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On Finding Integer Solutions to the Homogeneous Ternary Quadratic Diophantine Equation $3(x^2+y^2)-5xy=15z^2$

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ABSTRACT:

This paper concerns with the problem of obtaining non-zero distinct integer solutions to the homogeneous quadratic Diophantine equation with three unknowns given by$3(x^2+y^2)-5xy=15z^2$.Various sets of integer solutions are obtained. A few interesting properties among the solutions are given. Also, knowing a solution of the given equation, formulas for obtaining sequence of integer solutions based on the given solution are presented.

Keywords: Ternary quadratic, Homogeneous quadratic, Integer solutions.2010 Mathematics Subject Classification: 11D09

INTRODUCTION

The ternary quadratic Diophantine equation offers an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-17] for quadratic equations with two and three unknowns. This communication concerns with yet another interesting ternary quadratic equation $3(x^2+y^2)-5xy=15z^2$is analysed for its non-zero distinct integer solutions through different methods.

METHODS OF ANALYSIS

Theternary quadratic Diophantine equation to be solved for non-zero distinct integral

Solution

$$3(x^2+y^2)-5xy=15z^2 \quad (1)$$

Introduction of the linear transformations

$$x=u+v, \quad y=u-v, \quad u\neq v\neq 0 \quad (2)$$

in (1) leads to

$$u^2+11v^2=15z^2 \quad (3)$$

The above equation is solved for u, v and $z$through different methods

and using (2), the values of x and $y$satisfying (1), are obtained which are

illustrated below

Method I:

Write (3) in the form of ratio as

$$\frac{(u+2z)}{z+v}=\frac{11(z-v)}{u-2z}=\frac{\alpha}{\beta}, \quad \beta\neq 0 \quad (4)$$

which is equivalent to the system of double equations

$$\beta u-\alpha v+(2\beta-\alpha)z=0$$
$$-\alpha u-11\beta v+(11\beta+2\alpha)z=0$$

Solving the above system of double equations and using (2), the corresponding integer solutions to (1) are found to be

$$x = \alpha^2 - 11\beta^2 + 26\alpha\beta$$
$$y = 3\alpha^2 - 33\beta^2 + 18\alpha\beta$$
$$z = \alpha^2 + 11\beta^2$$

Note 1:

It is noted that (3) may also be written in the form of ratio as

(i) $\frac{u+2z}{11(z-v)} = \frac{(z+v)}{u-2z} = \frac{\alpha}{\beta}$

(ii) $\frac{u-2z}{(z+v)} = \frac{11(z-v)}{u+2v} = \frac{\alpha}{\beta}$

(iii) $\frac{u-2z}{11(z-v)} = \frac{11(z-v)}{u+2z} = \frac{\alpha}{\beta}$

(iv) $\frac{u-2z}{z-v} = \frac{11(z+v)}{u+2z} = \frac{\alpha}{\beta}$

For each of the above ratios, the corresponding integer solutions to (1) are exhibited below

Solutions obtained through (i)

$$x = -11\alpha^2 + \beta^2$$
$$y = -33\alpha^2 + 3\beta^2$$
$$z = -11\alpha^2 + \beta^2$$

Solutions obtained through (ii)

$$x = -3\alpha^2 - 33\beta^2 - 18\alpha\beta$$
$$y = \alpha^2 - 11\beta^2 - 26\alpha\beta$$
$$z = \alpha^2 + 11\beta^2$$

Solutions obtained through (iii)

$$x = -11\alpha^2 + \beta^2 + 26\alpha\beta$$
$$y = -33\alpha^2 + 3\beta^2 + 18\alpha\beta$$
$$z = -11\alpha^2 - \beta^2$$

Solution obtained through (iv)

$$x = 3\alpha^2 + 33\beta^2$$
$$y = \alpha^2 + 11\beta^2$$
$$z = -\alpha^2 - 11\beta^2$$

Method II:

Introducing the linear transformations

$Z = X + 11T, \quad v = X + 15T, \quad u = 2P$ (5)

In (3), it given

$X^2 = 165T^2 + P^2$ (6)

which is satisfied by

$$\left.\begin{aligned} T &= 2rs \\ P &= 165r^2 - s^2 \\ X &= 165r^2 + s^2 \end{aligned}\right\} \quad (7)$$

From (7), (5) & (2), we obtain the integer solutions to (1) as given below

$$x = 495r^2 - s^2 + 30rs$$
$$y = 165r^2 - 3s^2 - 30rs$$
$$z = 165r^2 + s^2 + 22rs$$

It is to be noted that (6) may be represented as the system of double equation

as shown in Table: 1

Table: 1 System of double equations

System	**1**	**2**	**3**	**4**	**5**
$X+P$	T^2	$11T^2$	$15T^2$	$3T^2$	$55T^2$
$X-P$	165	15	11	55	3

System	**6**	**7**	**8**	**9**	**10**
$X+P$	$33T^2$	$5T^2$	$11T$	$15T$	$165T$
$X-P$	5	33	15T	11T	T

System	**11**	**12**	**13**
$X+P$	$3T$	$55T$	T
$X-P$	$55T$	$3T$	$165T$

Solving each of the system of double equations in Table:1, the values of X, P & T are obtained, from (5) & (2), the corresponding solutions to (1) are found and they are exhibited below.

Solutions from system 1

$$x = 6k^2 + 36k - 66$$
$$y = 2k^2 - 28k - 262$$
$$z = 2k^2 + 24k + 94$$

Solutions from system 2

$$x = 66k^2 + 96k + 24$$
$$y = 22k^2 - 8k - 32$$
$$z = 22k^2 + 44k + 24$$

Solutions from system 3

$$x = 90^2 + 120k + 24$$
$$y = 30k^2 - 32$$
$$z = 30^2 + 52k + 24$$

Solutions from system 4

$$x = 18k^2 + 48k - 8$$
$$y = 6k^2 - 24k - 96$$
$$z = 6k^2 + 28k + 40$$

Solutions from system 8

$$x = 48k \quad + 24$$
$$y = -64k - 32$$
$$z = +48k + 24$$

Solutions from system 9

$$x = 64k$$
$$y = -48k$$
$$z = 48k$$

Solutions from system 12

$x = 192k + 96$

$$y = 16k + 8$$
$$z = 80k + 40$$

Method III:

Assume

$z = a^2 + 11b^2$ (8)

Case (i):

Write 15 as

$15 = (2 + i\sqrt{11})(2 - i\sqrt{11})$ (9)

Using (8) and (9) in (3) and employing the method of factorization, define

$$(u + i\sqrt{11}v) = (2 + i\sqrt{11})(a + i\sqrt{11}b)^2$$

Equating the real and imaginary parts, we get

$$u = 2a^2 - 22b^2 - 22ab$$
$$v = a^2 - 11b^2 + 4ab$$

In view of (2), we obtain

$$\left.\begin{aligned} x &= 3a^2 - 33b^2 - 18ab \\ y &= a^2 - 11b^2 - 26ab \end{aligned}\right\} \quad (10)$$

Thus (8) and (10) represent the integer solution to (1).

Case (ii):

One can write 15 as

$$15 = \frac{(7+i\sqrt{11})(7-i\sqrt{11})}{2^2} \quad (11)$$

Using (8) and (11) in (3) and applying the method of factorization, define

$$(u + i\sqrt{7}v) = \frac{(13 + i7\sqrt{7})}{4}(a + i\sqrt{7}b)^2$$

Equating the real and imaginary parts, we get

$$u = \frac{14a^2 - 154b^2 - 44ab}{4}$$

$$v = \frac{2a^2 - 22b^2 + 28ab}{4}$$

In view of (2), we obtain

$$\left.\begin{aligned} x &= \frac{16a^2 - 176b^2 - 16ab}{4} \\ y &= \frac{12a^2 - 132b^2 - 72ab}{4} \end{aligned}\right\} \quad (12)$$

To obtain the integer solutions, replacing a by 2A and b by 2B in (8) & (12), the corresponding integer solutions of (1) are given by

$$\left.\begin{aligned} x &= 16A^2 - 176B^2 - 16AB \\ Y &= 12A^2 - 132B^2 - 72AB \\ z &= 4A^2 + 44B^2 \end{aligned}\right\} \quad (13)$$

Method IV:

Equation (3) can be written as

$$u^2 + 11v^2 = 15z^2 * 1 \quad (14)$$

Write 1 on the R.H.S. of (14) as

$$1=\frac{(5+i\sqrt{11})(5-i\sqrt{11})}{6^2} \qquad (15)$$

Using (8), (9) & (15) in (14) and utilizing the method of factorization, define

$$(u+i\sqrt{11}v)=(2+i\sqrt{11})(a+i\sqrt{11}b)^2\left[\frac{(5+i\sqrt{11})}{6}\right]$$

Equating the real and imaginary parts, the values of u and v are obtained as

$$u=\frac{-6a^2+6b^2-984ab}{6}$$

$$v=\frac{42a^2-462b^2-12ab}{6}$$

Proceeding as in case (ii),we get

$$\left.\begin{aligned} x&=\frac{36a^2-456b^2-996ab}{6}\\ y&=\frac{-48a^2+468b^2-972ab}{6}\\ Z&=\frac{36a^2+396b^2}{4}\end{aligned}\right\} \qquad (16)$$

Thus (16) represent the non-zero distinct solution of (1)

Note 2:

It is seen that 1 is also represented as follows

$$1=\frac{(1+i3\sqrt{11})(1-i3\sqrt{11})}{10^2}$$

Following the above procedure, the solutions of (1) are obtained.

Method V:

Consider (3) as

$$15z^2-11v^2=u^2*1 \qquad (17)$$

Let

$$u=15a^2-11b^2 \qquad (18)$$

Consider 1 as

$$1=\frac{\left(\sqrt{15}+\sqrt{11}\right)\left(\sqrt{15}-\sqrt{11}\right)}{4} \qquad (19)$$

Using (18) & (19) in (17) and employing the method of factorization,

consider

$$\sqrt{15}z+\sqrt{11}v=\frac{1}{2}\left(\sqrt{15}+\sqrt{11}\right)\left(\sqrt{15}a+\sqrt{11}b\right)^2$$

Equating the coefficients of corresponding terms, we have

$$\left.\begin{aligned} z &= \frac{1}{2}\left(15a^2 + 11b^2 + 22ab\right) \\ v &= \frac{1}{2}\left(15a^2 + 11b^2 + 30ab\right) \end{aligned}\right\}$$

(20)

Replacing a by 2A, b by 2B in (18) & (20) the corresponding integer

solutions to (17) are given by

$$\left.\begin{aligned} u &= 60A^2 - 44B^2 \\ v &= 30A^2 + 22b^2 + 60AB \end{aligned}\right\} \quad (21)$$

$$z = 30A^2 + 22B^2 + 44AB \quad (22)$$

Substituting (21) in (2), we have

$$\left.\begin{aligned} x &= 90A^2 - 22B^2 + 60AB \\ y &= 30A^2 - 66B^2 - 60AB \end{aligned}\right\} \quad (23)$$

Then (22) & (23) give the integer solution to (1).

Method VI:

Consider (3) as

$$15z^2 - u^2 = 11v^2 \quad (24)$$

Let

$$v = 15a^2 - b^2 \quad (25)$$

Write 11 as

$$11 = \left(\sqrt{15} + 2\right)\left(\sqrt{15} - 2\right) \quad (26)$$

Using (25) & (26) in (24) and employing the method of factorization,

consider

$$\left(\sqrt{15}z + u\right) = \left(\sqrt{15} + 2\right)\left(\sqrt{15}a + b\right)^2 \quad (27)$$

Equating the coefficients of corresponding terms, we have

$$z = 15a^2 + b^2 + 4ab \quad (28)$$

$$u = 30a^2 + 2b^2 + 30ab \quad (29)$$

From (25) & (29) in (2), we have

$$\left.\begin{aligned} x &= 45a^2 + b^2 + 30ab \\ y &= 15a^2 + 3b^2 + 30ab \end{aligned}\right\} \quad (30)$$

Then, (28) & (30) gives the integer solution of (1).

Generation of Integer Solutions

Let (u_0, v_0, z_0) be any given integer solution to (3). We illustrate below the

method of obtaining a general formula for generating sequence of integer solutions based on the given solution.

Case (i)

Let

$$u_1 = -u_0 + 4h$$
$$v_1 = v_0 \qquad h \neq 0$$
$$z_1 = z_0 + h \tag{31}$$

be the second solution of (3).Substituting (31) in (3) & performing a few calculations, we have

$$h = 8u_0 + 30z_0$$

and then

$$u_1 = 31u_0 + 120z_0$$
$$z_1 = 8u_0 + 31z_0$$

This is written in the form of matrix as

$$\begin{pmatrix} u_1 \\ z_1 \end{pmatrix} = M \begin{pmatrix} u_0 \\ z_0 \end{pmatrix} \tag{32}$$

where $M = \begin{pmatrix} 31 & 120 \\ 8 & 31 \end{pmatrix}$

Repeating the above process, the general solution (u_n, z_n) to (3) is given by

$$\begin{pmatrix} u_n \\ z_n \end{pmatrix} = M^n = \begin{pmatrix} u_0 \\ v_o \end{pmatrix}$$

To find M^n, the eigen values of $M$are $\alpha = 31 + 8\sqrt{15}$, $\beta = 31 - 8\sqrt{15}$.

We know that $M^n = \frac{\alpha^n}{(\alpha-\beta)}(M - \beta I) + \frac{\beta^n}{(\beta-\alpha)}(M - \alpha I)$

Using the above formula, we have

$$M^n = \begin{pmatrix} \frac{\alpha^n+\beta^n}{2} & \frac{\sqrt{15}(\alpha^n-\beta^n)}{2} \\ \frac{\alpha^n-\beta^n}{2\sqrt{15}} & \frac{\alpha^n+\beta^n}{2} \end{pmatrix}$$

Thus the general solution (u_n, v_n, z_n) to (3) is given by

$$u_n = \left(\frac{\alpha^n + \beta^n}{2}\right)u_0 + \sqrt{15}\left(\frac{\alpha^n - \beta^n}{2}\right)z_0$$
$$v_n = v_0$$
$$z_n = \left(\frac{\alpha^n - \beta^n}{2\sqrt{15}}\right)u_0 + \left(\frac{\alpha^n + \beta^n}{2}\right)z_0$$

From (2) we have,

$$x_n = u_n + v_n$$
$$y_n = u_n - v_n$$

Thus the general solution (x_n, y_n, z_n) to (1) is given by

$$x_n = \left(\frac{\alpha^n+\beta^n}{2}\right)u_0 + v_0 + \left(\frac{\sqrt{15}(\alpha^n-\beta^n)}{2}\right)z_0$$
$$y_n = \left(\frac{\alpha^n + \beta^n}{2}\right)u_0 - v_0 + \left(\sqrt{15}(\alpha^n - \beta^n)\right)z_0$$
$$z_n = \left(\frac{\alpha^n - \beta^n}{2\sqrt{15}}\right)u_0 + \left(\frac{\alpha^n + \beta^n}{2}\right)z_0$$

Case (ii)

Let

$$u_1 = 4u_0$$
$$v_1 = 4v_0 + h$$
$$z_1 = -4z_0 + h \quad , h \neq 0$$

Repeating the process as in the case (i) the corresponding general solution (x_n, y_n, z_n) to (1) is given by

$$x_n = 4u_0 + \left(\frac{\alpha^n + \beta^n}{2}\right)v_0 + \left(\frac{\alpha^n - \beta^n}{2\sqrt{11}}\right)z_0$$
$$y_n = 4u_0 - \left(\frac{\alpha^n + \beta^n}{2}\right)v_0 - \left(\frac{(\alpha^n - \beta^n)}{2\sqrt{11}}\right)z_0$$
$$z_n = \left(\frac{\sqrt{11}(\alpha^n - \beta^n)}{\sqrt{11}}\right)v_0 + \left(\frac{\alpha^n + \beta^n}{2}\right)z_0$$

Case (iii)

Let

$$u_1 = -12u_0 + h$$
$$v_1 = -12v_0 + h$$
$$z_1 = 12z_0 \quad , h \neq 0$$

Repeating the process as in the case (i) the corresponding general solution (x_n, y_n, z_n) to (1) is given

$$x_n = \left(\frac{\alpha^n}{6}\right)u_0 + \frac{(11(\alpha^n - \beta^n))}{6}v_0$$
$$y_n = \frac{\beta^n}{6}u_0 - \frac{11\beta^n}{6}v_0$$
$$z_n = 12^n z_0$$

Case(iv):

Let

$$u_1 = 3x_0 + h$$
$$v_1 = 3y_0 + h$$
$$z_1 = h - 3z_0 \quad , \quad h \neq 0 \qquad (33)$$

Be the second solution to (3).substitute (33) in (3) and simplifying,

We have, $h = 2x_0 + 22y_0 + 30z_0$

From (33),

$$u_1 = 5x_0 + 22y_0 + 30z_0$$
$$v_1 = 2x_0 + 25y_0 + 30z_0$$
$$z_1 = 2x_0 + 22y_0 + 27z_0$$

Which is written in the matrix form as

$$(u_1, v_1, z_1)^t = M(u_0, v_0, z_0)^t$$

Where,

$$M = \begin{pmatrix} 5 & 22 & 30 \\ 2 & 25 & 30 \\ 2 & 22 & 27 \end{pmatrix}$$

Repeating the above process, one observes that

$$(u_2, v_2, z_2)^t = M^2(u_{0,}v_0, z_0)^t$$

Where,

$$M^2 = \begin{pmatrix} 129 & 1320 & 1620 \\ 120 & 1329 & 1620 \\ 108 & 1188 & 1449 \end{pmatrix}$$

Also, $(u_3, v_3, z_3)^t = M^3(u_0, v_0, z_0)^t$

Where,

$$M^3 = \begin{pmatrix} 6525 & 71478 & 87210 \\ 6498 & 71505 & 87210 \\ 5814 & 63954 & 78003 \end{pmatrix}$$

Following the procedure presented above, the general solution ($u_{n+1}, v_{n+1}, z_{n+1})$ to (3)is given by $(u_{n+1}, v_{n+1}, z_{n+1})^t = M^{n+1}(u_0, v_0, z_o)^t$

Where,

$$M^{n+1} = \begin{pmatrix} \frac{Y_n - 3^{n+1}}{12} + 3^{n+1} & \frac{11}{12}\left(Y_n - 3^{n+1}\right) & 15X_n \\ \frac{Y_n - 3^{n+1}}{12} & \frac{11}{12}\left(Y_n - 3^{n+1}\right) + 3^{n+1} & 15X_n \\ X_n & 11X_n & Y_n \end{pmatrix}$$,in which

$$X_n = X_0 Y_{n-1} + Y_0 X_{n-1}$$
$$Y_n = Y_0 Y_{n-1} + 180 X_0 X_{n-1}$$
$$Y_{-1} = 1, X_{-1} = 0$$

In view of (3), the corresponding general solution $(x_{n+1}, y_{n+1}, z_{n+1})$ to (1) is given by

$$x_{n+1} = \left(\frac{Y_n - 3^{n+1}}{6} + 3^{n+1} \right) u_0 + \left[\frac{11}{6} \left(Y_n - 3^{n+1} \right) + 3^{n+1} \right] v_0 + 30 X_n z_0$$

$$y_{n+1} = 3^{n+1} u_0 - 3^{n+1} v_0$$

$$z_{n+1} = X_n u_0 + 11 X_n v_0 + Y_n z_0$$

CONCLUSION:

In this paper, we have presented six different methods of obtaining infinitely many non-zero distinct integer solutions of the homogeneous cone given by$3(x^2 + y^2) - 5xy = 15z^2$. To conclude, one may search for other patterns of solutions and their corresponding properties.

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