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Control Chart for Variables with Specified Process Capability CPK Index Based on Proposed Downton's Estimator

[1] Ibrahim Abdulhakeem, [2] U. F. Abbas, [3] K. E. Lasisi

[1, 2] Federal Polytechnic Bauchi, Nigeria

[3] Department of Mathematical Sciences, Abubakar Tafawa Balewa University, Bauchi, Nigeria

ABSTRACT

The Shewhart charts, also known as statistical process control are statistical tools used to determine whether a production process is stable or not (i.e., in statistical control or not), while the capacity of the process is determined by evaluating the process capability indices (PCLs) so as to produce acceptable products that meet the customers specifications. However, the general idea is that process capability indices (PCls) can be evaluated only after it has been established that a process is in state of statistical control (by the use of Shewhart charts known as control charts). This is a two-stage procedure - the stability of a process has to be established by the help of control charts on one hand and then followed by computing the process capability indices to evaluate the capability of the process. This paper therefore proposes a single control chart procedure that determines stability of the process and assess its capability for variables simultaneously, based only on the specified process capability index Cpk, using the Downton Statistic as the estimate of the process standard deviation σ. Table containing constant factors for computing the proposed capability-based control charts for sample size ($n \leq 10$) is provided. The proposed procedure is effective for monitoring and assessing the capability of a process, as it collapses the two-stage method: stability and capability into one procedure where management can vary the specification limits so as to achieve stability and capability as long as the minimum capability index value is achieved. It has also been shown that the proposed method is simple to apply and does not require any difficult computations for both stability and capability and, it is effective for normal and non-normal process situations. The proposed method has also been demonstrated with real life data.

Keywords: Downton's Estimator, Control Charts, Process Capability Index, process mean, process standard deviation and Quality Characteristics

1.0 INTRODUCTION

Quality is indeed an important strategy in businesses, industries and manufacturing companies. Successful increase of quality results to the improvement of product output, increase customers acceptability and eventually leading to higher profitability. Every product possesses a number of elements that jointly describe what the user or consumer thinks of as quality and the elements are often called quality characteristics (Montgomery, 2009). Monitoring and controlling both the mean and the variability of the quality characteristics of a variable are done by the help of control chart for variable. Control of the process mean is usually done with the control chart for means while process variability can be monitored and controlled with either a control chart for the standard deviation, called the S control chart, or a control chart for the range, called an R control chart, though the R chart is more widely used (Montgomery, 2009).

To study the behaviour of the production process and to take a necessary action on the process, control charts for variables are very much used which require to collect random samples from the production process and to compute the estimate of the process characteristics (Subramani and Balamurali, 2012). Therefore, the control charts establish whether the process is under a statistical control by identifying any assignable causes of variability and where they occur, corrective measures are taken to remove them.

However, examining whether the process is capable of producing acceptable products that meet the customers specifications, so that the return of products is minimized, one has to show that the process is under the state of statistical control and then establish the capability of the process with the help of process capability indices. This however indicates two stage procedure: establishing the stability of the process by the help of control charts and capability of the process by the evaluation of process capability indices. (Subramani and Balamurali, 2012) presented a technique called process capability-based control charts which only combines the usual two stage procedures into a single stage procedure using range as the estimate of the process standard deviation σ where a specified process capability indices of Cp and Cpk values are used to determine the control charts for variable.

Downton (1966) introduced an estimator called Downton estimator to estimate the standard deviation of a normal population. It was also shown by (Barnett et al., 1967) that Downton estimator is an unbiased estimator of standard deviation. The Downton estimator is said to be a robust dispersion estimator that is not affected by non-normality (Abbasi and Miller, 2011).

Adeoti et al, (2016) derived the control chart limits based on the Downton estimator, $\bar{X}_D$, and proved to be more effective than the $\bar{X}$ chart limits based on the range statistic for monitoring process data when the normality assumption is violated.

Adeoti and Olaomi (2017) also developed a single procedure that can determine the stability of the process and assess its capability for variables, based only on the specified process capability index Cp, using the Downton Statistic as the estimate of the process standard deviation σ. His results shows that the proposed control chart performs better in monitoring and assessing processes, and eliminates the usual two stage procedure. Further study was suggested on other process capability indices.

This work therefore aims to further propose single procedure that can determine the stability of a process and assess its capability simultaneously, based on the specified process capability indices of Cpk using the Downton statistic as the estimate of the standard deviation σ.

2.0 MATERIALS AND METHOD

2.1 Downton Estimator

Downton (1966) introduced an estimator called Downton estimator to estimate the standard deviation of a normal population.

Let $X_1, X_2, X_3, \ldots, X_n$ represent random sample of size n from a normal distribution with mean, μ, and standard deviation σ, such that, X ~ N (μ, σ). If $X_1 \leq X_2 \leq \ldots \leq X_n$ denote the corresponding order statistics. The Downton estimator is defined as:

$$D = \frac{2\sqrt{\pi}}{n(n-1)} \sum \left[i - \frac{1}{2}(n+1)\right] X_i \quad (2.1)$$

The average of sample D's ($\bar{D}$) computed from a given number of random samples obtained from a stable process is an unbiased estimator of standard deviation σ (Downton, 1966; Barnett et al., 1967; Iglewicz, 1983; Abu-Shawiesh and Abdullah, 2000; Abbasi and Miller, 2011; Abbasi and Miller, 2013; Adeoti et al, 2016; Adeoti and Olaomi 2017).

2.2 Design of Proposed Control Chart with Specified Cpk

The control chart limits for the process variability using Downton estimator known as D chart as derived by (Abbasi and Miller, 2011) for σ is given as $\hat{\sigma} = \bar{D}$ are as follows:

$$\text{LCL} = Max(0, \bar{D} - 3z_3\bar{D}) = Z_3\bar{D} \quad (2.2)$$

$$\text{CL} = \bar{D} \quad (2.3)$$

$$\text{UCL} = \bar{D} + 3z_3\bar{D} = Z_4\bar{D} \quad (2.4)$$

Where $Z_3 = 1 - 3z_3$ and $Z_4 = 1 + 3z_3$

Also, the control chart limits for the process average using Downton estimator known as D chart as derived by (Adeoti et al, 2016) for σ is given as $\hat{\sigma} = \bar{D}$ are as follows:

$$\text{LCL} = \bar{X}_D - A\bar{D} \quad (2.5)$$

$$\text{CL} = \bar{X}_D \quad (2.6)$$

$$\text{UCL} = \bar{X}_D + A\bar{D} \quad (2.7)$$

Where $D = \frac{2\sqrt{\pi}}{n(n-1)} \sum \left[i - \frac{1}{2}(n+1)\right] X_i$ (2.8)

$\bar{D} = \frac{\sum_{j=1}^{m} D_j}{m}$ for m is number of subgroups (2.9)

$\hat{\sigma} = \bar{D}$ and (2.10)

$$\sigma_D = z_3\sigma \quad (2.11)$$

$$\text{Var}(D) = \frac{\sigma^2}{n(n-1)}\left[n\left(\frac{1}{3}\pi + 2\sqrt{3} - 4\right) + \left(6 - 4\sqrt{3} + \frac{1}{3}\pi\right)\right] \quad (2.12)$$

(Barnett et al., 1967) showed that from (2.12) and (2.13) we obtained

$$z_3 = \frac{1}{\sqrt{n(n-1)}}\sqrt{n\left(\frac{1}{3}\pi + 2\sqrt{3} - 4\right) + \left(6 - 4\sqrt{3} + \frac{1}{3}\pi\right)} \quad (2.13)$$

(Abbasi and Miller, 2011)

2.3 Cpk Index

The Cpk index takes process centering into account and it is simply the one-sided process capability index for the specification limit nearest to the process average (Montgomery, 2009). Also, it is a measure of process performance where it establishes the distance between the process average and the closest specification limits (Kane, 1986). It is defined as follows:

$$\text{Cpk} = min(Cpu, Cpl) = min\left(\frac{USL-\bar{\bar{X}}}{3\sigma}, \frac{\bar{\bar{X}}-LSL}{3\sigma}\right) \quad (2.14)$$

Where the process standard deviation σ is usually estimated using either the sample standard deviation or sample range.

By algebraic manipulation Cpk defined in (2.4) then becomes:

When $m \leq \bar{\bar{X}} \leq USL$

$$\frac{d-(\bar{X}-m)}{3\sigma} = \frac{\frac{USL-LSL}{2} - \left[\bar{X} - \left(\frac{USL+LSL}{2}\right)\right]}{3\sigma} = \frac{\frac{USL-LSL}{2} + \frac{USL+LSL}{2} - \bar{\bar{X}}}{3\sigma} = \frac{USL-\bar{\bar{X}}}{3\sigma} \quad (2.15)$$

When $LSL \leq \bar{\bar{X}} \leq m$

$$\frac{d-[-(\bar{X}-m)]}{3\sigma} = \frac{d+(\bar{X}-m)}{3\sigma} = \frac{\frac{USL-LSL}{2} + \left[\bar{\bar{X}} - \left(\frac{USL+LSL}{2}\right)\right]}{3\sigma} = \frac{\frac{USL-LSL}{2} - \frac{USL+LSL}{2} + \bar{\bar{X}}}{3\sigma} = \frac{\bar{\bar{X}} - LSL}{3\sigma} \quad (2.16)$$

$$Cpk = \frac{d-|\bar{X}-m|}{3\sigma} = min\left(\frac{USL-\bar{X}}{3\sigma}, \frac{\bar{X}-LSL}{3\sigma}\right) \quad (2.17)$$

Therefore, an unbiased estimator σ, as described by (Abbasi and Miller, 2011) is used in (2.18) to estimate the process standard deviation σ. Hence Cpk becomes:

$$\text{Cpk} = min\left(\frac{USL-\bar{X}}{3\bar{D}}, \frac{\bar{X}-LSL}{3\bar{D}}\right) = \frac{d-|\bar{X}-m|}{3\bar{D}} \quad (2.19)$$

Now, for a given If Cpk value, the control limits for mean and variability control charts using Downton estimator of σ can be derived.

2.4 The Derived Control Chart Limits for Mean with specified Cpk

$$\text{UCL} = \bar{X}_D + A^*_{pk}\frac{(d-|\bar{X}_D-M|)}{C_{pk}} \quad (2.20)$$

where $d = \frac{USL-LSL}{2}$, $M = \frac{USL+LSL}{2}$, $A^*_{pk} = \frac{A}{3}$ and $A = \frac{3}{\sqrt{n}}$

$$\text{CL} = \bar{X}_D \quad (2.21)$$

$$\text{LCL} = \bar{X}_D - A^*_{pk}\frac{(d-|\bar{X}_D-M|)}{C_{pk}} \quad (2.22)$$

2.5 The Derived Control Chart Limits for Variability with specified Cpk

$$\text{UCL} = Z^*_{4pk}\frac{(d-|\bar{X}_D-M|)}{C_{pk}} \quad (2.23)$$

where $Z^*_{4pk} = \frac{Z_4}{3}$

$$\text{CL} = \bar{D} = G^*_{4pk}\frac{(d-|\bar{X}_D-M|)}{C_{pk}} \quad (2.24)$$

where $G^*_{4pk} = \frac{1}{3}$

$$\text{LCL} = Z^*_{3pk}\frac{(d-|\bar{X}_D-M|)}{C_{pk}} \quad (2.25)$$

where $Z^*_{3pk} = \frac{Z_3}{3}$

Since Cpk establishes the distance between the process average and the closest specification limits, the above control limits can further be simplified if d and M are substituted which will lead to the following two cases:

2.6 Case 1: When $\bar{X}_D < M$

Control Limits for the proposed Mean Chart becomes:

$$\text{UCL} = \bar{X}_D + A^*_{pk}\frac{[\bar{X}_D-LSL]}{C_{pk}} \quad (2.26)$$

$$\text{CL} = \bar{X}_D \quad (2.27)$$

$$\text{LCL} = \bar{X}_D - A^*_{pk}\frac{[\bar{X}_D-LSL]}{C_{pk}} \quad (2.28)$$

Control Limits for the proposed Variability becomes:

$$\text{UCL} = Z^*_{4pk}\frac{[\bar{X}_D - LSL]}{C_{pk}} \quad (2.29)$$

$$\text{CL} = \bar{D} = G^*_{4pk}\frac{[\bar{X}_D - LSL]}{C_{pk}} \quad (2.30)$$

$$\text{LCL} = Z^*_{3pk}\frac{[\bar{X}_D - LSL]}{C_{pk}} \quad (2.31)$$

2.7 Case 2: When $\bar{X}_D > M$

Control Limits for the proposed Mean Chart becomes:

$$\text{UCL} = \bar{X}_D + A^*_{pk}\frac{[USL - \bar{X}_D]}{C_{pk}} \quad (2.32)$$

$$\text{CL} = \bar{X}_D \quad (2.33)$$

$$\text{LCL} = \bar{X}_D - A^*_{pk}\frac{[USL - \bar{X}_D]}{C_{pk}} \quad (2.34)$$

Control Limits for the proposed Variability Chart becomes:

$$\text{UCL} = Z^*_{4pk}\frac{[USL - \bar{X}_D]}{C_{pk}} \quad (2.35)$$

$$\text{CL} = \bar{D} = G^*_{4pk}\frac{[USL - \bar{X}_D]}{C_{pk}} \quad (2.36)$$

$$\text{LCL} = Z^*_{3pk}\frac{[USL - \bar{X}_D]}{C_{pk}} \quad (2.37)$$

The value of Z_3, z_3, A^*_{pk}, Z^*_{4pk}, Z^*_{3pk}, G^*_{4pk} and A for sample size n ($2 \le n \le 10$) are presented in Table 1. The proposed control charts can be used to assess the stability and measure the capability of the process simultaneously for a specified value of Cpk.

Table 1: Constant Values for Variable Control Charts Limits with specified Cpk

Sample Size n	A	A^*_{pk}	z_3	Z_3	Z_4	Z^*_{3pk}	Z^*_{4pk}
2	2.1213	0.7071	0.7555	0	3.2665	0	1.0888
3	1.7321	0.5774	0.5249	0	2.5746	0	0.8582
4	1.5	0.5	0.4247	0	2.274	0	0.758
5	1.3416	0.4472	0.3658	0	2.0973	0	0.6991
6	1.2247	0.4082	0.3259	0.02223	1.9778	0.0074	0.6593
7	1.1339	0.378	0.2967	0.1098	1.8902	0.0366	0.6301
8	1.0607	0.3536	0.2742	0.1775	1.8225	0.0592	0.6075
9	1	0.3333	0.2561	0.23183	1.7682	0.0773	0.5894
10	0.9487	0.3162	0.2411	0.27668	1.7233	0.0922	0.5744

3.0 Illustration Results and Discussion

As described by (Montgomery, 2009), a real-life data set pertaining to the manufacturing of Piston Rings for an automotive engine produced by a forging process of twenty-five samples, each of size five have been taken and the inside diameter is measured. This data set is applied to demonstrate the application of the proposed chart based on the Downton's estimator. The summary statistics for the twenty-five samples are given in the table 1 below.

Table 2: $\bar{X}$, R and Downton Values for Real-Life Data Set

Sample Number	$\bar{X}$	R	D
1	74.0102	0.038	0.01648
2	74.0006	0.019	0.00833
3	74.008	0.036	0.01613
4	74.003	0.022	0.0101
5	74.0034	0.026	0.01312

6	73.956	0.024	0.00922
7	74	0.012	0.00603
8	73.9968	0.03	0.01329
9	74.0042	0.014	0.00567
10	73.998	0.017	0.00691
11	73.9942	0.008	0.00301
12	74.0014	0.011	0.00461
13	73.9984	0.029	0.01117
14	73.9902	0.039	0.01666
15	74.006	0.016	0.00798
16	73.9966	0.021	0.00815
17	74.0008	0.026	0.01152
18	74.0074	0.018	0.00762
19	73.9982	0.021	0.00851
20	74.0092	0.02	0.00886
21	73.9998	0.033	0.01329
22	74.0016	0.019	0.00798
23	74.0024	0.025	0.01241
24	74.0052	0.022	0.00957
25	73.9982	0.035	0.01755
Average	**74.00118**	**0.02324**	**0.01017**

3.1 The Usual Two-Stage Procedure

In order to access the capability of the process using the specification limits $74.000 \pm 0.05\ mm$ where USL and LSL are given as 74.05 and 73.95 respectively. The usual two stage procedure is considered using the control chart to determine the stability of the process, then followed by the evaluation of capability index Cpk to access the capability of the process where range is used to estimate the standard deviation.

From table 1, the following values are obtained. $\bar{\bar{X}} = 74.00118$, $\bar{R} = 0.02324$,

$d_2 = 2.326$ $\sigma = \frac{\bar{R}}{d_2} = 0.010$ and $\bar{D} = 0.01017$

R-Chart Control Limits

The R-Chart Control Limits are determined as follows:

$\bar{R} = 0.02324$, $D_4 = 2.115$ and $D_3 = 0$

UCL= $D_4\bar{R} = 2.115 \times 0.02324 = 0.0492$

CL= $\bar{R} = 0.02324$

LCL= $D_3\bar{R} = 0 \times 0.02324 = 0$

$\bar{X}$-Chart Control Limits

$\bar{X}$-Chart Control Limits are determined as follows:

$\bar{\bar{X}} = 74.00118$, $\bar{R} = 0.02324$ and $A_2 = 0.557$

UCL= $\bar{\bar{X}} + A_2\bar{R} = 74.00118 + 0.557 \times 0.02324 = 74.01412$

CL= $\bar{\bar{X}} = 74.00118$

LCL== $\bar{\bar{X}} - A_2\bar{R} = 74.00118 - 0.557 \times 0.02324 = 73.98823$

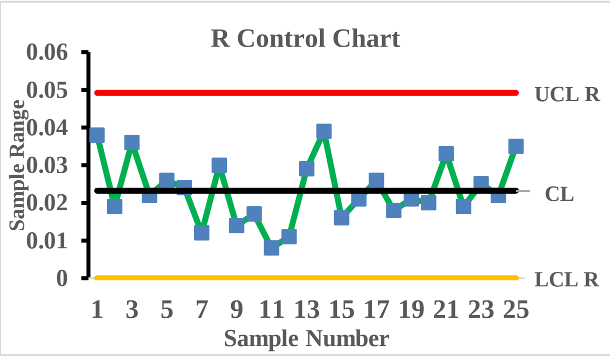

Figure 1: R Control Chart

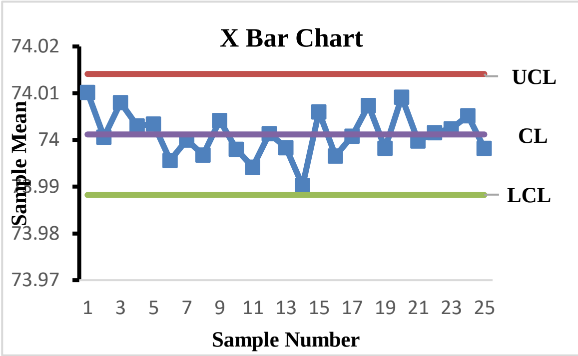

Figure 2: $\bar{X}$ Control Chart

Downton $\bar{X}_D$ Chart Control Limits

$\bar{X}_D$ Chart control Limits are obtained as follows:

$\bar{X}_D = 74.00118$, $\bar{D} = 0.01017$, and $A = 1.3416$

UCL= $\bar{X}_D + A\bar{D} = 74.00118 + (1.3416 \times 0.01017) = 74.01482$

CL= $\bar{X}_D = 74.00118$

LCL= $\bar{X}_D - A\bar{D} = 74.00118 - (1.3416 \times 0.01017) = 73.98754$

D-Chart Control Limits

D-Chart Control Limits are obtained as follows:

$Z_4 = 2.0973$, $Z_3 = 0$ and $\bar{D} = 0.01017$,

UCL= $Z_4\bar{D} = 2.0973 \times 0.01017 = 0.02133$

CL= $\bar{D} = 0.01017$

LCL= $Z_3\bar{D} = 0 \times 0.01017 = 0$

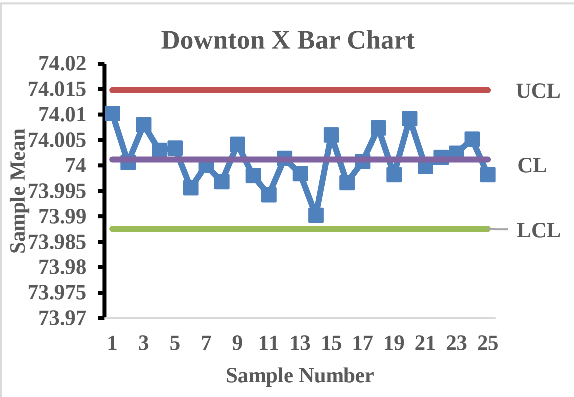

Figure 3: Downton $\bar{X}_D$ Control Chart

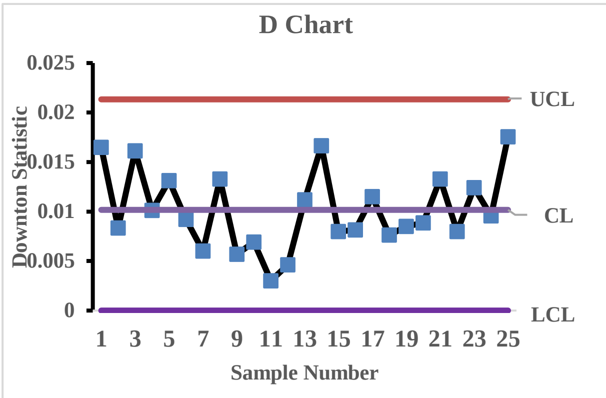

Figure 4: D Control Chart

R and $\bar{X}$ control charts in figure 1 and 2 respectively showed that the process is in state of statistical control (i.e., the process is stable). Similarly, Downton $\bar{X}_D$ and D control charts in figures 3 and 4 respectively also showed that the process is in state of statistical control as none of the sample points fall outside the control limits. Here, Downton estimator demonstrated that it is a good estimator sample standard deviation. Thereafter, a capability study is then evaluated on the process.

The Cpk index for $\bar{X}$ and R charts are thus obtained as follows:

Cpk $= min(Cpu, Cpl) = min\left(\frac{USL-\bar{X}}{3\sigma}, \frac{\bar{X}-LSL}{3\sigma}\right)$ where $\sigma = \frac{\bar{R}}{d_2}$ then,

Cpk $= min(Cpu, Cpl) = min\left(\frac{USL-\bar{X}}{3\frac{\bar{R}}{d_2}}, \frac{\bar{X}-LSL}{3\frac{\bar{R}}{d_2}}\right)$

$\bar{R} = 0.02324$, $d_2 = 2.326$, $\bar{D} = 0.01017$, $USL = 74.05$, $LSL = 73.95$, $3\frac{\bar{R}}{d_2} = 0.02997$

C$pu = \frac{74.05-74.00118}{0.02997} = 1.62896$, C$pl = \frac{74.00118-73.95}{0.02997} = 1.70771$

Cpk $= min(Cpu, Cpl) = 1.62896$

Similarly, the Cpk index for $\bar{X}_D$ and D charts is also obtained as follows:

Cpk $= min(Cpu, Cpl) = min\left(\frac{USL-\bar{\bar{X}}}{3\sigma}, \frac{\bar{\bar{X}}-LSL}{3\sigma}\right)$ where $\hat{\sigma} = \bar{D}$ then,

Cpk $= min(Cpu, Cpl) = min\left(\frac{USL-\bar{\bar{X}}}{3\bar{D}}, \frac{\bar{\bar{X}}-LSL}{3\bar{D}}\right)$ where $3\bar{D} = 3 \times 0.01017 = 0.03051$

C$pu = \frac{74.05-74.00118}{0.03051} = 1.60013$, C$pl = \frac{74.00118-73.95}{0.03051} = 1.67748$

Cpk $= min(Cpu, Cpl) = 1.60013$

The Cpk index value of 1.6 explains that the process is capable of producing products that meet the customers specification since Cpk index value≥ 1. (Kane, 1986; Kotz and Johnson, 2002 and Montgomery, 2009). If Cpk of index of 1.5 is the minimum customer's requirement of the process capability index value then one may conclude that the process is capable of meeting customer's specification.

3.2 The Proposed Single Procedure

The proposed single procedure of control chart limits with specified Cpk that determines simultaneously both the stability and the capability of the process at the same time are evaluated as follows:

Specification limits$= 74.000 \pm 0.05$

Upper specification limit (USL) $= 74.05$

Lower specification limit (LSL) $= 73.95$

Midpoint (M) $= \frac{74.05+73.95}{2} = 74.00$

Cpk $= 1.5$

The table below contains the Established control chart constant to construct capability Cpk index-based control chart.

Since $\bar{\bar{X}} > M$ then mean and variability control chart limits for $\bar{X}_D > M$ is used.

Mean Control Chart Limits

The Mean Control Chart Limits using the proposed method are obtained as follows:

$\bar{X}_D = 74.00118$, $A^*_{pk} = 0.4472$, USL $= 74.05$

UCL $= \bar{X}_D + A^*_{pk}\frac{(USL-\bar{X}_D)}{C_{pk}} = 74.00118 + 0.4472\frac{(74.05-74.00118)}{1.5} \cong 74.0157$

Centre Line (CL) $= \bar{X}_D = 74.00118$

LCL $= \bar{X}_D - A^*_{pk}\frac{(USL-\bar{X}_D)}{C_{pk}} = 74.00118 - 0.4472\frac{(74.05-74.00118)}{1.5} \cong 73.9866$

Variability Control Chart Limits

The Variability Control Chart Limits using the Proposed Method are obtained as follows:

USL $= 74.05$, $\bar{X}_D = 74.00118$, $Z^*_{4pk} = 0.6991$, Cpk $= 1.5$, $Z^*_{3pk} = 0$ and $G^*_{pk} = 0.3333$

UCL $= Z^*_{4pk}\frac{[USL-\bar{X}_D]}{C_{pk}} = 0.6991\frac{(74.05-74.00118)}{1.5} \cong 0.02275$

CL $= \bar{D} = G^*_{pk}\frac{[USL-\bar{X}_D]}{C_{pk}} = 0.3333\frac{(74.05-74.00118)}{1.5} = 0.01085$

LCL $= Z^*_{3pk}\frac{[USL-\bar{X}_D]}{C_{pk}} = 0\frac{(74.05-74.00118)}{1.5} = 0$

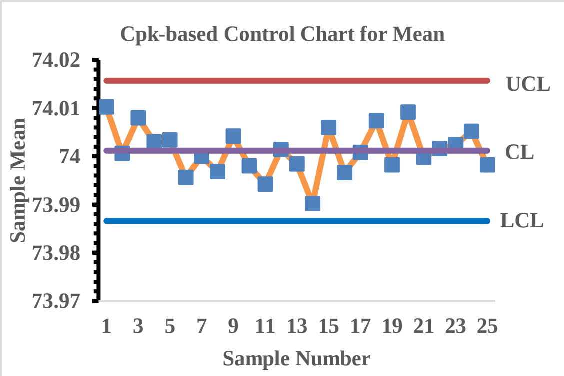

Figure 5: Cpk-based Control Chart for Mean

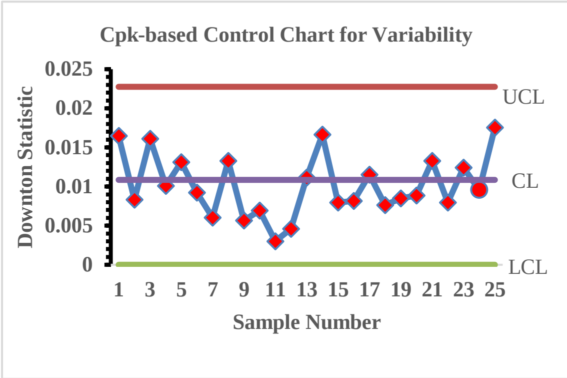

Figure 6: Cpk-based Control Chart for Variability

The above figures 5 and 6 are control charts to monitor the mean and variability of the process using the proposed control charts limits. If any of the process sample mean falls outside the control limits, then it is an indication that the manufacturing process data is not under statistical control and not capable of producing products that meet the customer's or engineering's specifications. But from the results obtained from the above proposed single procedure, it can be seen that none of the process sample mean or variation exceeded the control limits.

What this mean is that the single procedure establishes the stability of the process and the capability of the process simultaneously unlike the usual two procedures where the stability and capability of the process are obtained at different period. So, the process is under statistical control and as well capable of producing products that meets the organisation's specifications.

3.3 Simulation Study

To demonstrate this, a non-normal simulated data from Gamma (2,0.5) distribution consisting of 25 samples each of size $n = 5$ is generated using R package. where specification limits of 3 ± 2.5 is assumed with a desirable value of $Cpk \geq 1$.

According to (Alwan, 2000), the control charts are usually based on the assumption that the distribution of the quality characteristics is normal or approximately normal. However, in practice, the assumptions are often violated, which makes the chart to be affected by outliers. When the normality assumption is violated, one approach is to transform the data to normality using the Log transformation method.

Table 3: Transformed Simulated Data from Gamma $(2, 0.5)$ Distribution

SN	X1	X2	X3	X4	X5
1.	1.0247462	1.3696876	1.5014195	1.7409233	1.3526489
2.	1.4289093	1.6388859	1.0902093	1.5180276	1.2698694
3.	1.1199849	1.5831817	1.6234056	1.3481263	1.1376156
4.	1.6329184	1.5554461	1.6191468	1.2526874	1.2733446
5.	1.6580664	1.6171576	1.0849591	1.6202847	0.9693533
6.	1.2857099	2.0115161	1.3314304	0.8315873	0.5922224
7.	1.4199076	1.1106456	1.2672608	1.2537909	1.5349833
8.	1.2130951	1.8642770	1.4963960	1.7553197	1.7060303
9.	1.5935280	1.4010788	1.6322482	1.4836426	1.5491324
10.	1.6745425	2.0371943	1.6203082	1.5801485	1.3067178
11.	1.3697459	1.3895858	1.9797849	1.4620366	1.2319418
12.	1.8254477	1.7391518	1.1598728	1.3868941	2.0207735
13.	1.6767531	1.5913209	1.9164648	1.6272923	1.6002499
14.	1.6647959	1.2340997	0.4057076	1.3793700	1.6102386
15.	1.7404342	1.7842637	1.6791012	1.8895929	0.8735017
16.	1.6373379	2.0018210	1.4455970	1.2458796	0.5195388
17.	0.9715577	1.2913842	1.5407793	1.7166717	1.7800063
18.	1.4467788	1.2612883	1.6338516	1.6327062	1.8075055
19.	1.5986849	1.7412065	1.6712897	1.0127756	1.5788062
20.	1.4743363	1.9293574	1.0411359	2.0954667	1.6538421
21.	1.8397525	1.7445540	1.8216658	1.9784407	1.5223031
22.	1.7239121	1.6322747	1.7845991	1.3314526	1.7432113
23.	1.7920460	1.5743783	1.6447809	1.2159433	1.2455701
24.	1.9151880	1.2017065	1.1448186	1.7591268	1.5892798
25.	1.2079096	1.2863204	1.4974009	1.6782329	1.8233325

Table 4: $\bar{X}$, R and Downton Values for Simulated Data Set (in Table 3)

SN	D	X Bar	Range
1	0.280247	1.397885	0.716177
2	0.238486	1.38918	0.548677
3	0.257433	1.362463	0.503421
4	0.19608	1.466709	0.380231
5	0.339026	1.389964	0.688713
6	0.591721	1.210493	1.419294
7	0.179867	1.317318	0.424338
8	0.276731	1.607024	0.651182
9	0.101424	1.531926	0.231169
10	0.275678	1.643782	0.730477
11	0.281462	1.486619	0.747843
12	0.382913	1.626428	0.860901
13	0.12882	1.682416	0.325144
14	0.513004	1.258842	1.259088
15	0.378835	1.593379	1.016091
16	0.59484	1.370035	1.482282
17	0.361968	1.46008	0.808449
18	0.226787	1.556426	0.546217
19	0.274614	1.520553	0.728431
20	0.454401	1.638828	1.054331
21	0.17857	1.781343	0.456138
22	0.180299	1.64309	0.453147
23	0.274981	1.494544	0.576103
24	0.371889	1.522024	0.770369
25	0.287626	1.498639	0.615423
Average	**0.305108**	**1.498000**	**0.719745**

3.4 The Proposed Single Procedure on Simulated Data

The proposed single procedure of control chart limits with desirable specified Cpk ≥ 1 that determines simultaneously both the stability and the capability of the process are evaluated as follows:

Specification Limits$= 3 \pm 2.5$

Upper Specification Limit (USL) $= 5.5$

Lower Specification Limit (LSL) $= 0.5$

Midpoint (M) $= \frac{5.5+0.5}{2} = 3.0$

Cpk ≥ 1

$\bar{X}_D = 1.498000$

Since $\bar{\bar{X}} < M$ then Mean and Variability Control Chart Limits for $\bar{X}_D < M$ is used.

Mean Control Chart Limits using the Proposed Method

The Mean Control Chart Limits using the Proposed Method are obtained as follows:

$\bar{X}_D = 1.498000$, $A^*_{pk} = 0.4472$, LSL $= 0.5$

UCL $= \bar{X}_D + A^*_{pk}\frac{(\bar{X}_D - LSL)}{C_{pk}} = 1.498000 + 0.4472\frac{(1.498000-0.5)}{1} \cong 1.9443056$

Centre Line (CL) $= \bar{X}_D = 1.498000$

LCL $= \bar{X}_D - A^*_{pk}\frac{(\bar{X}_D - LSL)}{C_{pk}} = 1.498000 - 0.4472\frac{(1.498000-0.5)}{1} \cong 1.0516944$

Variability Control Chart Limits using the Proposed Method

The Variability Control Chart Limits using the Proposed Method are obtained as follows:

LSL $= 0.5,\ \bar{X}_D = 1.498000,\ Z^*_{4pk} = 0.6991,$ Cpk ≥ 1 , $Z^*_{3pk} = 0$ and $G^*_{pk} = 0.3333$

UCL $= Z^*_{4pk}\frac{[\bar{X}_D - LSL]}{C_{pk}} = 0.6991\frac{(1.498000-0.5)}{1} \cong 0.6977018$

CL $= \bar{D} = G^*_{pk}\frac{[\bar{X}_D - LSL]}{C_{pk}} = 0.3333\frac{(1.498000-0.5)}{1} = 0.3326334$

LCL $= Z^*_{3pk}\frac{[\bar{X}_D - LSL]}{C_{pk}} = 0\frac{(1.498000-0.5)}{1} = 0$

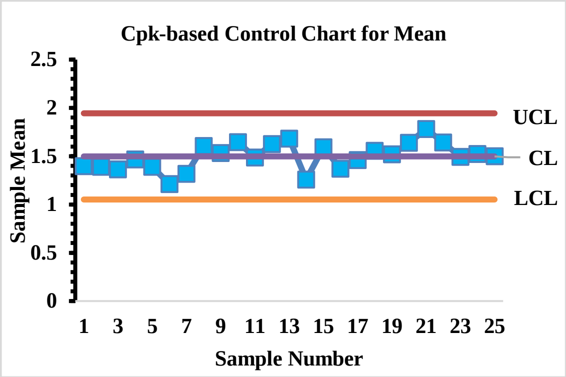

Figure 7: Cpk-based Control Chart for Mean

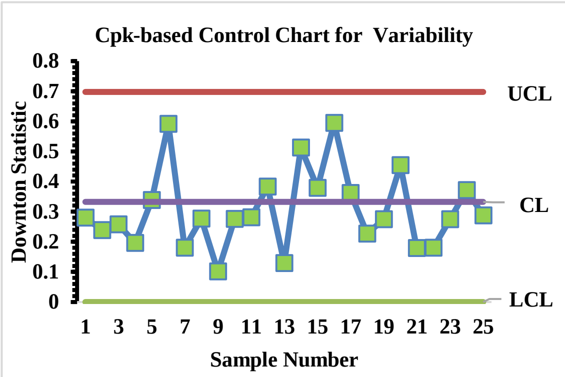

Figure 8: Cpk-based Control Chart for Variability Simulated Data

The control charts are usually based on the assumption that the distribution of the quality characteristics is normal or approximately normal. However, in practice, the assumptions are often violated, which makes the chart to be affected by outliers and sometimes sensitive to departure from normality. Specifically, the Shewhart X control chart is sensitive to outliers (Alwan, 2000). When the normality assumption is violated, one approach is to transform the data to normality.

According to (Adeoti et al, 2016), the study of departure from normality and effect of outliers on Shewhart control chart for process mean and variability has been undertaken by many authors. Some authors have studied this by constructing control chart based on robust statistic estimation for process mean and variability (Schiling and Nelson, 1976; Yourstone and Zimmer, 1992; Khoo, 2005; Abu-Shawiesh, 2008; Abu-Shawiesh,2009; Abbasi and Miller, 2013).

3.5 Conclusion

In this paper a single control chart was proposed to determine the stability of the process and assess its capability simultaneously based on the specified process capability index Cpk. This method involved using the Downton Statistic as the estimate of the process standard deviation σ. The proposed procedure is effective for monitoring and assessing the capability of a process, as it collapses the two-stage method: stability and capability into one procedure where management can vary the specification limits so as to achieve stability and capability as long as the minimum capability index value is achieved. It has also been shown that the proposed method is simple to apply and does not require any difficult computations for both stability and capability and, it is effective for normal and non-normal process situations. The proposed method has also been demonstrated with real life data.

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