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Mixed Convection Heat Transfer in the Presence of Magnetic Field at an Isothermal Vertical Plate with Variable Fluid Properties – Dual Solution

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A B S T R A C T

The present study is a brief analysis on magneto hydrodynamic mixed convection heat transfer in the case of opposing flow by assuming the fluid viscosity and thermal conductivity to be linear functions of temperature. Free stream is considered to be uniform. Using local similarity, the flow and heat transfer quantities of practical interest are found to be functions of five parameters namely Richardson number, Prandtl number, magnetic-interaction parameter, a viscosity variation parameter and a thermal conductivity variation parameter. Numerical solution of the governing problem is obtained by Nachtsheim-Swigert technique for saturated liquid Mercury appropriate at specific temperatures of the plate and the ambient fluid. It is observed that dual solutions exist for certain range of Richardson number.

Keywords: Magnetohydrodynamic (MHD) Mixed convection flow, Opposing flow, Dual solution, Local similarity, Prandtl number.

1. INTRODUCTION

The free and forced convection flow in an electrically conducting fluid is of prime interest to many researchers over the decades due to its applications in many transport and natural phenomena. A plethora of literature is available on mixed convection flows and is depicted in the work of G. Santhi et al [1]. Also, in recent days, the existence of dual solutions and multiple solutions in some convective flows increase the thirst of the researchers and as a consequence, the investigations on these flows are continued further. Here, the references from [1] to [12] reflects the Literature survey on the works of dual solutions. The present analysis is the extension of the work of G. Santhi et al [1] taking into account the effect of magnetic field.

2. FORMULATION

Consider a vertical stationary plate at the temperature T_w in a viscous incompressible electrically conducting fluid of ambient temperature T_∞. X-axis is taken vertically upwards along the plate and Y-axis perpendicular to it. The plate is aligned parallel to the uniform free stream with velocity U_∞. A magnetic field of uniform strength B_0 is applied transversely to the direction of free stream. Fluid viscosity and thermal conductivity are defined to be the linear functions of temperature as $\mu = \mu_f s_\mu(T)$ and$k = k_f s_k(T)$. Boussinesq's approximation is employed. Neglecting viscous dissipation and volumetric heat generation, the boundary layer equations governing the steady-state mixed convection flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \pm g\beta(T - T_\infty) - \frac{\sigma {B_0}^2 u}{\rho} \quad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) \quad (3)$$

Appropriate boundary conditions on the temperature and velocity fields are

at $y = 0$: $u = 0, v = 0, T = T_w$ (4)

as $y \to \infty$: $u \to U_\infty, T \to T_\infty$ (5)

Here, σ is the conductivity of the fluid. The remaining notations used are conventional symbols having their own usual meaning (refer [1]). Generally, free convection problems are dependent on Grashof numbers, forced convection on Reynolds numbers whereas mixed convection problems are based on Richardson number ξ or mixed convection buoyancy parameter.

Using local similarity method, the coordinates are transformed from (x, y) system to (ξ, η) system where $\eta = \eta(x, y)$ and $\xi = \xi(x)$. Here η, a pseudo-similarity variable and ξ, a mixed convection buoyancy parameter, also known as Richardson number, are defined as

$$\eta = \frac{y}{x}\sqrt{\left(\frac{Pe_x}{2}\right)} \quad (6)$$

$$\xi = Ri = \frac{Gr_x}{Re_x^2} \approx x \quad (7)$$

Introducing non-dimensional stream function $f(\xi,\eta)$ and non-dimensional temperature $\theta(\xi,\eta)$ through the relations $\left(2\alpha\left(\frac{Pe_x}{2}\right)^{1/2}\right) f(\xi,\eta) = \psi(x,y)$, $\theta(\xi,\eta) = \frac{T-T_\infty}{T_w-T_\infty}$

(where $\psi(x,y)$ is the conventional stream function defined by $u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}$)

into the governing equations, the following transformed equations in non-dimensional form are obtained.

$$S_\mu \frac{\partial^3 f}{\partial\eta^3} + \gamma_\mu \frac{\partial\theta}{\partial\eta}\frac{\partial^2 f}{\partial\eta^2} + \frac{1}{Pr}\left(2\xi\theta + f\frac{\partial^2 f}{\partial\eta^2}\right) - 2Na\frac{\partial f}{\partial\eta} = \frac{2x}{Pr}\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi\partial\eta} - \frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2}\right) \quad (8)$$

$$S_k\frac{\partial^2\theta}{\partial\eta^2} + \gamma_k\left(\frac{\partial\theta}{\partial\eta}\right)^2 + f\frac{\partial\theta}{\partial\eta} = 2x\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi} - \frac{\partial f}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right) \quad (9)$$

The boundary conditions become

$$\left.\begin{aligned} & f(\xi,0) = 0, \frac{\partial f}{\partial\eta}(\xi,0) = 0,\ \theta(\xi,0) = 1 \\ & \frac{\partial f}{\partial\eta}(\xi,\infty) = 1, \theta(\xi,\infty) = 0 \end{aligned}\right\} \quad (10)$$

The parameters S_μ, S_k, γ_μ and γ_k that arise in equations (8) and (9) will have the same definitions as in the reference [1]. The ξ derivatives are deleted from the equations (8) and (9), in accordance with the principle of local similarity. At any stream wise position along the plate, the quantity ξ may be regarded as an assignable constant parameter. As a consequence, equations (8) and (9) may be treated as a system of ordinary differential equations at each stream wise location of interest. Now the reduced system of ordinary differential equations is

$$Pr\left(S_\mu f''' + \gamma_\mu f''\theta'\right) + 2\xi\theta + ff'' - 2\,Na\,f' = 0 \quad (11)$$

$$S_k\theta'' + \gamma_k\theta'^2 + f\theta' = 0 \quad (12)$$

where a dash denotes differentiation with respect to η.

3. PARAMETERS OF THE PROBLEM

The parameters that govern the flow are *Pr*, γ_μ, γ_k, ξ and *Na*. Prandtl number *Pr* is a constant which represents a fluid. In this analysis, *Pr* = 0.01 is taken that correspond to Mercury at atmospheric pressure and at appropriate film temperatures.

Richardson number ξ also called mixed convection buoyancy parameter is a parameter that provides a measure of the influence of free convection in comparison to that of forced convection on the fluid flow. Solutions are obtained for a zero value of ξ that correspond to forced convection and selected negative values of ξ that correspond to opposing flow. In the opposing flow case, , it is obtained that there exists either a unique solution or dual solutions or no solution for some range of values of ξ.

Magnetic-Interaction parameter *Na* is defined as the ratio of Pondermotive force and Inertia force

given by $Na = \frac{M}{Re} = \frac{\sigma B_0{}^2\, x}{\rho\, u}$ where *M* is the magnetic parameter defined as $M = \frac{\sigma B_0{}^2\, x^2}{\mu}$ (*i.e.*, the ratio of Pondermotive force and viscous force) and *Re* is the Reynolds number. Pondermotive force is a force that measures the relative distribution of currents in a magnetic field.

The viscosity and thermal conductivity variation parameters, γ_μ and γ_k can assume both positive and negative values depending on the fluids and on specific temperatures T_w and T_∞. The limiting values for both the parameters are -2 and +2. Zero values of γ_μ, γ_k represent CFP (constant fluid properties) case whereas non-zero values of γ_μ, γ_k represent VFP (variable fluid properties) case. For the saturated liquid Mercury, at a particular reference temperature for *Pr* = 0.01 for γ_μ= 0.4, γ_k= -0.3 in the case of opposing flow.

4. SOLUTION OF THE PROBLEM

Equations (11) and (12), subject to boundary conditions (10) are solved by two methods (i) Runge-Kutta-Gill method coupled with a shooting technique and by (ii) Nachtsheim - Swigert technique. Excellent agreement is observed between the solutions obtained by the two techniques. Initially values of $f''(0)$ and '-$\theta'(0)$' are determined for different values of the parameters of the problem and then the viscous boundary layer thickness and thermal boundary layer thickness have been observed. The solutions are obtained for the fluid under consideration in the presence of magnetic interaction parameter. Much emphasis is given to opposing flow case because for negative values of ξ, two flow and heat transfer states are observed to exist and they are referred to as to as dual solutions. One of the solutions that correspond to a relatively larger value of $f''(0)$ is referred to as the upper solution and the other one as the lower solution. The transport quantities corresponding to the Mercury are presented in the table.1 below.

Table.1 Transport quantities in the presence of magnetic field for the saturated liquid Mercury *i.e.*, for Pr = 0.01, γ_μ= 0.4, γ_k= - 0.3, Na = 0.01.

ξ	$f''(0)$	$\theta'(0)$	ξ	$f''(0)$	$\theta'(0)$
0	3.479806374	-0.795810207	-0.051	0.327250583	-0.706522704
-0.01	3.005907605	-0.785069779		-0.638654405	-0.653402371
	-0.51351276	-0.481242275	-0.052	0.180380611	-0.700429398
-0.015	2.757233274	-0.779193446		-0.553364706	-0.660306411
	-0.657767656	-0.513609902	-0.053	-0.030449313	-0.690914322
-0.02	2.49882005	-0.772888269		-0.403930086	-0.670605457
	-0.768781861	-0.537753721	-0.0532	-0.101667151	-0.68744447
-0.03	1.943322477	-0.758533628		-0.344969512	-0.674228981
	-0.907856057	-0.57547908	-0.0533	-0.157486013	-0.684615083
-0.04	1.305204652	-0.74031561		-0.295276585	-0.67713482
	-0.92549359	-0.608566934	-0.05332	-0.174698524	-0.683721231
-0.05	0.450338996	-0.711358852	-0.05334	-0.200724958	-0.682349194
	-0.699972878	-0.647727657	-0.0533345	-0.212944186	-0.681696185

5. DISCUSSION OF THE RESULTS

The results are discussed both in the presence as well as in the absence of magnetic field from the representative Table.1 and from the results of the reference [1]. It is observed that in both the cases, dual solution exists. Also, the skin friction and wall heat transfer coefficient assume both positive and negative values in the presence as well as in the absence of magnetic field. The skin friction $f''(0)$ takes smaller values in the presence of magnetic field when compared to the values in the absence of magnetic field. Similar trend is exhibited with heat transfer coefficient up to certain range and opposite trend is exhibited outside this range. The critical point attained in the presence of magnetic field is faster than the critical point occurred in the absence of magnetic field. Beyond this critical point, no solution exists. As the Richardson number increases absolutely, the skin friction coefficient $f''(0)$ diminishes rapidly up to ξ = -0.053 and then increases. As a result, ξ = -0.053 become a critical point. The nature of dual solution continues its existence up to ξ = -0.0533 and unique solution exist after this range. Similarly, the Nusselt number or heat transfer coefficient '-$\theta'(0)$' diminishes with an absolute increase of Richardson number.

6. CONCLUSIONS

In opposing flow case, for the fluid under consideration, the hydrodynamic boundary layer thickness is more for lower solution than for upper solution. Boundary layer separation is observed with lower solution. Values of $f''(0)$ and '-$\theta'(0)$' of the VFP case are observed to deviate significantly from those of the CFP case and the deviations depend on the fluid and the mixed convection parameter.

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