial h}{\partial R}\right]^2 \partial R \partial Q = \frac{1}{32}[(2\pi R + \mathrm{Sin}2\pi R)(12\pi Q - 8\mathrm{Sin}2\pi Q + \mathrm{Sin}4\pi Q)]_0^1 = \frac{3\pi^2}{4} = 7.4022033008 \quad 1.76$$

$$k_2 = \int_0^1 \int_0^1 \left[\frac{\partial^2 h}{\partial R^2}\right]^2 \partial R \partial Q = -\frac{\pi^2}{32}[(-2\pi R + \mathrm{Sin}2\pi R)(12\pi Q - 8\mathrm{Sin}2\pi Q + \mathrm{Sin}4\pi Q)]_0^1 = \frac{3\pi^4}{4} = 73.05681828 \quad 1.77$$

$$k_{23} = \int_0^1 \int_0^1 \left[\frac{\partial^2 h}{\partial R \partial Q}\right]^2 \partial R \partial Q = -\frac{\pi^2}{8}[(2\pi R + \mathrm{Sin}2\pi R)(-4\pi Q + \mathrm{Sin}4\pi Q)]_0^1 = \pi^4 = 97.40909103 \quad 1.78$$

$$k_3 = \int_0^1 \int_0^1 \left[\frac{\partial^2 h}{\partial Q^2}\right]^2 \partial R \partial Q = \frac{\pi^2}{2}[(2\pi R - \mathrm{Sin}2\pi R)(4\pi Q + \mathrm{Sin}4\pi Q)]_0^1 = 4\pi^4 = 389.63636414 \quad 1.79$$

$$\bar{N} = \frac{\left[73.05681828 + 2\frac{1}{\beta^2}97.40909103 \quad + \frac{1}{\beta^4}389.63636414\right]}{7.4022033008} \quad 1.80$$

Table 3: Stiffness Coefficients for different Aspect ratios for CSSS Plate

b/a	k_2	k_{23}	k_3	k_N	K_t	$\bar{N}$
1	819.48355	1005.87435	2057.76597	83.03104	4888.99822	58.88157
1.1	819.48355	1005.87435	2057.76597	83.03104	3887.56763	46.82065
1.2	819.48355	1005.87435	2057.76597	83.03104	3208.89525	38.64693
1.3	819.48355	1005.87435	2057.76597	83.03104	2730.34846	32.88347
1.4	819.48355	1005.87435	2057.76597	83.03104	2381.53932	28.68252
1.5	819.48355	1005.87435	2057.76597	83.03104	2120.06638	25.53342
1.6	819.48355	1005.87435	2057.76597	83.03104	1919.31306	23.11561
1.7	819.48355	1005.87435	2057.76597	83.03104	1761.96744	21.22059
1.8	819.48355	1005.87435	2057.76597	83.03104	1636.41616	19.70849
1.9	819.48355	1005.87435	2057.76597	83.03104	1534.65446	18.48290
2	819.48355	1005.87435	2057.76597	83.03104	1451.03110	17.47577

Table 4: Stiffness Coefficients for different Aspect ratios for CSCS Plate

b/a	K_2	k_{23}	k_3	k_N	K_t	$\bar{N}$
1	73.05682	97.40909	389.63636	7.40220	657.51136	88.82644
1.1	73.05682	97.40909	389.63636	7.40220	500.19046	67.57319
1.2	73.05682	97.40909	389.63636	7.40220	396.25056	53.53143
1.3	73.05682	97.40909	389.63636	7.40220	324.75637	43.87293
1.4	73.05682	97.40909	389.63636	7.40220	273.87939	36.99971
1.5	73.05682	97.40909	389.63636	7.40220	236.60788	31.96452
1.6	73.05682	97.40909	389.63636	7.40220	208.61146	28.18235
1.7	73.05682	97.40909	389.63636	7.40220	187.11926	25.27886
1.8	73.05682	97.40909	389.63636	7.40220	170.30260	23.00701
1.9	73.05682	97.40909	389.63636	7.40220	156.92128	21.19927
2	73.05682	97.40909	389.63636	7.40220	146.11364	19.73921

Comparism between the two methods gives

Table 5: Comparison of Critical Buckling Loads for CSSS Plate

CSSS Plate			
a/b	(N_x) from present study	(N_x) from Ritz	Percentage Difference (%)
1.00	63.01	58.88	6.55
1.10	49.93	46.82	6.23
1.20	41.07	38.65	5.90
1.30	34.82	32.88	5.56
1.40	30.27	28.68	5.23
1.50	26.85	25.53	4.91
1.60	24.23	23.12	4.60
1.70	22.18	21.22	4.31
1.80	20.54	19.71	4.03
1.90	19.21	18.48	3.77
2.00	18.12	17.48	3.53

Table 6: Comparison of Critical Buckling Loads for CSCS Plate

CSCS Plate			
ASPECT RATIO	(N_x) from present study	(N_x) from Ritz	Percentage Difference (%)
1.00	102.01	88.83	12.92
1.10	77.21	67.57	12.48
1.20	60.82	53.53	11.99
1.30	49.55	43.87	11.46
1.40	41.53	37.00	10.91
1.50	35.65	31.96	10.35
1.60	31.24	28.18	9.79
1.70	27.85	25.28	9.24
1.80	25.20	23.01	8.70
1.90	23.09	21.20	8.19
2.00	21.39	19.74	7.70

Differences in Buckling loads were seen across the other plates under consideration and these differences ranged from 1.82% to 12.92%, with the highest variation in value occurring in the CSCS plate. It is seen that there are minor differences in values between the two. These differences can be ascribed to the fact that this study used trigonometric shape functions while Ibearugbulem.

CONCLUSIONS

From the work so far, the following conclusions were drawn:

i. The critical buckling loads derived herein can be used in confidence for buckling analysis of rectangular plates analysis of selected boundary conditions.
ii. The strong form of expression of plate equilibrium of forces derived in this study could be used in confidence to satisfactorily analyse real time rectangular thin plates of various boundary conditions under in-plane loadings.
iii. The results obtained herein are exact results as there were no presumptions of any kind made to arrive at the results.

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