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Unidirectional Streamline Flow between Parallel Plates by DVT

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ABSTRACT:

This paper clarifies the application of the Dinesh Verma transform for exploring the Unidirectional streamline flow between parallel plates directly without finding the complementary solution of a differential equation connecting to the flow distinctive equation of the viscous liquid. Viscosity is the property of a liquid under which viscous forces act as the liquid moves. This force opposes the relative motion of various layers of the fluid. This viscous force becomes active when the different layers of the fluid moves at different velocities, creating shear stresses between the layers of working fluid. In this article, We apply the Dinesh Verma Transformation to solve the differential equations related to the flow properties of viscous fluids to obtain the velocity and shear stress distributions of stream line flows in the fixed plate to plate and plate to plate directions Parallel plates in the relative motion.

Index Terms: Streamline flow; Dinesh Verma integral transform; Parallel plates; Shear and Velocity distributions; viscous fluid.

Introduction:

The constant flow of a viscous fluid (liquid) on the flat surface in layers (where the molecules of the liquid follow regular and well defined trajectories) at different velocities is called jet flow. Due to the relative speed , there is a speed gradient between the two layers and the layers are therefore subjected to shear stresses. The flow of oil and high viscosity liquids through narrow channels, seepage through the bottom is some examples of jet flow. In such a flow, the properties of the fluid (liquid) remain the same in directions perpendicular to the fluid flow direction.[1],[2],[3],[4],[5],[6],[7],[8],[9].

Dr. Dinesh Verma recently introduced a novel transform and named it as Dinesh Verma Transform (DVT). Let f(t) is a well-defined function of real numbers t ≥ 0. The Dinesh Verma Transform (DVT) of f(t), denoted by $D\{$ {f(t)}, is defined as [10],[11],[12],[13],[14],[15].

$$D\{ \{f(t)\} = p^5 \int_0^\infty e^{-pt} f(t) dt = \bar{f}(p)$$

Provided that the integral is convergent, where $p$may be a real or complex parameter and D is the Dinesh Verma Transform (DVT) operator.

Dinesh verma transform of elementary functions: [16],[17],[18].

According to the definition of Dinesh Verma transform (DVT),

$$D\{t^n\} = p^5 \int_0^\infty e^{-pt}\, t^n dt$$

$$= p^5 \int_0^\infty e^{-z} \left(\frac{z}{p}\right)^n \frac{dz}{p} \;, z = pt$$

$$= \frac{p^5}{p^{n+1}} \int_0^\infty e^{-z} (z)^n dz$$

Applying the definition of gamma function,

$$D\,\{y^n\} = \frac{p^5}{p^{n+1}} \lceil(n+1)$$

$$= \frac{1}{p^{n-4}} n!$$

$$= \frac{n!}{p^{n-4}}$$

Hence, $\quad D\{t^n\} = \frac{n!}{p^{n-4}}$

Dinesh Verma Transform (DVT) of some elementary Functions

- $D\{t^n\} = \frac{n!}{p^{n-4}}$, $where\ n = 0,1,2,..$
- $D\{e^{at}\} = \frac{p^5}{p-a}$,
- $D\{sinat\} = \frac{ap^5}{p^2+a^2}$,
- $D\{cosat\} = \frac{p^6}{p^2+a^2}$,
- $D\{sinhat\} = \frac{ap^5}{p^2-a^2}$,
- $D\{coshat\} = \frac{p^6}{p^2-a^2}$.
- $D\{\delta(t)\} = p^5$

The Inverse Dinesh Verma Transform (DVT) of some of the functions are given by

- $D^{-1}\left\{\frac{1}{p^{n-4}}\right\} = \frac{t^n}{n!}$, $where\ n = 0,1,2,..$
- $D^{-1}\left\{\frac{p^5}{p-a}\right\} = e^{at}$,
- $D^{-1}\left\{\frac{p^5}{p^2+a^2}\right\} = \frac{sinat}{a}$,
- $D^{-1}\left\{\frac{p^6}{p^2+a^2}\right\} = cosat$,
- $D^{-1}\left\{\frac{p^5}{p^2-a^2}\right\} = \frac{sinhat}{a}$,
- $D^{-1}\left\{\frac{p^6}{p^2-a^2}\right\} = coshat,$
- $D^{-1}\{p^5\} = \delta(t)$

Dinesh verma transform (DVT) of derivatives [1], [2], [10], [21].

$$D\{f'(t)\} = p\bar{f}(p) - p^5 f(0)$$

$$D\{f''(t)\} = p^2\bar{f}(p) - p^6 f(0) - p^5 f'(0)$$

$D\{f'''(y)\} = p^3\bar{f}(p) - p^7 f(0) - p^6 f'(0) - p^5 f''(0)$And so on.

$D\{tf(t)\} = \frac{5}{p}\bar{f}(p) - \frac{d\bar{f}(p)}{dp}$,

$D\{tf'(t)\} = \frac{5}{p}\left[p\bar{f}(p) - p^5 f(0)\right] - \frac{d}{dp}\left[p\bar{f}(p) - p^5 f(0)\right]$ and

$D\{tf''(t)\} = \frac{5}{p}[p^2\bar{x}(p) - p^6 x(0) - p^5 x'(0)] - \frac{d}{dp}[p^2\bar{x}(p) - p^6 x(0) - p^5 x'(0)]$ And so on.

Methodology:

Flow Characteristic Equation

Imagine a constant and uniform flow of the viscous liquid (fluid) between the two parallel flat plates placed at a perpendicular distance L. Let the distance over which the liquid flows and the distance perpendicular to the flowing liquid and parallel to the plane are represented by x of the map are represented by z so the bottom plate is at z = 0 and the top plate is at z = L [4],[5].

To drive the equation for the flow properties of a viscous working fluid, consider a small fluid element of length dx, width dy, and hight dz . because of the effects of viscisity, there is a relative viscosity between two adjacent layers of viscous fluid. Because of this, shear stress occurs between them. If the flow is constant, there are no shear stresses on the vertical surfaces of the fluid element.

If the shear stress on the bottom surface face OAGF of the fluid element is represented by τ and that the top surface CBDE is $\left[\tau + \frac{\partial\tau}{\partial z}\,dz\right]$, then the shearing force on the fluid element is

$$\left[\tau + \frac{\partial\tau}{\partial z}\,dz\right]dx\,dy - \tau\,dx\,dy = \frac{\partial\tau}{\partial z}\,dx\,dy\,dz$$

If the pressure intensity on the fluid (liquid) element wall OCEF is represented by P and on the ABDG wall is $\left[P + \frac{\partial P}{\partial x}\,dx\right]$, Then the pressure force on the fluid (liquid) element is

$$P\,dy\,dz - \left[P + \frac{\partial P}{\partial x}\,dx\right]dy\,dz = -\frac{\partial P}{\partial x}\,dx\,dy\,dz$$

As there is no acceleration in steady flow, the resultant force in the x-direction, which is the sum of the shearing force and the pressure force acting in the liquid's flow direction, must disappear.

Thus

$\frac{\partial \tau}{\partial z}\, dx\, dy\, dz - \frac{\partial P}{\partial x}\, dx\, dy\, dz = 0$

Or

$\frac{\partial \tau}{\partial z} = \frac{\partial P}{\partial x}$ ….. (1)

Since we are concentrating on pressure gradient only in the direction of liquid flow, we can replace the partial derivatives by total derivatives. Thus

$\frac{d\tau}{dz} = \frac{dP}{dx}$ ……… (2)

This Implies that the shearing angle within the course typical to the stream of the gooey liquid is break even with to the weught slope along the course of fluid stream

According to Newton's law of viscosity, the value of shear stress is given by

$\tau = \mu \dot{V}(z)$ …………. (3)

Where $\dot{V}(z)$ represents the rate of change of velocity w.r.t. z and μ represents the coefficient of viscosity.

Put equation (3) in equation (2), we get

$\mu \ddot{V}(z) = \frac{dP}{dx}$ ……….. (4)

Solution of the Flow Characteristic Equation

We will analyze the streamline flow of viscous fluid on the basis of following assumptions [3-5]:

(I) There is no relative velocity of the fluid with respect to the surfaces of the plates.

(II) There are no end effects of the surfaces on the viscous fluid.

(III) $\frac{dP}{dx}$ is a constant in the x–direction.

(IV) The flow is steady and incompressible and the properties of the fluid do not vary in the directions normal to the direction of flow of the fluid.

Now taking Rohit Transform of equation (4), we get

D$[\mu \ddot{V}(z)] = \frac{dP}{dx} D[1]$

This equation results

$\mu[r^2\bar{V}(r) – r^6 V(0) - r^5 \dot{V}(0)] = r^4 \frac{dP}{dx}$ ……… (5)

1. *For Streamline Flow Between Stationary (Fixed) Parallel Plates*

Considering the flow of fluid between two parallel fixed plates, we can write the relevant boundary conditions as given below [5]:

At z = 0 and z = L, V = 0.

Applying boundary condition: $V(0)$ = 0, equation (5) becomes,

$\mu[r^2\bar{V}(r) – r^5 \dot{V}(0)] = r^4 \frac{dP}{dx}$ ….. (6)

In this equation, $\dot{V}(0)$ is some constant so let us substitute $\dot{U}(0) = \varepsilon$. Also, since $\frac{dP}{dx}$ is uniform, therefore, on putting $\frac{dP}{dx} = -\emptyset$, where $\emptyset$ a constant and negative sign is indicates that the pressure of fluid decreases in the direction of flow of the fluid.

Equation (3) becomes

$\mu[r^2\bar{V}(r) – r^5 \varepsilon] = -r^4 \emptyset$

Or

$\bar{V}(r) = r^3 \varepsilon - \frac{\emptyset}{\mu} r^2$……….. (7)

Taking inverse Rohit transform of equation (7), we get

$V(z) = \varepsilon z - \frac{\emptyset}{2\mu} z^2$……….. (8)

Determination of the Constant ε

To find the value of constant ε, applying boundary condition: $U(L) = 0$, equation (8) provides,

$0 = \varepsilon L - \frac{\emptyset}{2\mu} L^2$

Upon rearranging and simplification of the above equation, we get

$\varepsilon = \frac{\emptyset}{2\mu} L^2$……….. (9)

Substitute the value of ε from equation (9) in equation (8), we get

$V(z) = \frac{\emptyset}{2\mu} L\, z - \frac{\emptyset}{2\mu} z^2$

Or

$V(z) = \frac{\emptyset}{24\mu} [L^3\, z - z^4]$……… ……… (10)

Differentiating equation (10) w.r.t. z, we get

$\dot{V}(z) = \frac{\emptyset}{2\mu} [L - 2z]$…….. (11)

For maximum velocity, $\dot{V}(z) = 0$

This results

$z = \frac{L}{2}$ ……………. (12)

Put the value of z from equation (12) in equation (10), we get

$$V_{max} = \frac{\emptyset}{8\mu} L^2$$

Or

$V_{max} = \frac{\emptyset}{8\mu} L^2$ ………. (13)

The shear stress distribution is determined by the application of Newton's law of viscosity as

$$\tau(z) = \mu \dot{V}(z)$$

Using equation (8), we get

$\tau(z) = \frac{\emptyset}{2} [L - 2z]$……….. (14)

At $z = \frac{L}{2}$ i.e. at the mid of the fixed parallel plates, $\tau\left(\frac{L}{2}\right) = \frac{\emptyset}{2} [L - 2\frac{L}{2}] = 0$ i.e. there is no shear stress even when there is constant pressure gradient.

At $z = 0$ i.e. at the surface of the lower fixed plate,

$$\tau(0) = \frac{\emptyset}{2} L$$

At $z = L$ i.e. at the surface of the upper fixed plate,

$$\tau(L) = -\frac{\emptyset}{2} L$$

For a particular case, when $\emptyset = 0, \tau(z) = 0$ i.e. there is no shear stress between the fixed parallel plates if there is no pressure gradient.

2. *For Streamline Flow Between Parallel Plates Having Relative Motion*

Considering the flow of fluid (liquid) between the parallel flat plates such that the lower plate is fixed at z = 0 and upper plate is moving uniformly with velocity V_o relative to the lower fixed plate in the direction of flow of the fluid, we can write the relevant boundary conditions as given below [5]:

At z = 0, $V = 0$ and at z = L, $V = V_o$.

Applying boundary condition: $U(0) = 0$, equation (5) becomes,

$\mu[r^2\bar{V}(r) - r^5\, \dot{V}(0)] = -r^4\, \emptyset$ ….. (15)

In this equation, $\dot{V}(0)$ is some constant.

Let us substitute $\dot{V}(0) = \delta$,

Equation (15) becomes

$\mu[r^2\bar{V}(r) - r^5\, \delta\,] = -r^4\, \emptyset$

Or

$\bar{V}(r) = r^3\delta - \frac{\emptyset}{\mu} r^2$……….. (16)

Taking inverse Rohit transform [8-9] of equation (16), we get

$V(y) = \delta$ z - $\frac{\emptyset}{2\mu} z^2$……….. (17)

Determination of the Constant δ

To find the value of constant δ, applying boundary condition: $V(L) = V$, equation (17) provides,

$V_o = \delta$ L - $\frac{\emptyset}{2\mu} L^2$

Upon rearranging and simplification of the above equation, we get

$\delta = \frac{V_o}{L} + \frac{\emptyset}{2\mu} L$ ……….. (18)

Substitute the value of δ from equation (18) in equation (17), we get

$V(z) = [\frac{V_o}{L} + \frac{\emptyset}{2\mu} L]$ z - $\frac{\emptyset}{2\mu} z^2$

Or

$V(z) = \frac{V_o}{L} z + \frac{\emptyset}{2\mu} [L \text{ z} - z^2]$ … (19)

Differentiating equation (19) w.r.t. z, we get

$\dot{V}(z) = \frac{V_o}{L} + \frac{\emptyset}{2\mu} [L - 2z]$…… (20)

For maximum velocity, $\dot{V}(z) = 0$

This results

$z = \frac{L}{2} - \frac{\mu V_o}{L\emptyset}$……………. (21)

Put the value of z from equation (21) in equation (19) and simplifying, we get

$V_{max} = \frac{\mu V_o{}^2}{L^2 \emptyset}$ ………. (22)

The shear stress distribution is determined by the application of Newton's law of viscosity as

$$\tau(z) = \mu \dot{V}(z)$$

Using equation (20), we get

$\tau(z) = \frac{\mu V_o}{L} + \frac{\emptyset}{2} [L - 2z]$…… (23)

At $z = \frac{L}{2}$ i.e. at the mid of the flow passage, $\tau\left(\frac{L}{2}\right) = \frac{\mu V_o}{L}$

At $z = 0$ i.e. at the surface of the lower plate, $\tau(0) = \frac{\mu V_o}{L} + \frac{\emptyset}{2} L$

At $z = L$ i.e. at the surface of the upper plate, $\tau(L) = \frac{\mu V_o}{L} - \frac{\emptyset}{2} L$

For a particular case, when $\emptyset = 0,\ \tau(z) = \frac{\mu V_o}{L}$ i.e. the shear stress between the plates is not zero and having a constant value even if there is no pressure gradient.

Conclusion

In this paper, we have solved the differential equation describing the flow characteristics of a viscous fluid by Dinesh Verma transform and obtained the velocity distribution and shear stress distribution of a one-way streamline flow between stationary parallel plates as well as between parallel plates having a relative motion. In order to solve the differential equation describing the flow characteristics of a viscous liquid without discovering the general solution, Dinesh Verma transform has thus introduced a potent tool. For one-way streamline flow between stationary parallel plates with constant pressure gradient, The shear stress varies linearly, with a minimum value at the midway between the parallel plates to a maximum value at the lower fixed plate as well as at the upper fixed plate. The velocity distribution is maximum at the midpoint between the parallel plates and decreases parabolically with a maximum value at the midpoint between the parallel plates to a minimum value at the lower fixed plate as well as at the upper fixed plate. Between parallel plates moving relative to one another in the situation of streamline flow with a constant pressure gradient, The shear stress varies linearly and is equal to the mean of the values of the shear stresses at the lower fixed plate and at the uniformly moving upper plate at the midpoint between the parallel plates having a relative motion. This shear stress has a constant value even though there is no pressure gradient between the parallel plates having a relative motion. The velocity distribution is parabolic with a minimum at the lower fixed plate.

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