



Problem-Based Learning Model Learning Framework for Solving Integer Addition Problems Based on Multi-Technique and Assisted by Digital Worksheets

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ABSTRACT

The rapid and massive digital development has penetrated all aspects of life, including education. Digital technology plays an important role in learning and facilitates access to information and learning materials. One of them is digital worksheets which provide convenience in accessing and learning materials anywhere and anytime with the availability of an internet connection. The development of digital worksheets with a problem-based learning model for mathematics with the subject of integer addition aims to solve problems related to integer addition, thus improving critical thinking skills. Thus far, integer addition has been considered an abstract and difficult subject to understand. The techniques used in this digital worksheet are based on several references and instructional videos available on YouTube, such as the number line technique and the positive-negative sign technique. These techniques are still considered abstract and difficult to comprehend. To make it easier to understand the concept of integer addition, the worksheet introduces the technique of pairing bottle caps of two different colors, resulting in the Pairing Numbers technique. This technique can be implemented in various integer addition problems with larger numbers.

Keywords: Digital Worksheets, Problem-Based Learning Model, Integer Addition, Multi-Technique

1. Introduction

The rapid and massive development of information and communication technology has affected all aspects of life. According to Budiman (2017), technological development has spread across the world, including in education. Therefore, teachers must be able to utilize technology effectively. Wardhana (2020) explains that the rapid development of technology demands that teachers use technology to enhance learning, making it easier to process information and creating more interactive and effective learning experiences. Andri (2017) states that information and communication technology plays an important role in improving the quality of education by utilizing it in learning, making it more efficient and effective, and facilitating the achievement of educational goals. Given the current state of rapid and massive technological development, learning materials as a source of learning play a crucial and strategic role in education. According to Ardina & Sa'dijah (2016), digital-based LKPD (Student Worksheets) can actively engage students in learning, thereby facilitating the achievement of learning objectives. Gazali (2016), Majid (2012), Marsa et al. (2016) stated that LKPD is a learning resource that facilitates students' understanding of the material through independent and systematic learning. To ensure that the digital LKPD created involves students in learning and provides meaningful learning, it needs to be designed with an appropriate learning model that can enhance critical thinking skills and problem-solving abilities. One learning model that can enhance critical thinking skills is Problem-Based Learning (PBL).

The research conducted by Khainingsiha et al. (2022) titled "Design of Electronic LKPD Based on PBL Model with Malay Culture Context in Comparison Materials to Facilitate Mathematical Critical Thinking Skills" showed that developing electronic LKPD using the ADDIE model with the PBL learning model can enhance students' mathematical critical thinking skills. According to Fathurrohman (2015), the Problem-Based Learning (PBL) model of learning presents authentic and contextual problems experienced in daily life, which requires students to solve problems and develop their critical thinking skills to gain new knowledge. Hamzah (2003), Janawi (2013), Rahmawati et al. (2020), Soden (2014), Wardhani (2010), and Wena (2011) agree that a problem-based approach involves finding solutions using prior knowledge to achieve learning goals and gaining meaningful experiences. Wijnen et al. (2017) assert that PBL is an effective model, while Pamungkas et al. (2019) argue that it enhances the understanding of concepts, and Sariastuti & Mawardi (2021) highlight its potential to develop thinking skills. Siswono (2016) defines critical thinking as the effective use of thinking skills to gather information, select and decide appropriately, and make decisions that can be used in problem-solving. Miele & Wigfield (2014) describe critical thinking as the idea of an individual or group accompanied by real evidence related to the problem at hand, in order to evaluate or assess. Asriningtyas et al. (2018) define critical thinking as high-level thinking, while Alawiyah & Marlina (2019) and Johnson (2011) describe it as systematic, open, and analytical, allowing for scientific and well-informed decisions rather than relying on personal opinions. Rachmadtullah (2015) adds that critical thinking includes

evaluative thinking, demonstrating the ability of humans to distinguish between reality and truth by referring to ideal things. Developing critical thinking skills is essential in mathematics education. This is supported by the study conducted by Fitriyah & Ghofur (2021) in their research entitled "Development of Android-based E-LKPD with Problem Based Learning (PBL) Model to Improve Students' Critical Thinking" where they developed an android-based LKPD using the 4D development model. The results showed that the developed LKPD can encourage students' learning motivation, facilitate independent learning, measure abilities independently, and improve critical thinking skills. However, the developed LKPD still has weaknesses, such as not including videos related to the presented material and only accessible on Android-based smartphones. The study conducted by Gabriella & Mitarli (2021) entitled "Development of Problem-Based Learning (PBL) Oriented LKPD to Improve Critical Thinking Skills in Hydrocarbon Material" using the 4D development model also showed that the developed LKPD is feasible, practical, and effective in developing critical thinking skills. To address the above issues, a problem-solving approach is needed to find breakthroughs in finding the addition operation of integers using multi-techniques, developing digital LKPD with problem-based learning model, and capturing the phase of students' critical thinking skills. To solve these problems, we conducted research on the development of digital LKPD with problem-based learning model to improve students' critical thinking skills in solving addition operation of integers using multi-techniques.

2. Methods

This study is a research and development study aimed at producing a product and testing the validity, effectiveness, and practicality of the product. According to Sugiyono (2011), to produce a product, an analysis of needs, design, and development is necessary, followed by research to test the validity, effectiveness, and practicality of the product so that the resulting product is truly of quality and functions well. Soenarto (2005), Borg & Gall (1983) stated that research and development is a process for developing products that can be materials, media, or tools, and testing their validity for implementation in education to solve learning problems. This research and development study used the ADDIE model, which consists of five stages: analysis, design, development, implementation, and evaluation. According to Gustafson et al. (2002), the core stages of the ADDIE model are analyzing the needs of learners, creating quality product designs that suit the learning context, developing and managing materials in the learning process, implementing them for testing, and evaluating them for improvement. This is in line with Tegeh et al. (2014) who also developed the ADDIE model with stages of analysis, product design, development, and implementation in the field. During the implementation, the problem-based learning model was used.

3. Results

Research on the development of digital worksheets with the problem-based-learning model to solve multi-technique-based integer addition problems with the ADDIE model, as follows:

3.1 Analisis

In this stage, the analysis of the K-13 curriculum on KI-3 and KI-4 is carried out, which will be used in this development research, specifically for Mathematics in grade VI of the 2013 curriculum with competencies 3.2 and 4.2, particularly for the topic of addition of integers. According to Aldoobie (2015), the first step in this analysis stage is to establish the development objectives.

3.2 Analysis of Teaching Materials and Some References from the Internet

Based on the analysis of teaching materials and various internet references, the techniques used to solve addition of integers include:

Number Line Technique

Example:

$$3 + (-5)$$

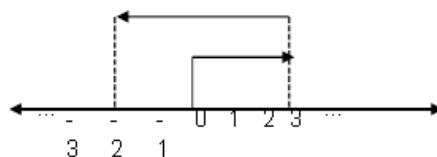


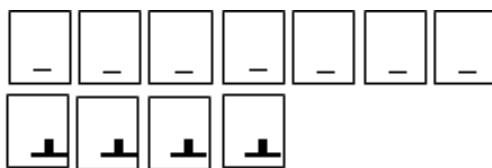
Figure 1. The example of number line technique

The weakness of this technique is that students still have difficulty in making arrow lines towards the specified numbers. The technique is still abstract for students, so they still have difficulty understanding the concept of adding integers.

Positive-Negative Sign Technique

Example:

$-7 + 4$



The weakness of this technique is that students have to make many positive-negative signs when the numbers are large, making it difficult for them to add numbers with minimal tens. The technique is still abstract, so students still have difficulty understanding the concept of adding integers.

Addition-Subtraction Techniques

Example:

a. $-26 + 12$

$26 - 12 = 14$, the rule for the sign of the result in addition of integers is to follow the sign of the larger absolute value.

$-26 + 12 = -14$

b. $-27 + (-13)$

$27 + 13 = 40$, if both numbers have negative signs, then the result will follow the sign of the second number.

$-27 + (-13) = -40$

The weakness of this technique is that students still have difficulty when subtracting or adding two numbers. The technique is still abstract, so it cannot draw conclusions for its implementation on larger numbers.

Design

In the design phase, the next step is to create a draft of the LKPD in Word format, which will later be converted to PDF before being uploaded to Kvisoft. According to Woo (2018), the design phase involves creating a development design and examining its strengths and weaknesses.

Development

The development phase, according to Alodwan & Almosa (2018), involves developing the product design and conducting trials in the learning process. The designed LKPD in pdf format was developed into a Kvisoft-based digital LKPD. This was done by uploading the pdf file to fliphtml5. The digital LKPD resulting from the development can be accessed through the following link: <https://online.fliphtml5.com/zeiwo/cyuv/>

Implementation

The next step is to implement the digital LKPD in the learning process. According to Aldoobie (2015), the implementation stage is the stage of applying the development product into the learning process. The implementation of the digital LKPD using the problem-based learning model with the syntax of the problem-based learning model include:

Learning Syntax of Problem Based-Learning Model

Learning syntax of problem based learning model can be seen in Table 1, Table 2, Table 3, Table 4, Table 5, and Table 6.

Table 1. Phase 1

Phase 1	Activity
The students are oriented towards the problem.	<ul style="list-style-type: none"> The teacher reviews the material on integer numbers. The teacher reminds the students about the digital learning media that has been sent. The teacher presents a problem related to the addition of integer numbers.



Figure 2. Photo of collecting mineral water bottle caps activity

Table 2. Phase 2

Phase 2	Activity
Organizing students for learning	<ul style="list-style-type: none"> The teacher forms groups of four students The students prepare the tools and materials for the activity.

Table 3. Phase 3

Phase 3	Activity
Guiding students to perform activities in groups.	<ul style="list-style-type: none"> The students engage in group discussions as part of the activity. The teacher facilitates the group discussions

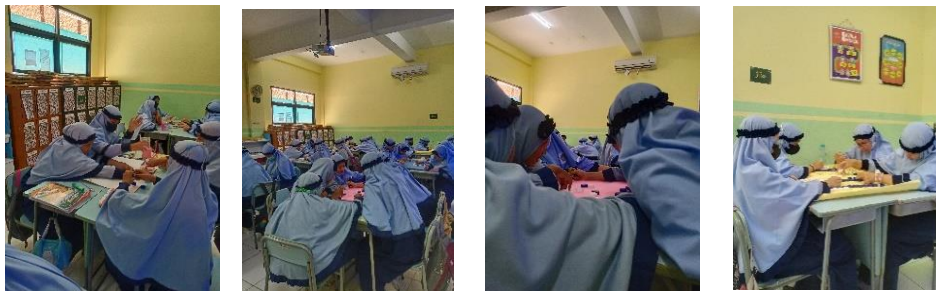


Figure 3. Photo of group discussion activity

Table 4. Phase 4

Phase 4	Activity
Presenting the results of group work	Each group presents the result of their group discussion.



Figure 3. Photo of group discussion activity

Table 5. Phase 5

Phase 5	Activity
Evaluation.	<ul style="list-style-type: none"> The teacher, along with the students, conducts a reflection on the learning process. The teacher provides reinforcement of the materials that have been learned The teacher, along with the students, draws a conclusion.

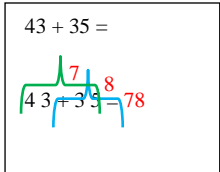
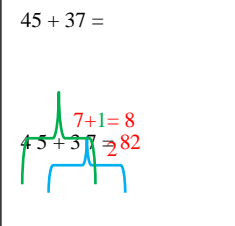
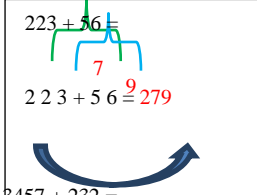
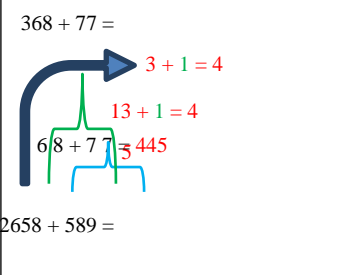
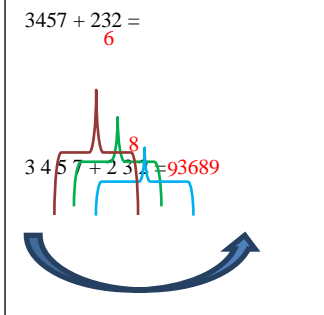
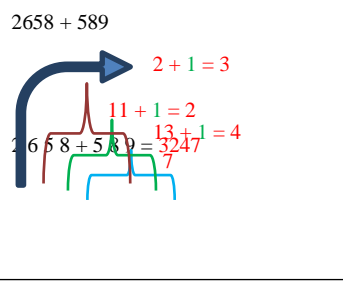
The results of this study show several new techniques for adding integers. Some of these new techniques for adding integers include the following: hierarchical technique, layered technique, table technique, and paired-number technique. We present some of these techniques below.

Tiered Technique

Addition of Integers with the same sign (positive-positive)

Addition of Integers with the same sign (positive-positive) can be seen in Table 6.

Table 6. Addition of Integers with the same sign

Addition of Positive-Positive Numbers Without Regrouping.	Addition of Positive-Positive Numbers with Carrying.
<p>43 + 35 =</p> 	<p>45 + 37 =</p> 
<p>223 + 56 =</p>  <p>3457 + 232 =</p>	<p>368 + 77 =</p>  <p>2658 + 589 =</p>
<p>3457 + 232 =</p> 	<p>2658 + 589 =</p> 

The Technique:

- Add the digits from the rightmost position (i.e., units, tens, hundreds, etc.).
- Draw a line from the digit in the units place of the number on the left to the corresponding digit on the right, and write the sum on the line connecting the two digits.
- Draw a line from the tens digit on the left to the corresponding tens digit on the right, and repeat for each subsequent digit until the last digit is reached.
- If the sum of two digits is greater than 9, write only the rightmost digit on the line and carry the leftmost digit over to the next line.
- If there is an unmatched digit at the end of one of the numbers, simply copy it to the answer.
- Write the answer above the line, starting from the leftmost or uppermost digit.
- Finish.

The addition of signed integers with the same sign (negative-negative)

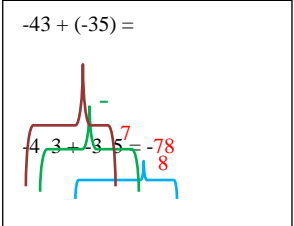
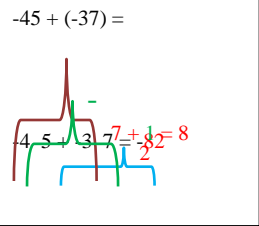
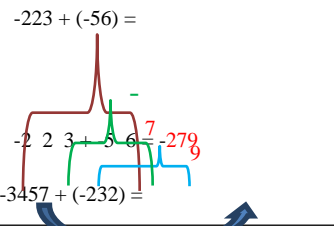
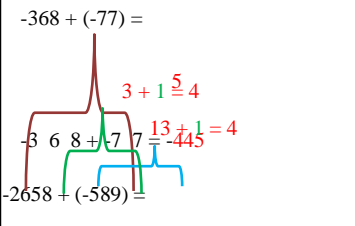
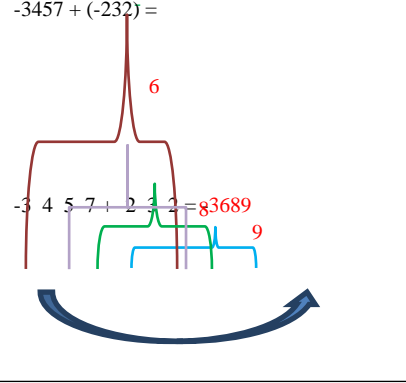
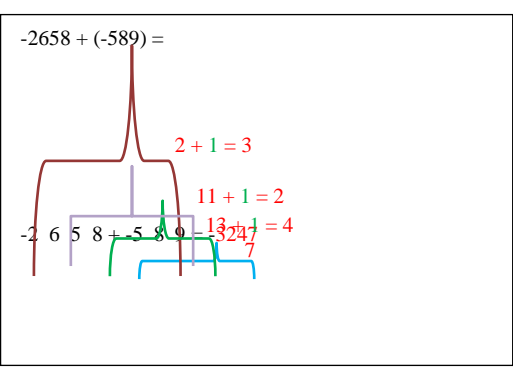
The technique is as follows:

- Add the numbers from the rightmost digit in the order (ones, tens, hundreds, etc.).
- Draw a line from the ones place in the number on the left to the ones place in the number on the right, and write the sum on the line connecting the two numbers.

- Draw a line from the tens place on the left to the tens place on the right, and so on until the final digit.
- If the sum is a two-digit number, write the last digit above the line and carry the digit to the left, writing it on the line above.
- If there is a final digit with no partner to add to, copy it directly to the answer, writing it on the leftmost side.
- Write the answer above the line, starting from the left or topmost digit.
- The sign (positive/negative) of the sum follows the signs of the two original numbers being added.
- Finished.

The addition of signed integers with the same sign (negative-negative) can be seen in Table 7.

Table 7. Addition of Integers with the same sign (negative-negative)

The addition of negative-negative numbers without borrowing.	Addition of negative numbers by saving
<p>$-43 + (-35) =$</p>  <p>$-223 + (-56) =$</p>	<p>$-45 + (-37) =$</p>  <p>$-368 + (-77) =$</p>
<p>$-223 + (-56) =$</p>  <p>$-3457 + (-232) =$</p>	<p>$-368 + (-77) =$</p>  <p>$-2658 + (-589) =$</p>
<p>$-3457 + (-232) =$</p> 	<p>$-2658 + (-589) =$</p> 

Addition of Integers with Different Signs (negative-positive/positive-negative)

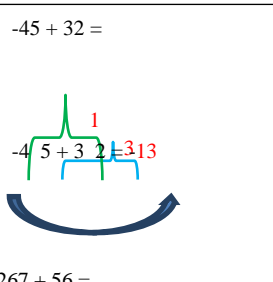
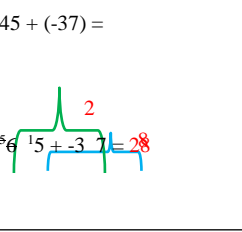
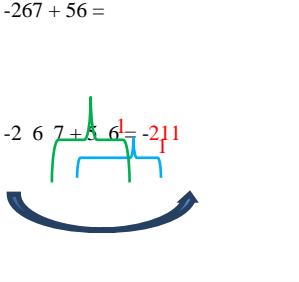
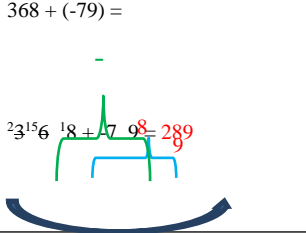
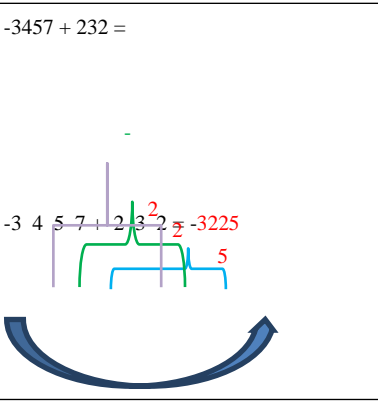
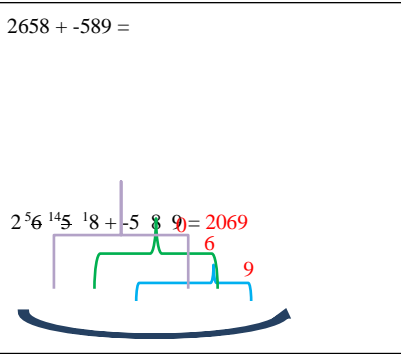
The technique:

- Subtract the numbers by starting from the rightmost digit (ones, tens, hundreds, etc.)
- Draw a line connecting the digit in the left number's ones place to the corresponding digit in the right number's ones place. Write the result of the subtraction on the line.

- Draw a line connecting the digit in the left number's tens place to the corresponding digit in the right number's tens place, and so on until the last digit.
- If the subtrahend is greater than the minuend, add 1 to the minuend and continue the subtraction.
- If there is an unmatched digit, bring it down to the answer line.
- Write the answer above the line starting from the leftmost digit.
- The sign (positive/negative) of the answer follows the sign of the larger number being subtracted.
- Done.

The addition of signed integers with the same sign (negative-negative) can be seen in Table 8.

Table 7. Addition of Integers with the same sign (negative-positive/positive-negative)

Addition of Negative-Positive Numbers	Addition of Positive-Negative Numbers
<p>$-45 + 32 =$</p> 	<p>$65 + (-37) =$</p> 
<p>$-267 + 56 =$</p> 	<p>$368 + (-79) =$</p> 
<p>$-3457 + 232 =$</p> 	<p>$2658 + (-589) =$</p> 

Multilevel Engineering

Adding integers with the same sign

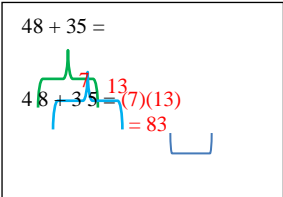
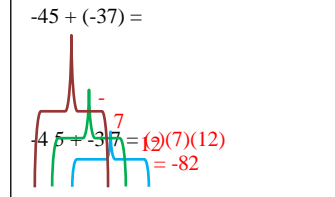
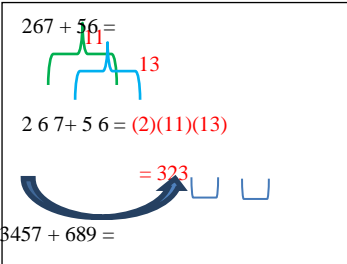
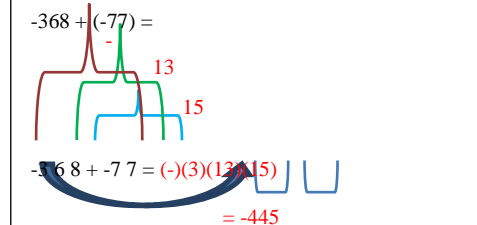
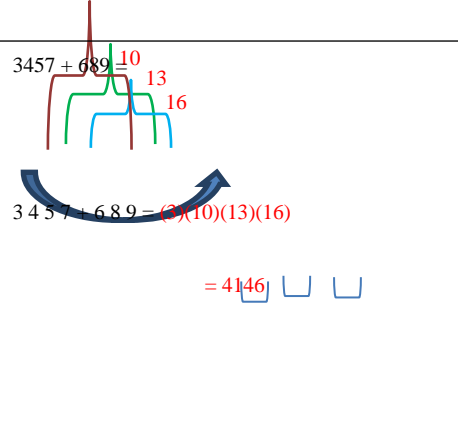
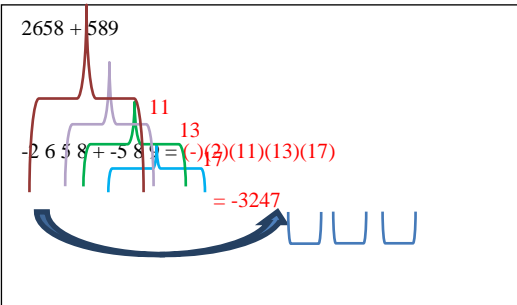
Technique:

Layered Technique: Multi-level Addition of Same/Positive Signed Integers with Carrying

- Start by adding the digits from the rightmost end in order (ones, tens, hundreds, etc.).
- Draw a line from the rightmost digit to the digit to its left, and write the sum on top of this line.

- Draw a line from the left digit of each pair to the right digit of the pair to its left, and repeat until you reach the leftmost digit.
- If the sum of two digits is greater than or equal to 10, carry the 1 to the next pair of digits to the left.
- Write the final answer above the line, starting from the leftmost digit.
- Combine the sums of the pairs of digits to the left of the ones place.
- The sign (positive/negative) of the final answer follows the sign of the original numbers.
- Finish.

Table 8. Adding integers with the same sign

Addition of positive numbers by saving	Addition of negative numbers by saving
<p>48 + 35 =</p> 	<p>-45 + (-37) =</p> 
<p>267 + 56 =</p> 	<p>-368 + (-77) =</p> 
<p>3457 + 689 =</p> 	<p>-2658 + (-589) =</p> 

Technique: Multi-level Addition of Same/Negative Signed Integers with Carrying

- Start by adding the digits from the rightmost end in order (ones, tens, hundreds, etc.).
- Draw a line from the rightmost digit to the digit to its left, and write the sum on top of this line.
- Draw a line from the left digit of each pair to the right digit of the pair to its left, and repeat until you reach the leftmost digit.
- If the sum of two digits is greater than or equal to 10, carry the 1 to the next pair of digits to the left.
- Write the final answer above the line, starting from the leftmost digit.

- Combine the sums of the pairs of digits to the left of the ones place.
- The sign (positive/negative) of the final answer is negative, since the original numbers are negative.
- Finish.

Layered Technique

Addition of Integers with Different Signs

Table 9. Addition of Integers with Different Signs

Addition of Positive-Negative Numbers	Addition of Negative-Positive Numbers
$48 + (-35) = \overbrace{3 + 1}^4 = 4 \quad 5$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> $48 + (-35) =$ $4 - 4 = 0$ $\overbrace{48 + (-35)} = 13 \quad 8 + 5 = 13$ </div> $267 + (-56) = \overbrace{5 + 1}^6 = 6 \quad 4$	$-48 + 35 = \overbrace{3 + 1}^4 = 4 \quad 5$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> $-48 + 35 =$ $4 - 4 = 0$ $\overbrace{-48 + 35} = -13 \quad 8 + 5 = 13$ </div>
$267 + -56 =$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> $267 + -56 =$ $6 - 6 = 0$ $\overbrace{267 + -56} = 211 \quad (2)(0)(11)$ $= 211$ </div>	$-267 + 56 = \overbrace{5 + 1}^6 = 6 \quad 4$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> $-267 + 56 =$ $6 - 6 = 0$ $\overbrace{-267 + 56} = -211 \quad 7 + 4 = 11$ $-267 + 56 = (-2)(0)(11)$ $= -211$ </div>
$897 + (-629) = \overbrace{2 + 1}^3 = 3 \quad 1$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> $897 + (-629) =$ $8 - 6 = 2$ $9 - 3 = 6$ $\overbrace{897 + (-629)} = 268 \quad 7 + 1 = 8$ </div>	$-897 + 629 = \overbrace{2 + 1}^3 = 3 \quad 1$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> $-897 + 629 =$ $8 - 6 = 2$ $9 - 3 = 6$ $\overbrace{-897 + 629} = -268 \quad 7 + 1 = 8$ </div>

The Technique: Two-Digit Addition with Different Signs (Positive-Negative)

- Pair up the ones place digits to make 10s in the subtrahend (for example, 9 with 1, 8 with 2, and so on).
- Add 1 to the tens place digit in the subtrahend.
- For the digits being subtracted, add the ones place digit to the paired 10s above, and continue this process for the tens and hundreds place digits.
- If there is a final digit without a paired addition, then that digit is moved directly to the answer and placed in the front/leftmost position.

- Write the answer above the line starting from the leftmost or topmost digit.
- Add the paired digits before the ones place digits.
- The sign (positive/negative) of the answer follows the sign of the largest digit being added.
- Finished.

The Table Technique:

Addition with Same Signs

The Table Technique: Addition of Integers with Same Signs

- Create a table with columns equal to the number of digits in the integers, with the last column representing the ones place.
- Add the corresponding digits in each column.
- If the sum of any column results in two digits, then add the digit in front of it to the next column.
- If there is a final digit without a paired addition, then that digit is moved directly to the answer and placed in the front/leftmost position.
- Add the paired digits before the ones place digits.
- The sign (positive/negative) of the answer follows the signs of the original integers.
- Finished.

Table 10. Addition of Integers with Same Signs

Addition of Positive-Positive Numbers	Addition of Negative-Negative Numbers																																																				
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Addition of Different Signs

Table 11. Addition of Different Signs

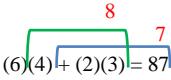
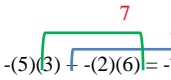
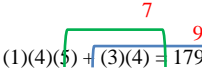
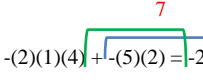
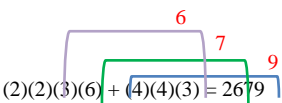
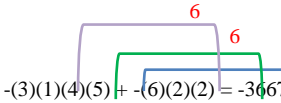
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The Table Technique: Addition of Integers with Same Signs

- Create a table with columns equal to the number of digits in the integers, with the last column representing the ones place.
- Pair the ones place digits to make multiples of 10 (e.g. 9 with 1, 8 with 2, etc.).
- Add 1 to the digit in the column to the left of the paired ones place digits.
- For the number being added to, add the ones place digit to the paired multiple of 10, and repeat for the tens and hundreds digits.
- Add the corresponding digits in each column.
- If the sum of any column results in two digits, then add the digit in front of it to the next column.
- If there is a final digit without a paired addition, then that digit is moved directly to the answer and placed in the front/leftmost position.
- Write the answer above the line starting from the leftmost or top digit.
- Add the paired digits before the ones place digits.
- The sign (positive/negative) of the answer follows the sign of the largest original integer.
- Finish

The Pair Technique: Addition of Integers with Same Signs

Table 12. Addition of Integers with Same Signs

Addition of Positive-Positive Numbers without Saving	Addition of Negative-Negative Numbers without Saving
$64 + 23 =$ 	$-53 + (-26) =$ 
$145 + 34 =$ 	$-214 + (-52) =$ 
$2236 + 443 =$ 	$-3145 + (-622) =$ 

The Digit-by-Digit Technique: Addition of Integers

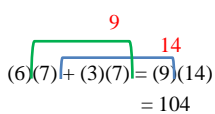
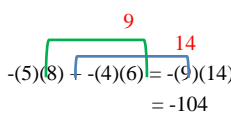
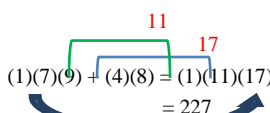
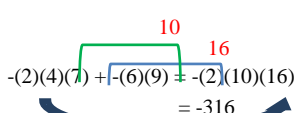
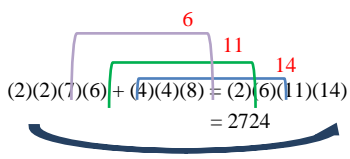
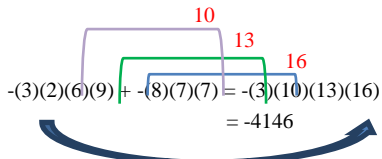
- Start by adding the digits in the ones place (rightmost) of the two integers.
- Draw a line from the ones digit of the integer on the left to the ones digit of the integer on the right, and write the sum of the ones digits above the line.
- Add the digits in the tens place of the two integers, and draw a line from the tens digit of the integer on the left to the tens digit of the integer on the right, writing the sum above the line.
- Continue this process for each place value (hundreds, thousands, etc.) until you reach the leftmost digit.
- If there is a digit without a corresponding digit to add it to, write it directly in the answer, at the leftmost position.
- Write the final answer above the line, starting from the leftmost digit.
- Finished.

Addition of Integers with Saving

The technique:

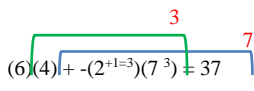
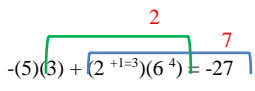
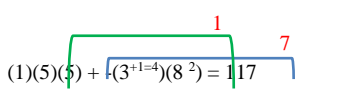
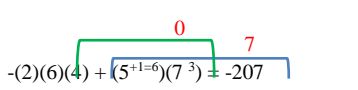
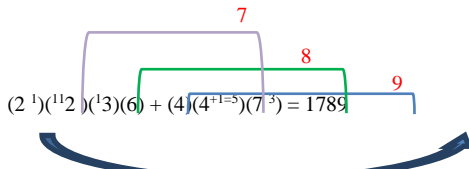
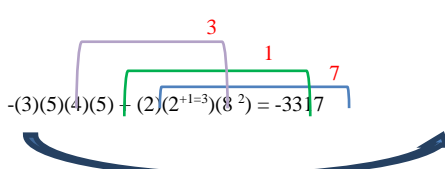
- Sum the digits from the rightmost/end digits in order (ones, tens, hundreds, etc.)
- Draw a line from the ones digit on the left to the ones digit on the right, and write the sum on the line connecting the two digits.
- Draw a line from the tens digit on the left to the tens digit on the right, and continue until the last digit.
- If the sum of any two digits results in a two-digit number, write the last digit above the line and carry the first digit to the next addition.
- If there is a final digit that does not have a partner to add to, bring it directly to the answer, placing it on the leftmost side.
- Write the answer above the line, starting from the leftmost digit or top.
- Done.

Table 13. Addition of Integers with Saving

Addition of Positive-Positive Numbers with Saving	Addition of Negative-Negative Numbers with Saving
$67 + 38 =$ 	$-58 + (-46) =$ 
$179 + 48 =$ 	$-247 + (-69) =$ 
$2276 + 448 =$ 	$-3269 + (-877) =$ 

Addition of Different Signs

Table 14. Addition of Different Signs

The sum is marked Positive-Negative	The sum is marked Negative-Positive
$64 + (-27) =$ 	$-53 + 26 =$ 
$155 + (-38) =$ 	$-264 + 57 =$ 
$2236 + (-447) =$ 	$-3545 + (228) =$ 

Technique:

Tiered Technique: Addition of Different Signs (positive-negative)

- Pair the ones digits to become a multiple of 10 in the subtrahend (for example, 9 with 1, 8 with 2, etc.).
- Add 1 to the tens digit in the subtrahend.
- For the minuend, add the ones digit to the corresponding multiple of 10 in the subtrahend above, and so on for the tens and hundreds.
- If there is a last digit that does not have a pair to add, then it should be directly transferred to the answer, placed in front of it or to the far left.

- Write the answer above the line, starting from the leftmost or topmost digit.
- Add the paired digits before the ones digit.
- The sign (positive/negative) of the sum follows the sign of the largest number.
- Finished.

3.3 Evaluation

The evaluation stage of digital teaching materials development is carried out formatively, which means that after the development is validated by content/material experts, design experts, and language experts, their feedback and suggestions are used as a basis for improving the developed digital teaching materials. After implementation, feedback and suggestions from teachers and students are also used to make improvements to the digital teaching materials developed.

3.4 Discussion

The development of digital teaching materials (LKPD) using a problem based-learning model is crucial and beneficial in developing students' critical thinking skills, especially in solving multi-technique addition problems. This is in line with the research conducted by Ramadayani et al. (2021) who conducted a study titled Development of Problem-Based Learning Based Teaching and Learning Materials (LKPD). Based on the results of this research, several new techniques have been developed with their own advantages and disadvantages. The use of hierarchical techniques, incremental techniques, table techniques, and paired number techniques make it easier to perform addition without borrowing or carrying, indirectly helping students solve addition problems in their daily lives without having to remember borrowing or carrying. Some of these techniques also make it easier for students to perform addition with larger numbers. However, these techniques require careful attention and precision when dealing with large and complex addition problems. Nevertheless, the advantages of these techniques are that they are practical, easy to apply, and can be used with large numbers. Consequently, this implies that addition calculations, both with like and unlike signs, can be performed more quickly than with traditional techniques such as number lines and positive-negative signs.

4. CONCLUSION

This study presents the development of new techniques for addition of integers that have not been used before. These new techniques have their own advantages and disadvantages, which can help us add integers, both with like and unlike signs, quickly and accurately. However, some of these new techniques need to be further developed so that students have more options in solving addition problems with integers. Therefore, the development of new techniques is essential for the future.

5. ACKNOWLEDGEMENT

We gratefully acknowledge the assistance and support of Universitas Terbuka 2023.

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