



## Study Adaptive Backstepping Control for My Quadrotor

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### ABSTRACT

Controlling the flight of a quadrotor is a challenging task due to the complexity of the flight trajectory program and the uncertain changes in aerodynamic parameters. These factors contribute to the system's nonlinear and complicated nature. Flight path angle tracking control involves various channels, such as a pitch controller, a roll controller, and a yaw controller. Traditionally, these channels are designed using linear control methods, which may not always be satisfactory. A more effective approach to controller design is the use of backstepping. Backstepping control laws not only provide certain gain margins for the controllers but also handle model errors in the description of aerodynamics.

This paper proposes a backstepping nonlinear controller design method for quadrotor control systems. Simulation results demonstrate that the control strategy presented in this paper is effective and has strong robustness in the presence of disturbances and parameter uncertainty. The proposed approach addresses the challenges associated with traditional linear control methods and offers a more efficient and robust solution to quadrotor flight control.

**Keywords:** Quadrotor, Backstepping, nonlinear controller

### 1. Introduction

A quadrotor is an unmanned aerial vehicle that utilizes vertical take-off and landing. It consists of two pairs of propellers arranged symmetrically, with each pair rotating in opposite directions as illustrated in Figure 1. The quadrotor's structural components include mechanical, transmission, automatic control, sensing, receiving, and transmitting information systems.

The quadrotor is a control object with a dynamic model represented by nonlinear equations containing uncertain parameter components. The motion of the quadrotor can be divided into various channels such as tilt, flip, rotation, angular velocity, and angular acceleration in the three XYZ axes.

The backstepping control method is a recently developed nonlinear design method that can selectively handle nonlinear factors [1]. Adaptive backstepping control can provide convergence assessment for unknown parameters [4]. In this article, the authors conduct research on designing adaptive backstepping and backstepping controllers for quadrotors to enhance control quality, expand the working range, and increase adaptability in the presence of uncertain factors. The proposed approach offers a promising solution for improving the performance of quadrotor systems.

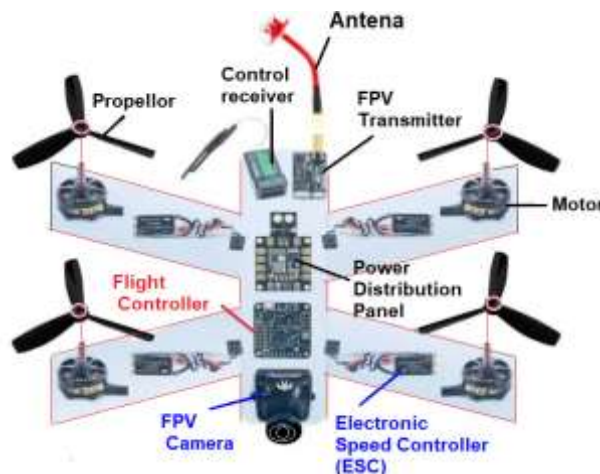


Fig.1. Basic structure of a quadrotor

## 2. Building a mathematical model for my quadrotor

The control object can be viewed as being split into two systems that work in conjunction with each other. The first system is the engine model of the propeller described by the system of equations (1), and the second system is the propeller dynamics system described by the system of equations (2.a). These two systems are related to each other through the relationship between the rotational speed  $(\omega_1, \omega_2, \omega_3, \omega_4)$  of the four motors, the lifting force  $F_z$ , and the torque  $(T_x, T_y, T_z)$ .

### 2.1. A mathematical model of the propeller engine

The most common type of motor currently applied to quadrotors is the BLDC motor. According to [4], the system of equations describing the mathematical model of a DC motor has the following form:

$$\begin{cases} j\ddot{\omega} = K_t \cdot I_a - b \cdot \dot{\omega} - T_{load} \\ V_a = R_a \cdot I_a + L_a \cdot \frac{dI_a}{dt} + K_b \cdot \omega \end{cases} \quad (1)$$

The first equation describes the relationship between the force moment and the electromagnetic moment. The second equation describes the relationship between the voltage applied to the motor and the current;  $j$  is the moment of inertia of the propeller motor;  $\ddot{\omega}$  is the angular acceleration of the motor;  $I_a$  is the supply current;  $b$  is the drag torque coefficient;  $K_t$  is the motor torque coefficient;  $T_{load}$  is the load torque;  $V_a$  is the supply voltage;  $R_a$  is the impedance of the motor;  $L_a$  is the motor inductance; and  $K_b$  is the reactance coefficient.

### 2.2. The dynamic model of the Quadrotor

To establish a mathematical model for the quadrotor, we need to consider some related coordinate systems, as shown in Figure 2. In which,  $Oxyz$  is the inertial coordinate system and  $Bxyz$  is the associative coordinate system, which is closely tied to the fixed frame of the quadrotor. Call the quadrotor's three Euler rotations around the three axes, respectively, as shown in Figure 2, the tilt angle  $\Phi$ , the pitch angle  $\theta$  and the direction angle  $\Psi$ . And  $F_1, F_2, F_3, F_4$  are the thrust caused by the four propellers, respectively.

To analyze the dynamics of the quadrotor, we consider it an absolutely rigid body moving freely through space. From there, we divide the motion of the quadrotor into two components: the translational motion of the center of mass and the rotation motion of the quadrotor about its three axes. The translational motion is caused by the drag and lift force components, while the rotational motion is caused by the force moments.

For convenience and simplicity when considering the dynamic properties of the quadrotor, we do not consider the part of the DC motor that rotates the propeller, and the control signal of the system is converted to the force as well as the torque to be generated. Therefore, the principle of controlling the quadrotor is as follows: The motion of the quadrotor is controlled through four components: the total force  $F$  along the axis  $Bz$ ; the torque  $T_x$  on the axis  $Bx$ ; the torque  $T_y$  on the  $By$  axis; and the torque  $T_z$  on the axis  $Bz$ ; and  $-F_z = U_1$ . It is the  $F$  component that makes the quadrotor changeable in height, the  $T_x$  and  $T_y$  components that cause edge motion along the  $Bx$  and  $By$  axes, and the  $T_z$  component that helps to change the direction around the  $Bz$  axis.

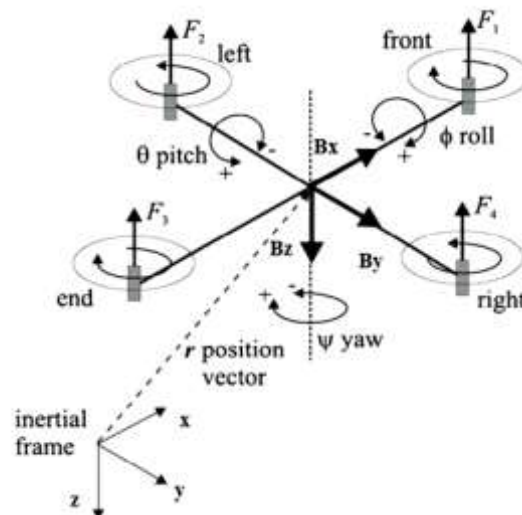


Fig.2. Coordinate systems of the quadrotor

It is assumed that the Euler rotations are small and have little effect on the rotation of the quadrotor, while ignoring the effect of the gyro effect on the quadrotor caused by the four propeller motors. We get the complete system of nonlinear kinematic equations for the quadrotor:

$$\begin{aligned}
 \ddot{\Phi} &= \dot{\Psi} \cdot \dot{\theta} \cdot \frac{J_{yy} - J_{zz}}{J_{xx}} + \frac{T_x}{J_{xx}} \\
 \ddot{\theta} &= \dot{\Psi} \cdot \dot{\Phi} \cdot \frac{J_{zz} - J_{xx}}{J_{yy}} + \frac{T_y}{J_{yy}} \\
 \ddot{\Psi} &= \frac{T_z}{J_{zz}} \\
 \ddot{x} &= (\cos\Phi \cdot \sin\theta \cdot \cos\Psi + \sin\Phi \cdot \sin\Psi) \cdot \frac{1}{m} \cdot (-F_L) \\
 \ddot{y} &= (\cos\Phi \cdot \sin\theta \cdot \sin\Psi - \sin\Phi \cdot \cos\Psi) \cdot \frac{1}{m} \cdot (-F_L) \\
 \ddot{z} &= g + \cos\Phi \cdot \cos\theta \cdot \frac{1}{m} \cdot (-F_L)
 \end{aligned}
 \tag{2a}$$

### 2.3. A quadrotor dynamics model, taking into account uncertainties

The system of equations (2.a) is the result of calculation after applying a few assumptions to simplify, such as: consider the tilt angle  $\Phi$  and the slope angle  $\theta$  as two small motion angles, eliminating the influence of gyro torque on channel  $\Phi$ . In fact, the mathematical model is much more complex with the cross-relationship between the channels  $\Phi$ ,  $\theta$  and  $\Psi$ . There is also an additional gyro-torque component. Therefore, the actual mathematical model of the object will contain uncertain components:

$$\begin{aligned}
 \ddot{\Phi} &= \dot{\Psi} \cdot \dot{\theta} \cdot \frac{J_{yy} - J_{zz}}{J_{xx}} + \frac{T_x}{J_{xx}} + f_1 \cdot \dot{\theta} + f_2 \cdot \dot{\theta} + f \\
 \ddot{\theta} &= \dot{\Psi} \cdot \dot{\Phi} \cdot \frac{J_{zz} - J_{xx}}{J_{yy}} + \frac{T_y}{J_{yy}} + f_3 \cdot \dot{\Phi} + f_4 \cdot \dot{\Phi} + f \\
 \ddot{x} &= (\cos\Phi \cdot \sin\theta \cdot \cos\Psi + \sin\Phi \cdot \sin\Psi) \cdot \frac{1}{m} \cdot (-F_L) - f_x \cdot \dot{x} \\
 \ddot{y} &= (\cos\Phi \cdot \sin\theta \cdot \sin\Psi - \sin\Phi \cdot \cos\Psi) \cdot \frac{1}{m} \cdot (-F_L) - f_y \cdot \dot{y} \\
 \ddot{z} &= g + \cos\Phi \cdot \cos\theta \cdot \frac{1}{m} \cdot (-F_L) + f_z
 \end{aligned}
 \tag{2b}$$

### 3. Designing a controller for my quadrotor

Below, the author synthesizes adaptive backstepping and backstepping controllers as follows:

The main content of the synthesis of the backstepping controller is to give the control law for each control channel, provided that the parameters in the kinematic model of the quadrotor are clear. Below are the steps to synthesize the backstepping controller according to the z-height channel. The synthesis of the backstepping controller for x and y channels and according to tilt angle  $\Phi$ , direction angle  $\Psi$ , pitch angle  $\theta$  is completely similar.

For the z channel, we have a system of kinematic equations:

$$\ddot{z} = g + \cos\Phi \cdot \cos\theta \cdot \frac{1}{m} \cdot (-F_L)$$

Set  $X_1 = z$ ;  $X_2 = \dot{z}$ ; and  $F_L = U_1$ , so  $\dot{X}_1 = \dot{z}$  and  $\dot{X}_2 = \ddot{z}$ . From this, the model of tight backpropagation along the z-channel is:

$$\begin{cases} \dot{X}_1 = X_2 \\ \dot{X}_2 = g + \cos\Phi \cdot \cos\theta \cdot \frac{1}{m} \cdot (-U_1) \end{cases}
 \tag{3}$$

Call the desired altitude signal  $z_d$ , and  $X_1 = z$ .

Step 1: Set  $Z_1 = X_1 - z_d$ .

So  $\dot{Z}_1 = \dot{X}_1 - \dot{z}_d = X_2 - \dot{z}_d$ .

Select the control function Lyapunov  $V_1(Z_1) = \frac{1}{2} Z_1^2$ ; the condition for signal  $X_1 \rightarrow z_d$  is:

$$\dot{V}_1(Z_1) = Z_1 \cdot \dot{Z}_1 = Z_1 \cdot (X_2 - \dot{z}_d) < 0 \quad \forall Z_1 \neq 0$$

Considering  $x_2$  is a virtual control signal, deducing the virtual control law to ensure the stability criterion of Lyapunov is:  $X_{2d} = -C_1 \cdot Z_1 + \dot{z}_d$  with  $C_1 > 0$ .

Step 2: Set  $Z_2 = X_2 - z_{2d}$ . So

$$\ddot{Z}_2 = \ddot{X}_2 - \ddot{X}_{2d} = g + \cos\phi \cdot \cos\theta \cdot \frac{1}{m} \cdot (-U_1) + C_1 \cdot \ddot{Z}_1 - \ddot{z}_{2d}$$

Inferred  $\dot{V}_1(Z_1) = Z_1 \cdot (-C_1 \cdot Z_1 + Z_2)$

Select the control function Lyapunov  $V_2(Z_1, Z_2) = V_1 + \frac{1}{2}Z_2^2$ ; the condition for signal  $X_2 \rightarrow X_{2d}$  is:

$$\dot{V}_2 = \dot{V}_1 + Z_2 \cdot \dot{Z}_2 < 0 \quad \text{" } Z_1, Z_2 \rightarrow 0$$

$$\ddot{V}_2 = -C_1 \cdot Z_1^2 + Z_1 \cdot Z_2 + Z_2 \cdot \left( g + \cos\phi \cdot \cos\theta \cdot \frac{1}{m} \cdot (-U_1) + C_1 \cdot \ddot{Z}_1 - \ddot{z}_{2d} \right)$$

From the Lyapunov stability criterion, to ensure the system is globally asymptotically stable, the control law U1 has the following form:

$$U_1 = m \frac{1}{\cos\phi \cdot \cos\theta} \left( Z_1 + g + C_1 \cdot \ddot{Z}_1 - \ddot{z}_{2d} + C_2 \cdot Z_2 \right)$$

Ignoring the higher-order derivatives, we get that the control law for the z channel is:

$$U_z = m \frac{1}{\cos\phi \cdot \cos\theta} Z_1 + g + C_2 \cdot Z_2 + C_1 \cdot Z_2 - C_1^2 \cdot Z_1$$

Similarly, we have the control law for the channels x, y, and  $\Phi, \theta, \Psi$  as:

$$U_x = m \frac{1}{U_z} (Z_3 + C_4 \cdot Z_4 + C_3 \cdot Z_4 - C_3^2 \cdot Z_3)$$

$$U_y = m \frac{1}{U_z} (Z_5 + C_6 \cdot Z_6 + C_5 \cdot Z_6 - C_5^2 \cdot Z_5)$$

$$U_\phi = J_{xx} \cdot \left( -Z_7 - \Psi \cdot \theta \cdot \frac{J_{yy} - J_{zz}}{J_{xx}} - C_8 \cdot Z_8 - C_7 \cdot Z_8 + C_7^2 \cdot Z_7 \right)$$

$$U_\theta = J_{yy} \cdot \left( -Z_9 - \Psi \cdot \phi \cdot \frac{J_{zz} - J_{xx}}{J_{yy}} - C_{10} \cdot Z_{10} - C_9 \cdot Z_{10} + C_9^2 \cdot Z_9 \right)$$

$$U_\psi = J_{zz} \cdot (-Z_{11} - C_{12} \cdot Z_{12} - C_{11} \cdot Z_{12} + C_{11}^2 \cdot Z_{11})$$

where  $C_i$  are positive design constants.

#### 4. Backstepping controller design for propeller motor

As analyzed, the control of the quadrotor must be through the actuators, which are DC motors with propellers. The control quality of the quadrotor is directly affected by the control quality of these four motors. As with these motors, the faster the response, the better the maneuverability of the quadrotor, and vice versa.

The dynamic model of the motor used in this study is an independently excited DC motor. The author also applies the backstepping control algorithm to stabilize the motor's rotation speed in accordance with the required load torque.

The system of dynamic equations for a DC motor, according to (1), is:

$$\begin{cases} j\dot{\omega} = K_t \cdot I_a - b \cdot \omega - T_{load} \\ V_a = R_a \cdot I_a + L_a \cdot \frac{dI_a}{dt} + K_b \cdot \omega \end{cases}$$

Putting  $X_1 = \omega$ ,  $X_2 = I_a$ ,  $U = V_a$ , converting the above system to tight backpropagation, we get:

$$\begin{cases} \dot{X}_1 = -\frac{b}{j} \cdot X_1 - \frac{K_M}{j} \cdot X_1^2 + \frac{K_t}{j} \cdot X_2 \\ \dot{X}_2 = -\frac{K_b}{L_a} \cdot X_1 - \frac{R_a}{L_a} \cdot X_2 + \frac{1}{L_a} U \end{cases}$$

Let the desired rotational speed of the motor be  $\omega_d$ . Set  $Z_1 = X_1 - \omega_d$ , when the rotation speed signal  $\omega \rightarrow \omega_d$ , then  $Z_1 \rightarrow 0$ . On the other hand, with  $\dot{X}_1 = \dot{Z}_1 + \dot{\omega}_d$ , substituting into the first equation of the system, we have:

$$\dot{Z}_1 = -\frac{b}{j} \cdot X_1 - \frac{K_M}{j} \cdot X_1^2 + \frac{K_t}{j} \cdot X_2 - \dot{\omega}_d$$

Similar to applying the backstepping algorithm to the state channels discussed above, we need a virtual control signal,  $X_2 \rightarrow X_{2d}$ . Finally, according to the Lyapunov asymptotic stability criterion, for the system to be globally stable, the necessary control signal U has the following form:

$$U = L_a \cdot \left[ -\frac{K_t}{j} \cdot Z_1 - f_2(Z_1, Z_2) - C_2 \cdot Z_2 \right]$$

In which:  $C_1, C_2 > 0$

$$f_2(Z_1, Z_2) = -\left[\frac{b}{K_t} + \frac{K_M}{K_t} \cdot 2 \cdot X_1 - \frac{j}{K_t} C_1\right] \cdot \left[-C_1 \cdot Z_1 + \frac{K_t}{j} \cdot Z_2\right] - \frac{K_b}{L_a} X_1 - \frac{R_a}{L_a} \cdot X_2$$

Each motor should have a rotary speed stabilizer, as shown above.

## 5. Adaptive Backstepping Controller Design for Quadrotor

### 5.1. Consider the $\Phi$ -angle channel

The mathematical model of the channel  $\Phi$  when considering the uncertainty components in the tight backpropagation form is as follows:

$$\begin{cases} \overset{\square}{X}_7 = X_8 \\ \overset{\square}{X}_8 = \psi \cdot \theta \cdot \frac{J_{yy} - J_{zz}}{J_{xx}} + f_1 \cdot \theta + f_2 \cdot \theta + f + \frac{U_2}{J_{xx}} \end{cases}$$

Let the desired tilt angle signal  $\Phi$  be  $\Phi_d$ .

Set  $Z_7 = X_7 - \Phi_d$ , if  $X_7 \rightarrow \Phi_d$  then  $Z_7 \rightarrow 0$ .

Set  $Z_8 = X_8 - X_{8d}$ , if  $X_8 \rightarrow X_{8d}$  then  $Z_8 \rightarrow 0$ .

Transforming the above system in terms of  $Z_7$  and  $Z_8$ , we get:

$$\begin{cases} \overset{\square}{Z}_7 = X_8 - \Phi_d \\ \overset{\square}{Z}_8 = \psi \cdot \theta \cdot \frac{J_{yy} - J_{zz}}{J_{xx}} + f_1 \cdot \theta + f_2 \cdot \theta + f + \frac{U_2}{J_{xx}} - X_{8d} \end{cases}$$

From the results of the synthesis of the backstepping control law for channel  $\Phi$ , done in the previous section, the adaptive control law has the form:

$$U_\Phi = J_{xx} \cdot \left(-Z_7 - \psi \cdot \theta \cdot \frac{J_{yy} - J_{zz}}{J_{xx}} - C_7 \cdot Z_8 + C_7^2 \cdot Z_7 - C_8 \cdot Z_8 - \hat{f}_1 \cdot \theta - \hat{f}_2 \cdot \theta - \hat{f}\right)$$

where  $\hat{f}_1$  and  $\hat{f}_2$  are estimates of the uncertainty component.

Let the estimated errors of  $f_1$ ,  $f_2$ , and  $f_3$  be:  $\tilde{f}_1 = f_1 - \hat{f}_1$ ;  $\tilde{f}_2 = f_2 - \hat{f}_2$ ;  $\tilde{f} = f - \hat{f}$

Putting the estimated error components into the Lyapunov control function component:

$$V_8(Z_7, Z_8, \tilde{f}_1, \tilde{f}_2, \tilde{f}) = V_8(Z_7, Z_8) + \frac{1}{2 \cdot \gamma_1} \cdot \tilde{f}_1^2 + \frac{1}{2 \cdot \gamma_2} \cdot \tilde{f}_2^2 + \frac{1}{2 \cdot \gamma} \cdot \tilde{f}^2$$

where  $\gamma_1, \gamma_2, \gamma$  are the adaptation coefficients. If we differentiate with respect to time, we get:

$$\overset{\square}{V}_8 = -C_7 Z_7^2 + Z_8 \cdot \left[ Z_7 + \psi \cdot \theta \cdot \frac{J_{yy} - J_{zz}}{J_{xx}} + f_1 \cdot \theta + f_2 \cdot \theta + f + \frac{U_2}{J_{xx}} + C_7 \cdot Z_7 \right] + \frac{1}{\gamma_1} \cdot \tilde{f}_1 \cdot \dot{\tilde{f}}_1 + \frac{1}{\gamma_2} \cdot \tilde{f}_2 \cdot \dot{\tilde{f}}_2 + \frac{1}{\gamma} \cdot \tilde{f} \cdot \dot{\tilde{f}}$$

Substituting  $U_\Phi$ , get:

$$\overset{\square}{V}_8 = -C_7 Z_7^2 + Z_8 \cdot \left[-C_8 \cdot Z_8 + \tilde{f}_1 \cdot \theta + \tilde{f}_2 \cdot \theta + \tilde{f}\right] + \frac{1}{\gamma_1} \cdot \tilde{f}_1 \cdot \dot{\tilde{f}}_1 + \frac{1}{\gamma_2} \cdot \tilde{f}_2 \cdot \dot{\tilde{f}}_2 + \frac{1}{\gamma} \cdot \tilde{f} \cdot \dot{\tilde{f}}$$

Using the same reasoning as in the non-adaptive case, in order to ensure that the system is globally asymptotically stable according to the Lyapunov criterion, deduce the correction rule for the estimation of the uncertain parameters:

$$\begin{cases} \frac{1}{\gamma_1} \cdot \tilde{f}_1 + Z_8 \cdot \theta = 0 \\ \frac{1}{\gamma_2} \cdot \tilde{f}_2 + Z_8 \cdot \theta = 0 \\ \frac{1}{\gamma} \cdot \tilde{f} + Z_8 = 0 \end{cases} \Rightarrow \begin{cases} \dot{\tilde{f}}_1 = -Z_8 \cdot \dot{\theta} \cdot \gamma_1 \\ \dot{\tilde{f}}_2 = -Z_8 \cdot \dot{\theta} \cdot \gamma_2 \\ \dot{\tilde{f}} = -Z_8 \cdot \dot{\gamma} \end{cases} \Rightarrow \begin{cases} \dot{\hat{f}}_1 = Z_8 \cdot \dot{\theta} \cdot \gamma_1 \\ \dot{\hat{f}}_2 = Z_8 \cdot \dot{\theta} \cdot \gamma_2 \\ \dot{\hat{f}} = Z_8 \cdot \dot{\gamma} \end{cases}$$

### 5.2. Considering the $\theta$ -angle channel

The same reasoning as above, we have the adaptive backstepping control law for channel  $\theta$  as follows:

$$U_\theta = J_{yy} \cdot \left(-Z_9 - \psi \cdot \Phi \cdot \frac{J_{zz} - J_{xx}}{J_{yy}} - C_{10} \cdot Z_{10} - C_9 \cdot Z_{10} + C_9^2 \cdot Z_9 - \hat{f}_3 \cdot \Phi - \hat{f}_4 \cdot \Phi - \hat{f}\right)$$

The correction law is defined as follows:

$$\begin{cases} \hat{f}_3 = Z_{10} \cdot \hat{\Phi} \cdot \gamma_3 \\ \hat{f}_4 = Z_{10} \cdot \hat{\Phi} \cdot \gamma_4 \\ \hat{f} = Z_{10} \cdot \gamma \end{cases}$$

Where  $\gamma_3, \gamma_4, \gamma$  are the adaptation coefficients.

5.3. Consider the channels x, y, and z

The same reasoning as above, we have adaptive backstepping control laws for x, y, and z channels as follows:

$$U_x = m \frac{1}{U_1} \left( Z_3 + C_4 \cdot Z_4 + C_3 \cdot Z_4 - C_3^2 \cdot Z_3 - \hat{f}_x \cdot (-C_3 \cdot Z_3 + Z_4) \right)$$

$$U_y = m \frac{1}{U_1} \left( Z_5 + C_6 \cdot Z_6 + C_5 \cdot Z_6 - C_5^2 \cdot Z_5 - \hat{f}_y \cdot (-C_5 \cdot Z_5 + Z_6) \right)$$

$$U_z = m \frac{1}{\cos\Phi \cdot \cos\theta} \left( Z_1 + g + C_2 \cdot Z_2 + C_1 \cdot Z_2 - C_1^2 \cdot Z_1 + \hat{f}_z \right)$$

The correction law is defined as follows:

$$\hat{f}_x = (-Z_4^2 + C_3 \cdot Z_3 \cdot Z_4) \cdot \gamma_x;$$

$$\hat{f}_y = (-Z_6^2 + C_5 \cdot Z_5 \cdot Z_6) \cdot \gamma_y;$$

$$\hat{f}_z = Z_2 \cdot \gamma_z$$

Where  $\gamma_x; \gamma_y; \gamma_z$  are the adaptation coefficients.

6. Simulation results

The parameters of the motor and the initial parameters are selected as follows:

$j=6.10^{-5}$  kg.m<sup>2</sup>;  $b=3,36.10^{-5}$ kg.m<sup>2</sup>/s;  $K_t=0,0052$ ;  $R_a=0,9\Omega$ ;  $L_a=2$ mH;  $b=0,0057$ ;  $K_m=7,5.10^{-7}$ ;  $K_f=3,13.10^{-5}$ ;  $m=0,65$ kg;  $g=9,81$ m/s<sup>2</sup>;  $i_{xx}=7,5.10^{-3}$  kg.m<sup>2</sup>;  $i_{yy}=7,5.10^{-3}$ kg.m<sup>2</sup>;  $i_{zz}=1,3.10^{-2}$  kg.m<sup>2</sup>;  $L=0,23$ m;  $u_{10}=m \cdot g$  N;

6.1. Backstepping controller review

The backstepping control diagram is as follows:

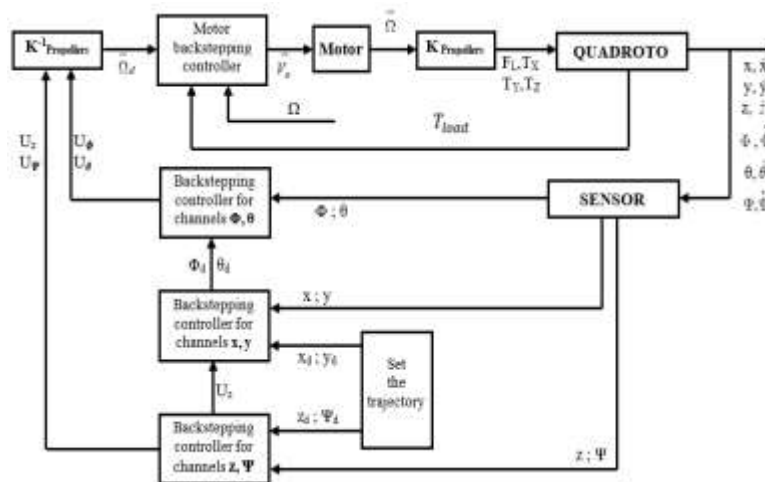


Fig.3. Backstepping control diagram for quadrotor

Consider the stable mode of all 3 parameters  $x$ ,  $y$ , and  $z$  of the quadrotor at the same time to evaluate the control quality of backstepping when there is a phenomenon of crossover channels.

The wanted input signals ( $x_d$ ,  $y_d$ , and  $z_d$ ) all have the form of a step function, which is fed into the system at the same time. Through simulation and comparison, it can be concluded that the control quality of the backstepping orbital stabilizer is superior to that of the PID orbital stabilizer, which is shown by:

- First, the control quality in the backstepping transition is better than that of the PID, as demonstrated by the shorter transient time, less oscillation, and smaller overshoot.

- Second, the working band on the backstepping channels is wider than that of the PID, which increases the quadrotor's mobility in trajectory control and tracking, and at the same time helps the quadrotor not to be unstable when encountering large disturbances causing changes. sudden change in the kinematic parameters of the quadrotor.

- Third, backstepping still ensures control quality in cases of crossover channels.

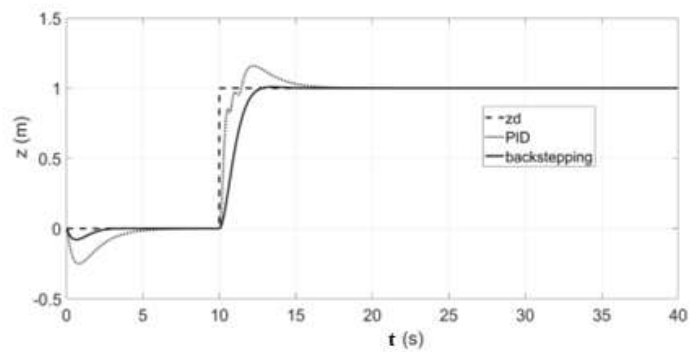


Fig.4. Control results on channel Z

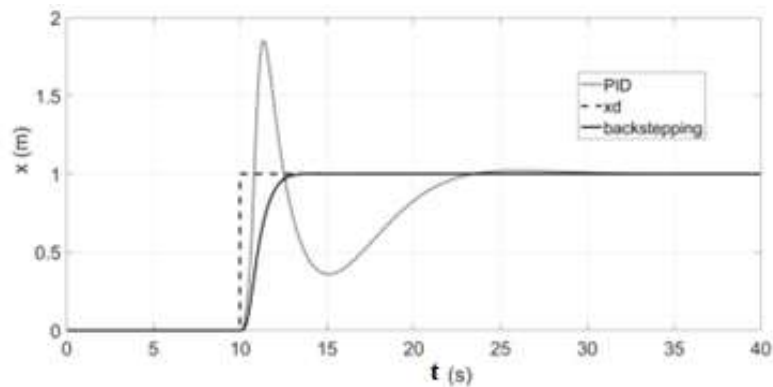


Fig.5. Control results on channel X

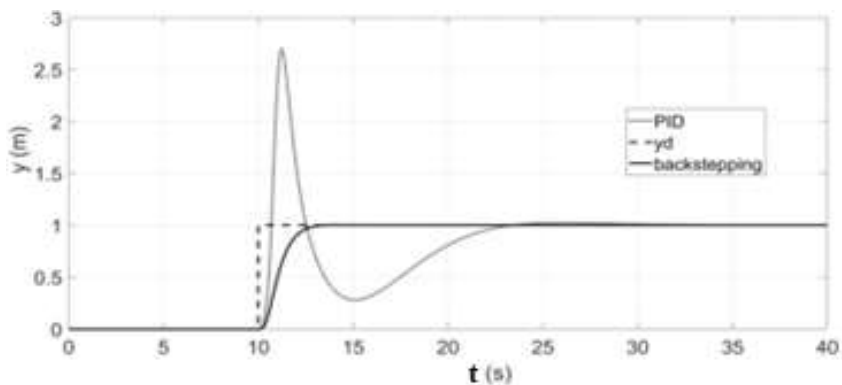


Fig.6. Control results on channel Y

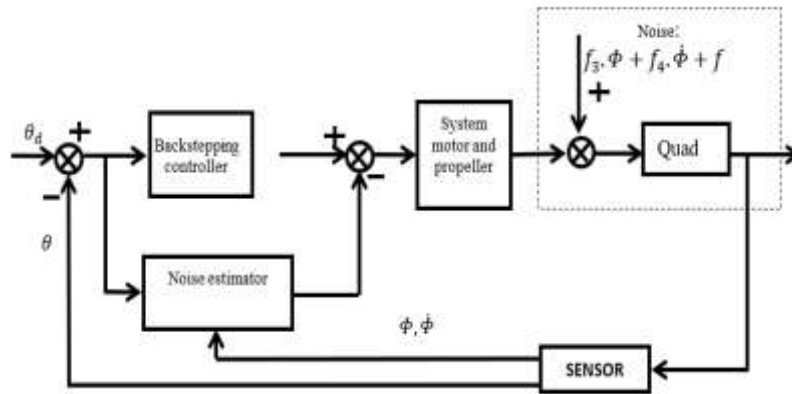
**6.2. Adaptive backstepping controller evaluation**

In the above, a mathematical model of a quadrotor has been given with possible uncertain components due to the existence of an uncertain dynamic component, changes in the weight of the quadrotor, noise, or drag... Below, the author analyzes the effect of an adaptive backstepping orbital stabilizer on control quality on channels  $\theta$  and X when there is noise on channel  $\theta$ . The angle channel  $\Phi$ , y channel, and z channel can be done similarly.

The mathematical model of channel  $\theta$  taking into account the uncertainty component has the following form:

$$\ddot{\theta} = \frac{J_{zz} - J_{xx}}{J_{yy}} \dot{\Phi} + \frac{T_y}{J_{yy}} + f_3 \cdot \Phi + f_4 \cdot \dot{\Phi} + f$$

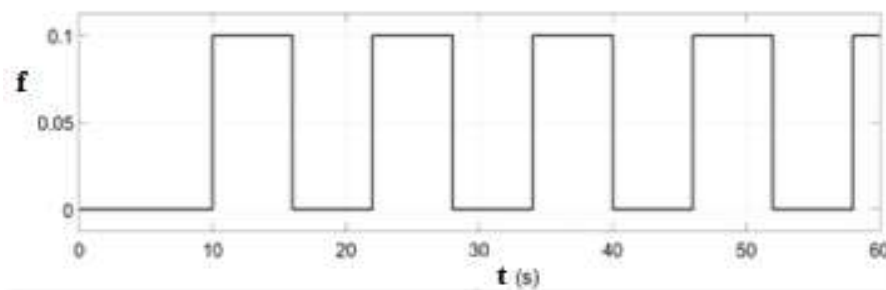
The adaptive backstepping control scheme for the Picht channel  $\theta$  is depicted in Figure 7 below.



**Fig.7.** Adaptive backstepping control diagram for the  $\theta$ -channel

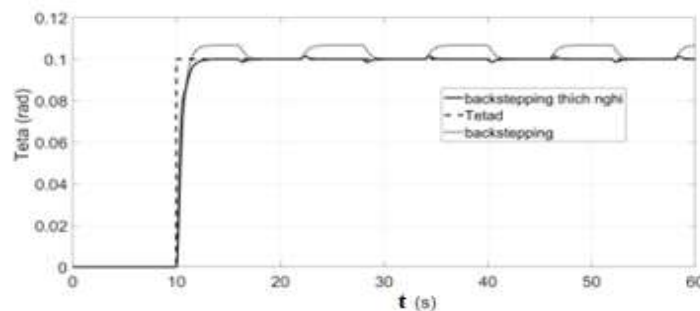
To evaluate the control quality of the system when applying adaptive backstepping, the author conducts a survey in the following cases:

- *Case 1:* Consider the impact noise as an uncertain component  $f$  on channel  $\theta$  with pulse shape as shown in Figure 8. The input signal of channel  $\theta$  is  $\theta_d$  in the form of a step function.



**Fig.8.** Noise in the form of pulses

Figure 9 depicts the control results on  $\theta$ -channel when available uncertainty noise in the form of pulses.



**Fig.9.** The control results on  $\theta$ -channel when available uncertainty noise in the form of pulses

- *Case 2:* Consider the impact noise as an uncertain component  $f$  on channel  $\theta$  with the form of a sine function, with the value shown in Figure 10. The input signal of channel  $\theta$  is  $\theta_d$  in the form of a step function.



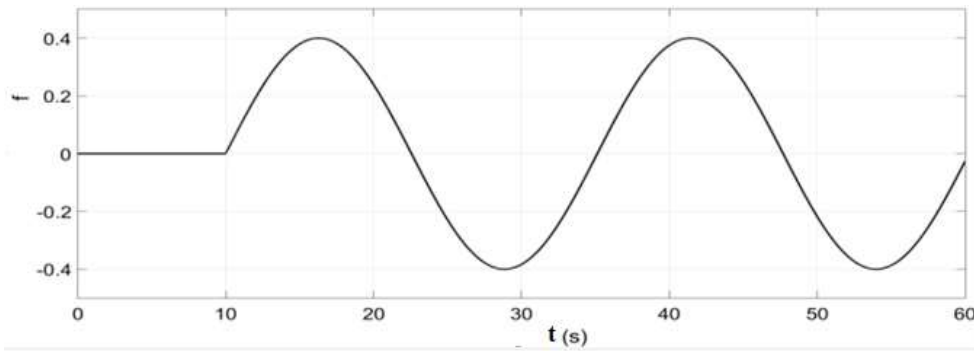


Fig.10. Sinusoidal noise

Figure 11 depicts the control results on  $\theta$ -channel when available uncertainty noise in the form of Sinusoidal.

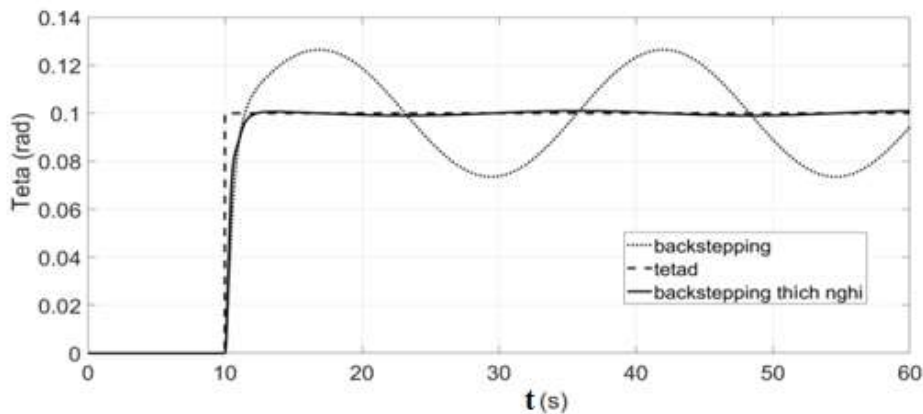


Fig.11. The control results on  $\theta$ -channel when available uncertainty noise in the form of Sinusoidal

- Case 3: Consider the impact noise as the uncertainty component caused by the gyro torque  $-J_r \cdot \ddot{\Phi} \cdot \Omega_r / J_{yy}$  acting on channel  $\theta$ . The author investigates the adaptive backstepping controller in the following modes: On the  $Oy$  axis, the control quadrotor oscillates around the equilibrium point  $y_0=0$ . The input signal  $y_d$  is pulsed. On the  $Ox$  axis, the quadrotor control stabilizes at  $x_0=0$ .

Theoretically, when there is movement on the  $Oy$  axis, there will be a change in rotation angle  $\Phi$  and rotation angle speed  $\dot{\Phi}$ . On the other hand, the angular momentum of the propeller system occurs due to the difference in rotational speeds of the two pairs of propellers that are always rotating in opposite directions. The angular momentum of the propeller system is calculated as follows:  $\vec{H} = J_r \cdot \vec{\Omega}_r$

The angular momentum combined with the variation of the tilt angle  $\Phi$  gives rise to a gyro moment affecting the channel  $\theta$ . The gyro moment is calculated as follows:

$$\vec{M}_{gi} = \vec{H} \cdot \ddot{\Phi}$$

Notice that the  $\vec{M}_{gi}$  vector has a similar form to the considered uncertainty component  $f_3 \cdot \dot{\Phi}$ . This uncertainty component acts on the channel  $\theta$ , leading to a change in  $\theta$  and a variation of  $x$  around the equilibrium position  $x_0$ .

Figure 12 depicts the variation of the gyro torque acting on the channel  $\theta$ .

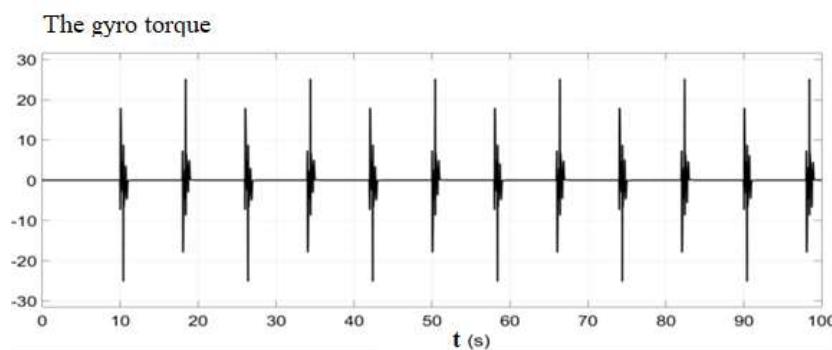
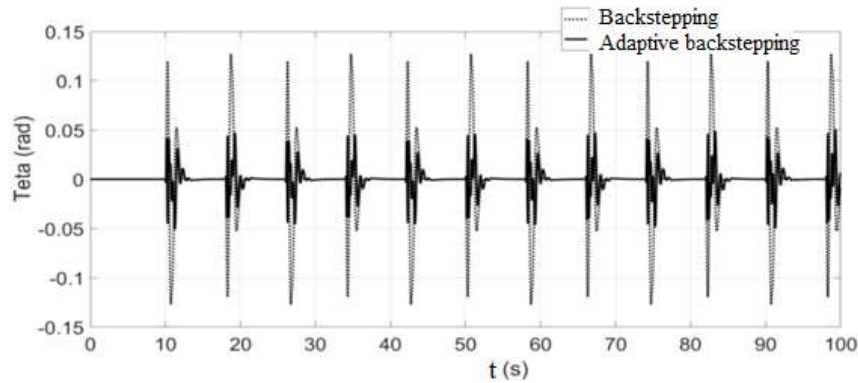


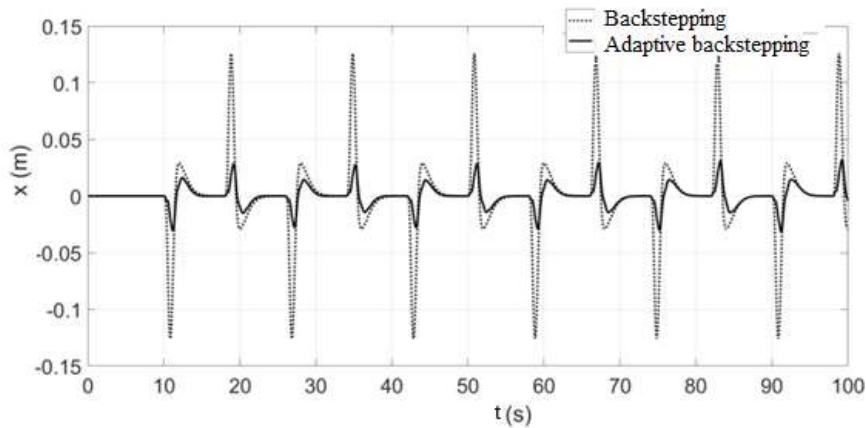
Fig.12. The gyro torque acting on the channel  $\theta$

Figure 13 depicts the variation of the angle  $\theta$  due to the influence of the gyroscope torque.



**Fig.13.** The variation of the angle  $\theta$  due to the influence of the gyroscope torque

Due to the variation of the pitch angle  $\theta$ , the quadrotor oscillates around the equilibrium position  $x_0$  on the  $0x$  axis, as depicted in Figure 14.



**Fig.14.** Variation of  $x$  due to influence of gyro torque

From the results in Figures 13 and 14, it is shown that in the orbital stabilization mode at  $x_0=0$ , when affected by the gyro torque, the adaptive backstepping controller gives better results than the backstepping controller. This is shown by the fact that the amplitude of oscillation on the  $x$  channel is significantly reduced.

#### 4. Conclusion

In this study, the authors have made a new contribution by dividing the mathematical modeling of the quadrotor system into two systems that are bound together, resulting in a closer representation of reality. The first system is characterized by a nonlinear mathematical model describing quadrotor kinematics, while the second system is characterized by a mathematical model of a DC motor and the relationship between propeller thrust and rotational speed. This is an important mathematical basis that ensures the synthesis of quality adaptive backstepping and backstepping controllers.

The simulation results have shown the outstanding advantages of the backstepping controller compared to the PID controller, namely the fast setting time and maneuverability of the quadrotor at a larger working range. Additionally, the application of an adaptive backstepping controller to stabilize the trajectory produces better results than a conventional backstepping controller. It helps to suppress the influence of uncertain components and reduce the impact of noise during quadrotor operation.

When dealing with strong nonlinear systems such as quadrotors, the control law is usually designed using the adaptive backstepping method, which is capable of compensating for uncertainties in the model and improving the stability of the control system. However, one limitation of the adaptive backstepping method is that the calculation of the virtual control law can be very complex, and control law synthesis is only possible under suitable conditions. On the other hand, the sliding controller is invariant and stable to internal uncertainties and external disturbances. Therefore, combining the sliding control mode with the backstepping technique can provide an effective solution for developing a robust controller for strict-feedback nonlinear systems with a simple design procedure that takes advantage of the benefits of both methods. This is an area of focus for the authors' future research.

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**ACKNOWLEDGEMENT**

*This work is supported by: Department of Information Technology, Faculty of General Education, University of Labour and Social Affairs in Hanoi Vietnam. Faculty of Fundamental Technics, AD-AF Academy of Viet Nam.*

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