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Formulation of Numerical Relation to Second Order Ramanujan Numbers

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ABSTRACT:

This communication focuses on formulating numerical relation to second order Ramanujan numbers.

Keywords: Second order Ramanujan number, Numerical relation

Introduction:

It is well-known that, if a non-zero positive integer n is expressed as the sum of two squares in two different ways, then, the integer n is called Second order Ramanujan number. Mathematically, a non-zero positive integer n is said to be Second order Ramanujan number provided $n = a^2 + b^2 = c^2 + d^2$, where a, b, c, d are non-zero distinct integers. The above relation is known as the numerical relation to the Second order Ramanujan number and the integers a, b, c, d are called base integers. In [1], various methods to solve the above numerical relation for obtaining the integer values to the base integers a, b, c, d are illustrated. In [2], different approaches to formulate numerical relation for different Second order Ramanujan numbers from the given numerical relation are illustrated. In [3, 4], the authors have obtained the numerical relation to second order Ramanujan numbers from the integer solutions of the pell equation through employing the method of factorization. Here this communication focuses on formulating numerical relation to second order Ramanujan numbers through employing the well-known identity $(A-B)^2 \equiv A^2 + B^2 - 2AB$.

Method of analysis:

Consider the well-known identity

$$(A-B)^2 \equiv A^2 + B^2 - 2AB$$

and write it as

$$(A-B)^2 + 2AB = A^2 + B^2 \qquad (1)$$

The above equation (1) will represent the numerical relation to the Second order Ramanujan number provided the product 2*A*B is a perfect square.

Choice 1:

Let

$$A = (2k^2 + 12k + 18)B$$

$$(1) \Rightarrow (2k^2 + 12k + 18)^2 + 1 = (2k^2 + 12k + 17)^2 + (2k + 6)^2$$

Choice 2:

Let

$$A = k^{2t} 2^{2s-1} \alpha^{2n},\ B = \beta^{2m},\ 2AB = 2^{2s} k^{2t} \alpha^{2n} \beta^{2m}$$

$$(1) \Rightarrow (k^{2t} 2^{2s-1} \alpha^{2n} - \beta^{2m})^2 + (2^s k^t \alpha^n \beta^m)^2 = (k^{2t} 2^{2s-1} \alpha^{2n})^2 + (\beta^{2m})^2$$

Choice 3:

Let

$$(1) \Rightarrow (k^{2t} \alpha^{2n} - 2^{2s-1} \beta^{2m})^2 + (2^s \beta^m k^t \alpha^n)^2 = (k^{2t} \alpha^{2n})^2 + (2^{2s-1} \beta^{2m})^2$$

References:

[1] M. Meyyappan, Ramanujan Numbers, S. Chand & Company limited.

[2] S. Vidhyalakshmi and MAG, Second order Ramanujan numbers and Applications, 2017 Omni Scriptum Lab Lamber & Academic Publishing GmbH & co, KG, Germany.

[3] J. Shanthi, P. Deepalakshmi, M. A. Gopalan, “A study on the positive pell equation $y^2 = 42x^2 + 7$, IRJEdT, Volume-1, Issue-5, 107-118, January 2021.

[4] T. Mahalakshmi, J. Shanthi, S. Vidhyalakshmi, M. A. Gopalan, “Observations on the pell equation $x^2 = 6(y^2 + y) + 4$, IJCRT, Volume-9, Issue-4, 626-630, April 2021.