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Mathematical Study of Ferrohydrodynamic Fluids (FHD)

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ABSTRACT

A phenomenological treatment is given for the fluid dynamics and thermodynamics of strongly polarizable magnetic fluid continuing in the presence of magnetic fields. This study involves the areas of analysis, namely and ferrohydrodynamics (FHD). The ferrohydrodynamic (FHD) is the important effects of a magnetic field in a liquid. An analytical solution is found for the problem of source flow with heat addition in order to display the thermomagnetic and magnetomechanical effects attendant to simultaneous heat addition and fluid motion in the presence of a magnetic field. In this paper, emphasis is given to the study of the effect of the FHD interaction parameters on the flow field.

Key words:Ferrohydrodynamics, Mathematical methods, Thermo magnetic effect, Magneto mechanical effect.

1. INTRODUCTION

Ferrofluid is a liquid that is attracted to the poles of a magnet. It is a colloidal liquid made of nanoscale ferromagnetic or ferrimagnetic particles suspended in a carrier fluid (usually an organic solvent or water). Each magnetic particle is thoroughly coated with a surfactant to inhibit clumping. Large ferromagnetic particles can be ripped out of the homogeneous colloidal mixture, forming a separate clump of magnetic dust when exposed to strong magnetic fields. Ferrohydrodynamics (FHD) is defined as fluid dynamics and heat transfer process associated with the motion of incompressible, magnetically polarizable fluids in the presence of magnetic field. Ferrohydrodynamics (FHD) treats the flow of strongly magnetized fluid media. The most common fluid media are colloidal ferrofluids. Magnetic particles suspended in a fluidized bed provide another example. The forces that arise in either case are due to magnetic field acting on magnetizable matter, normally in the absence of any electric current or electric charge in the matter. One particular interest in Ferrohydrodynamics arises from an energy conversion in which heat added to the ferrofluid in the presence of a magnetic field is converted into useful work [1].

In this paper, fluid flow of the Ferrohydrodynamics (FHD) is studied and also thermomagnetic and magnetomechanical effects are analyzed. For the mathematical formulation of the problem for Ferrohydrodynamics (FHD) are taken into account and consequently principle of Ferrohydrodynamics (FHD) is adopted.

2. MATHEMATICAL FORMULATION AND PROBLEM DESCRIPTION

We consider the flow of an incompressible Newtonian fluid through a rectangular miniature cylinder with the z-axis being the pivotal way. The differential conditions overseeing the flow incorporate the coherence condition and the Navier–Stirs conditions as follows

$$\frac{\partial U_j}{\partial x_j} = 0 \qquad (1)$$

$$p\left(\frac{\partial y_j}{\partial x_j} + y_i \frac{\partial y_j}{\partial x_i}\right) = -\frac{\partial p}{\partial x_j} + \mu \frac{\partial^2 y}{\partial x_i \partial x_j} + p g_{i,} [i = 1,2,3; j = 1,2,3] \qquad (2)$$

Where p and yi are respectively the fluid pressure and velocity vector, gj is the gravitational acceleration, ρ and µ are respectively the fluid density and viscosity and xi denotes coordinates. As the flow is axially symmetric, the velocity components in the x and y directions vanish, namely

$y_1 = y_x = 0$ and $y_2 = y_y = 0$. Thus the continuity equation (1) becomes

$$\frac{\partial y_3}{\partial x_3} = \frac{\partial y_z}{\partial z} = 0$$

Which gives rise to y_3= v = v (x, y, t)

As the flow is horizontal, $g_3 = g_z = 0$, and hence eq. (2) becomes

$$p\left(\frac{\partial v}{\partial t}\right) = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial p}{\partial z}$$

We consider the liquid stream driven by the weight field with a pressure angle q(t) which can be communicated by a Fourier arrangement, to be specific

$$\frac{\partial p}{\partial z} = q(t) = a_0 + \sum_{n=1}^{\infty}[a_n cos(n\omega t) + b_n sin(n\omega t)] \qquad (3)$$

As the issue is pivotally symmetric, we just need to think about a quadrant of the cross-segment in the calculation.

By applying the Navier slip conditions in the primary quadrant of the rectangular cross segment, as in the paper by Wu *et al*., (2008) [2] for each time t, we have

$$\frac{\partial v}{\partial y}(x,0) = 0; 0 \le x \le a$$
$$\frac{\partial v}{\partial x}(0,y) = 0; 0 \le y \le b$$
$$v(x,b) + l\frac{\partial v}{\partial y}(x,b) = 0; 0 \le x \le a$$
$$v(a,y) + l\frac{\partial v}{\partial x}(a,y) = 0; 0 \le y \le b \qquad (4)$$

3. MAGNETO MECHANICAL EFFECT

From electromagnetic theory, the force per unit volume in mks units on a piece of magnetized materials of Magnetization M (i.e., dipole moment per unit volume) in the field of magnetic intensity H is given by $\mu_0(M.\nabla)H$ where μ_0 is the free space permeability [3]. Thus, unlike magneto hydrodynamic theory, where the Lorentz force can be non zero even with the application of uniform magnetic field, Ferrohydrodynamics (FHD) interactions require the existence of spatially varying fields. If the direction of magnetization of a fluid elements is always in the direction of local magnetic field, then

$$\mu_0\,(M.\nabla) = \left(\frac{\mu_{0M}}{H}\right)(H.\nabla)H$$

Using the vector identity

$(H.\nabla)H = \frac{1}{2}\,\nabla(H.\mathrm{H}) - HX(\nabla XH),$

And assuming the fluid is electrically non conducting and that the displacement current is negligible so that(∇XH)= 0, we obtain

$$\mu_0\,(M.\nabla) = \left(\frac{\mu_{0M}}{H}\right)\frac{1}{2}\,\nabla(H.\mathrm{H}) = \mu_0 M\nabla H - - - - - (5)$$

This result is good, when M is parallel to H, should be good approximation for the flows of fluid dispersions considered here since the individual suspended particles are mechanically align with the field in times small compared with the characteristic convective time. The assumption $MXH = 0$, therefore, forms a basis of the theory which follows and it is also assumed that the fluid is electrically non-conducting.

4. THERMO MAGNETIC EFFECT

The thermodynamic state of incompressible magnetic substance is specified when temperature (T) and magnetic moment are known. Now the term $-\mu_0 HdM$ is dimensionally equivalent to work per unit volume, and in addition, is zero unless a state change occurs. This term may be identified through application of Faraday's law of induction as minus the external electrical circuit that magnetizes the material from M to M+dM if the material is uniform ring. The element of flowing fluid is not surrounded by an induction coil, the reversible work term appropriate thermodynamic analysis may be chosen as

$$dW_R = -\mu_0 HdM - - - - - - - (6)$$

Reversible heat addition may accompany either a change of temperature or change of local field intensity. Hence we may write

$$dQ = c(T,H)dT + g(T,H)dH - - - - - (7)$$

Where the coefficients c (T, H) and g (T, H) are to be determined. There will be an equation of state

$$M = M(T,H) - - - - - - - (8)$$

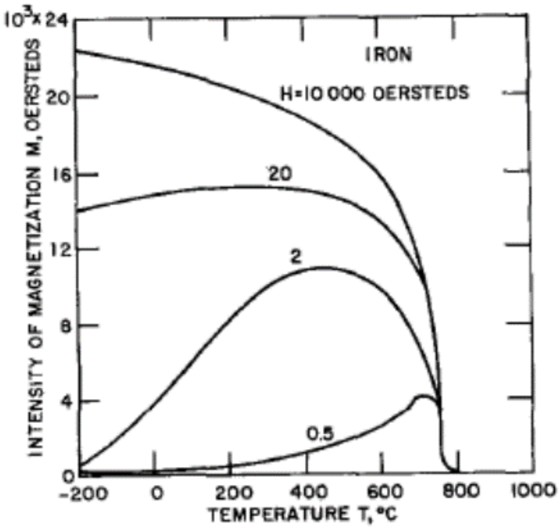

Fig.1. Magnetic equation of state for iron giving dependence of intensity of magnetization on Temperature

An example of a ferromagnetic equation of state is that shown in Fig. 1. Under moderate applied fields (H<< 10,000 Oe), the iron approaches technical saturation and the magnetization is sensibly temperature dependent only [4]. Then, there are, in general, appreciable region over which a linear dependence of M on T are only valid. From the existence of equation of state,

$$dM = \left(\partial M/_{\partial T}\right)_H dT + \left(\partial M/_{\partial H}\right)_T dH$$

And the work term may be expressed in terms of independent variables H and T as

$$dW_R = -\mu_0 H\left(\partial M/_{\partial T}\right)_H dT - \mu_0 H\left(\partial M/_{\partial H}\right)_T dH$$

Neither the heat added dQ, nor the work performed dW_R, are exact differentials but their difference, the internal energy is

$$dU = dQ - dW_R = \left(c + \mu_0 H\left(\partial M/_{\partial T}\right)_H dT\right) + \left(c + \mu_0 H\left(\partial M/_{\partial H}\right)_T dH\right) ------(9)$$

According to the second law of thermodynamics, the entropy is also perfect differential.

$$dS = \frac{dQ}{T} = \frac{c}{T}\,dT + \frac{g}{T}dH$$

$$dQ = cTdT - \mu_0 KT\,dH ------- (10)$$

$$dW = -\mu_0 H\,dM = \mu_0\,KH\,dT --------(11)$$

$$dU = (cT - \mu_0\,KH)\,dT - \mu_0 KT\,dH ------- (12)$$

$$dS = c\,dT - \mu_0 K\,dH ----------(13)$$

CONCLUSIONS

The Ferrohydrodynamic (FHD) is the important effects of a magnetic field in a liquid. The flow of Ferrohydrodynamic fluids are studied by Mathematical methods, thermo magnetic and magneto mechanical effect and also effect of the FHD interaction different parameters on the flow field are analyzed.

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