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Modification of Ratio Estimators for Finite Population Variance

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ABSTRACT

This research work is to improve the efficiency and reduce the biasedness of some ratio estimators of population variance under transformation of sample variance we discovered that alternative to yadav and pandy (2013) and olufadi and kadilar (2014) ratio estimators for finite population variance based on the transformation of auxiliary variables were proposed. The properties (biases and, mean square errors) up to first order approximation of the modified estimators were obtained using Taylor's series expansion technique. Also the conditions for the efficiency of the modified estimators over some existing estimators are better are more efficient than the confessional.

1.1 GENERAL INTRODUCTION

Sampling is a method or technique of drawing sample from the population. It is used whenever the population is large and the complete enumeration is very time consuming and costly. Parameters of the population are estimated through their appropriate estimators using the information supplied by the sample and their large sample properties are studied up to a certain order of approximation.

The problem of estimating the population variance assumes importance in various fields such as industry, agriculture, medical and biological sciences etc. In sample surveys, auxiliary information on the finite population under investigation is quite often available from previous experience, census or administrative databases. It is well known that the auxiliary information in the theory of sampling is used to increase the efficiency of the estimators of the parameters such as mean or total, variance, coefficient of variation etc. Out of many, ratio and regression methods of estimation are good examples in this context. In many situations of practical importance, the problem of estimating the population variance $\boldsymbol{S}_y^2$of the study variable $y$deserves special attention.

Variations are present everywhere in our daily life. It is the law of nature that no two things or individuals are exactly alike. For instance, a physician needs a full understanding of variations in the degree of human blood pressure, body temperature, and pulse rate for adequate prescription. A manufacturer needs constant knowledge of the level of variations in people's reaction to his product to be able to know whether to reduce or increase his price or improve the quality of his product. An agriculturist needs an adequate understanding of the variations in climatic factors especially from place to place (or time to time) to be able to plan on when, how, and where to plant his crop.

The use of auxiliary information increases the precision of an estimator when study variable Y is highly correlated with auxiliary variable X. When the variable under study y is highly positively correlated with the auxiliary variable x, then the ratio type estimators are used to estimate the population parameter and product estimators are used when the variable under study y is highly negatively correlated with the auxiliary variable x for improved estimation of parameters of variable under study. But there are situations when information on auxiliary variable is not available in quantitative form but in practice, the information regarding the population proportion possessing certain attribute ψ is easily available (Jhajj et.al.1980), which is highly correlated with the study variable Y.

Estimation of population variance is significantly important in the theory of estimation. Efficient variance estimation under auxiliary information has been widely discussed by various authors such as Das and Tripathi (1978), Srivenkatramana (1980), Isaki (1983), Singh et al. (1988), Singh and Katyar (1991), Rao and Shao (1992), Sarndal (1992), Agrawal and Sthapit (1995), Rao and Sitter (1995), Garcia and Cebrain (1996), Arcos et al. (2005), Kadilar and Cingi (2006, 2006a), Solanki and Singh (2013),Yadav&Kadilar (2013),Olufadi&Kadilar (2014), Adewara (2005) and Adewara (2006).

1.2 DEFINITIONS OF TERMS USED

- Population: Is the entire set of individuals or objects to which findings of a survey to be made.

- Sample: Is any part or subset of a population.
- Sampling: Is the procedure for selection of a part or function of a population and observing the selected part with respect to some property of interest and then drawing some conclusions about the population.
- Sampling unit (element): Is a single individual or object of a population whose characteristics are to be measured.
- Sampling frame: Is a list or map or any accepted material from which sample is selected.
- Variable: An observation quantitative characteristic of an elementary unit that vary from unit to unit.
- Auxiliary information (Variable): Is a variable which is not variable of interest in a survey but its employed to improve the sampling and enhance estimation of the variable of interest.
- Population parameter: Is any function of the values of all population units.
- Statistic: Is any function of values of sample unit.
- Correlation: Is the measure of the degree of dependence or association between two variables.
- Estimate: A judgment or calculation of approximately how large or how great something is.
- Ratio estimators: Is used when the auxiliary variable X correlated with study variable Y.
- Bias: The bias of an estimator is the difference between estimator's expected value and the true value of the parameters being estimated.
- Mean square error: The mean square error (MSE) of an estimator is one of many ways to quantify the difference between values implies by an estimator and the true values of the quantity being estimated.

2.1 LITERATURE REVIEW

In this section, we will briefly review the literature of some ratio estimators for finite population variance, which we have used in the present study.

The use of auxiliary information in the estimation of population values of the study variable has been a common phenomenon in the sampling theory of surveys. Auxiliary information may be fruitfully utilized either at the planning stage or at the design stage or at the information stage to arrive at an improved estimator compared to those, not utilizing auxiliary information. The use of auxiliary information for forming ratio and the regression method of estimation were introduced during the 1930's with a comprehensive theory provided by Cochran.

In 1942, Cochran made a particular important contribution to the modern sampling theory by suggesting methods of utilizing the auxiliary information for the purpose of estimation of the population mean in order to increase the precision of the estimates (Cochran 1942). It is common experience that uses of auxiliary information gives rise to some estimators that are known today, and in under certain conditions, these estimators are more efficient than the estimators based on simple random sampling. The use of auxiliary information in the estimation of population values of the study variable has been a common phenomenon in the sampling theory of surveys.

Let $\Omega = (1\ 2, 3 \ldots N)$be a population of size NandY, X be two real valued functions having values$(Y_i, X_i) \in \mathbb{R}^+ > 0$ on the i^{th} unit of $U(1 \leq i \leq N)$.. Let $\bar{Y}$ and $\bar{X}$ be the population means of Y and X respectively with C_y and C_x as coefficients of variation Y and$X$. We assume positive correlation

$\rho > 0$ between the study variable $Y$and auxiliary variable XLet S_y^2 and S_x^2be the finite population variance of Y and Xrespectively and s_y^2 and s_x^2be respective sample variances based on the random sample of size $n$drawn without replacement.

Isaki (1983)suggtesed ratio type estimators for population variance of the study as:

$$t_1 = s_y^2 \left(\frac{S_x^2}{s_x^2} \right) \qquad 2.1$$

$$MSE\ (t_1) = fS_y^4[(\lambda_{40} - 1) + (\lambda_{04} - 1) - 2(\lambda_{22} - 1)] \qquad 2.2a$$

$$MSE\ (t_1) = fS_y^4[(\lambda_{40} - 1) + (\lambda_{04} - 1)(1 - 2C)] \qquad 2.2b$$

$$where\ C = \frac{(\lambda_{22} - 1)}{(\lambda_{04} - 1)}$$

Upadhyaya and Singh (1986) suggested an alternative estimator forS_y^2 as

$$t_2 = s_y^2 \left(\frac{s_x^{*2}}{S_x^2} \right) \qquad 2.3$$

Where $s_x^{*2} = \frac{NS_x^2 - ns_x^2}{N-n} = (1 + g)S_x^2 - gs_x^2\ with\ g = \frac{n}{N-n}$

with the mean squared error (MSE) of the estimator t_2 up to the first order of approximation as

$$MSE\ (t_2) = fS_y^4[(\lambda_{40} - 1) + g^2(\lambda_{04} - 1) - 2g(\lambda_{22} - 1)] \quad 2.4a$$

$$MSE\ (t_2) = fS_y^4[(\lambda_{40} - 1) + g(\lambda_{04} - 1)(g - 2C)] \quad 2.4b$$

$$where\ C = \frac{(\lambda_{22} - 1)}{(\lambda_{04} - 1)}$$

Yadav and Pandey (2016)suggested the following ratio-cum-dual to ratio type estimator for the population variance of the study variable as

$$t_3 = s_y^2\left[\alpha\left(\frac{S_x^2}{s_x^2}\right) + (1 - \alpha)(\left(\frac{s_x^{*2}}{S_x^2}\right)\right] \quad 2.5$$

whereα is suitably chosen constant to be determined such that MSE of the estimator $t_R^d$is minimum, For$\alpha = 1$. We get the minimum $MSE\ (t_R^d)$ as

$$MSE_{min}(t_3) = fS_y^4\left[(\lambda_{40} - 1) - \frac{(\lambda_{22} - 1)^2}{(\lambda_{04} - 1)}\right] \quad 2.6$$

which is equal to the variance of the linear regression estimator of population variance, $\hat{S}_{lr}^2 = s_y^2 + b(S_x^2 - s_x^2)$with the sample regression coefficient$b = \frac{s_y^2(\lambda_{22}-1)}{s_x^2(\lambda_{04}-1)}$

Srivenkataramana and Tracy (1980) gave a *dual to ratio* estimator for variance estimator in sample surveys. Under non-response it can be modified as:

$$t_4 = s_{y(r)}^2\left(\frac{S_{x(N)}^2 - fs_{x(r)}^2}{(1 - f)S_{x(N)}^2}\right) \quad 2.7$$

The M.S.E of t_{SV} is given by:

$$MSE\ (t_4) = M_1S_y^4[(\lambda_{40} - 1) + g^2(\lambda_{04} - 1) - 2g(\lambda_{22} - 1)] \quad 2.8$$

Where $g = \frac{f}{1-f}$

Yadav and Kadilar (2013)suggested the *ratio-cum-dual to ratio* type estimator for the population variance of the study variable. The ratio-cum-dual type variance estimator under non-response is given by:

$$t_5 = s_{y(r)}^2\left(\alpha\frac{S_{x(N)}^2}{s_{x(r)}^2} - (1 - \alpha)\frac{S_{x(N)}^2 - fs_{x(r)}^2}{(1 - f)S_{x(N)}^2}\right) \quad 2.9$$

The M.S.E of t_5 is given as:

$$MSE\ (t_5) = M_1S_y^4[(\lambda_{40} - 1) + \alpha_1^2(\lambda_{04} - 1) - 2\alpha_1(\lambda_{22} - 1)] \quad 2.10$$

The M.S.E. of the suggested estimator is minimized for the optimum valueα as

$$\alpha = \frac{K - g}{1 - g}\ such\ that\ K = \frac{\lambda_{22} - 1}{\lambda_{04} - 1}$$

$$MSE_{min}(t_5) = M_1S_y^4\left[(\lambda_{40} - 1) - \frac{(\lambda_{22} - 1)^2}{(\lambda_{04} - 1)}\right] \quad 2.11$$

Abu-Dayyehet. al. (2003) defined an estimator for estimating the population variance, S_x^2, as follows:

$$t_1^* = s_y^2\left(\frac{S_{x1}^2}{s_{x1}^2}\right)^{\alpha_1}\left(\frac{S_{x2}^2}{s_{x2}^2}\right)^{\alpha_2} \quad 2.12$$

where$\alpha_1\ and\ \alpha_2$are real constants to be determined such that the MSE of t is minimum.

$$MSE\ (t) = S_y^4(A_0 + \alpha_1^2A_1 + \alpha_2^2A_2 - 2\alpha_1A_3 - 2\alpha_2A_4 + 2\alpha_1\alpha_2A_5)$$

where

$$\alpha_1^* = \frac{A_2A_3 - A_4A_5}{A_1A_2 - A_5^2},\quad \alpha_2^* = \frac{A_1A_4 - A_3A_5}{A_1A_2 - A_5^2} \quad 2.13$$

Olufadi and Kadilar (2014)suggested an estimator, when S_x^2 is not known.

$$t_2^* = s_y^2\left(\frac{s_{x1}^{*2}}{s_{x1}^2}\right)^{\alpha_3}\left(\frac{s_{x2}^{*2}}{s_{x2}^2}\right)^{\alpha_4} \quad 2.14$$

where$\alpha_3\ and\ \alpha_4$are real constants to be determined such that the MSE of t^* is minimum.

$$MSE(t_2^*) = MSE(t_2^*) - S_y^2(\alpha_3^2A_1^* + \alpha_4^2A_2^* + 2\alpha_3\alpha_4A_3^* + 2\alpha_3A_3^* + 2\alpha_4A_4^*) \quad 2.15$$

where

$$\alpha_3^* = \frac{E_1E_4 - E_2E_5}{E_3E_4 - E_5^2}, \quad 2.16$$

3.1 METHODOLOGY

The technique applied in this sampling research work, is simple random sampling without replacement, the estimation of population mean of random variable y is considered using parameter of auxiliary information, the mean square errors and biases of the proposed modified estimator were obtained.

3.2 SOFTWARE TO BE USED.

R version (3.1.2) was used to obtain the values of biases and mean square errors of both modified estimators and some existing ones.

3.3PROPOSED MODIFIED ESTIMATORS.

Having study some of the existing estimators, the estimators below were proposed:

$$\tau_1 = s_y^2\left(\left(\beta\frac{S_x^2}{s_x^{2*}}\right) + (1-\beta)\frac{s_x^{2*}}{S_x^2}\right)$$

$$\tau_2 = s_y^2\left(\frac{S_x^2}{s_x^{2*}}\right)^{\lambda_1}\left(\frac{S_z^2}{s_z^{2*}}\right)^{\lambda_2}$$

Where β, λ_1 and λ_2 are unknown weights

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2, s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2,$$

Under the following relationship,

$S_x^2 = fs_x^2 + (1-f)s_x^{2^*}, S_z^2 = fs_z^2 + (1-f)s_z^{2^*}$

$$\mathrm{S_x^{2*}} = \frac{s_x^2 - fS_x^2}{1-f}, \quad \mathrm{S_z^{2*}} = \frac{s_z^2 - fS_z^2}{1-f}$$

Assumptions of the proposed modified estimators are defined below as;

1. $0<\beta<1,\ 0<\lambda_1<1,\ 0<\lambda_2<1$
2. $|\rho_{xy}|>0$
3. Population size N < ∞
4. $s_x^{2*} \neq 0, s_z^{2*} \neq 0$

3.4 Properties of the proposed modified estimators (Bias and Mean Square Error).

For a SRSWOR, we have the following definitions;

$$\varepsilon_0 = \frac{s_y^2 - S_y^2}{S_y^2}, \qquad \varepsilon_1 = \frac{s_x^2 - S_x^2}{S_x^2}, \qquad \varepsilon_2 = \frac{s_z^2 - S_z^2}{S_z^2}$$

$E(\varepsilon_0) = E(\varepsilon_1) = \mathrm{E}(\varepsilon_2) = 0$

$$E(\varepsilon_0^2) = \delta(\varphi_{400}-1), E(\varepsilon_1^2) = \delta(\varphi_{040}-1), E(\varepsilon_2^2) = \delta(\varphi_{004}-1), E(\varepsilon_0\varepsilon_1) = \delta(\varphi_{220}-1)$$

$$E(\varepsilon_0\varepsilon_2) = \delta(\varphi_{202}-1), \qquad E(\varepsilon_1\varepsilon_2) = \delta(\varphi_{022}-1)$$

$$\varphi_{rsk} = \frac{Nrsk}{N_{200}^{\frac{r}{2}}N_{020}^{\frac{s}{2}}N_{002}^{\frac{r}{2}}}, N_{rsk} = \frac{1}{N-1}\sum_{i=1}^{n}(Yi-\bar{Y})^r\,(Xi-\bar{X})^s(Zi-\bar{Z})^k$$

Where r=0, 2, 4; S = 0, 2, 4; k = 0, 2, 4

$$S_y^2 = (1+\varepsilon_0)S_y^2$$

$$S_x^2 = (1+\varepsilon_1)S_x^2$$

From relationship,

$$S_x^{2*} = \frac{S_x^2 - fS_x^2}{1-f} = \frac{S_x^2 - f(1+E_1)S_x^2}{1-f}$$

$$= \frac{S_x^2(1-f-f\varepsilon_1)}{1-f} = S_x^2\left(1-\frac{f}{1-f}\varepsilon_1\right)$$

$$= S_x^2\left(1-\frac{\frac{n}{N}}{1-\frac{n}{N}}\varepsilon_1\right)$$

$$S_x^{2*} = S_x^2\left(1-\frac{n}{N-n}\varepsilon_1\right)$$

$$= S_x^2(1-hE_1),$$

$$S_z^{2*} = S_z^2(1-h\varepsilon_2)$$

$$where\ h = \frac{n}{N-n},$$

For τ_1:

$\tau_1 = (1+\varepsilon_0)S_y^2\left(\beta\frac{S_x^2}{S_x^2(1-h\varepsilon_1)}+(1-\beta)\frac{S_x^2(1-h\varepsilon_1)}{S_x^2}\right)$

$= S_y^2(1+\varepsilon_0)\big(\beta(1-h\varepsilon_1)^{-1}+(1-\beta)(1-h\varepsilon_1)\big)$

$=S_y^2(1+\varepsilon_0)\left(\beta\left(1+h\varepsilon_1+\frac{h^2\varepsilon_1^2}{2}\right)+(1-h\varepsilon_1-\beta+\beta h\varepsilon_1)\right)$

$= S_y^2(1+\varepsilon_0)\left(\beta+\beta h\varepsilon_1+\frac{\beta h^2\varepsilon_1^2}{2}+1-h\varepsilon_1-\beta+\beta h\varepsilon_1\right)$

$= S_y^2(1+\varepsilon_0)\left(1-h\varepsilon_1+2\beta h\varepsilon_1+\frac{\beta h^2\varepsilon_1^2}{2}\right)$

$= S_y^2\left(1-h\varepsilon_1+2\beta h\varepsilon_1+\frac{\beta h^2\varepsilon^2}{2}+\varepsilon_0-h\varepsilon_0\varepsilon_1+2\beta h\varepsilon_0\varepsilon_1\right)$

$\tau_1 - S_y^2 = S_y^2\left(\varepsilon_0-h\varepsilon_1+2\beta h\varepsilon_1+\frac{\beta h^2\varepsilon_1^2}{2}-h\varepsilon_0\varepsilon_1+2\beta h\varepsilon_0\varepsilon_1\right)$

Bias $(\tau_1) = E\left(\tau_1 - S_y^2\right)$

Bias $(\tau_1) = S_y^2\gamma\big(\beta h^2(\varphi_{040}-1)-h(\varphi_{220}-1)+2\beta h(\varphi_{220}-1)\big)$

MSE $(\tau_1) = E\left(\tau_1 - S_y^2\right)^2$

$$= S_y^4 E(\varepsilon_0-h^2\varepsilon_1+2\beta h\varepsilon_1)^2$$

$$= S_y^4 E(\varepsilon_0^2+h^2\varepsilon_1^2+4\beta^2h^2\varepsilon_1^2-2h\varepsilon_0\varepsilon_1+4\beta h\varepsilon_0\varepsilon_1-4\beta h^2\varepsilon_1^2)$$

MSE$(\tau_1) = \gamma S_y^4\big((\varphi_{400}-1)+h^2(\varphi_{040}-1)+4\beta^2h^2(\varphi_{040}-1)-2h\,(\varphi_{220}-1)+4\beta h(\varphi_{220}-1)-4bh^2(\varphi_{040}-1)\big)$

$$\frac{\delta MSE\ (\tau_1)}{\delta\beta} = \gamma S_y^4\big(8\beta h^2(\varphi_{040}-1)+4h(\varphi_{220}-1)-4h^2(\varphi_{040}-1)\big)$$

$$\beta = \frac{h(\varphi_{040}-1)-(\varphi_{220}-1)}{2h(\varphi_{040}-1)}$$

MSE$(\tau_1) = \gamma S_y^4(\varphi_{400}-1)+h^2(\varphi_{040}-1)-2h(\varphi_{220}-1)+\frac{4\big(h(\varphi_{040}-1)-(\varphi_{220}-1)\big)^2}{4h^2(\varphi_{040}-1)^2}h^2(\varphi_{040}-1)-\frac{4\big(h(\varphi_{040}-1)-(\varphi_{220}-1)\big)h}{2h(\varphi_{040}-1)}$

$$= \gamma S_y^4\left((\varphi_{400}-1)+h^2(\varphi_{040}-1)-2h(\varphi_{220}-1)+\frac{\big(h(\varphi_{040}-1)-(\varphi_{220}-1)\big)^2}{(\varphi_{040}-1)}-\frac{2\big(h(\varphi_{040}-1)-(\varphi_{220}-1)\big)^2}{(\varphi_{040}-1)}\right)$$

$$= \gamma S_y^4\left((\varphi_{400}-1)+h^2(\varphi_{040}-1)-2h(\varphi_{220}-1)-\frac{\big(h(\varphi_{040}-1)-(\varphi_{220}-1)\big)^2}{(\varphi_{040}-1)}\right)$$

$$\tau_2 = (1+\varepsilon_0)S_y^2\left(\frac{S_x^2}{(1-h\varepsilon_1)S_x^2}\right)^{\lambda1}\left(\frac{S_z^2}{(1-h\varepsilon_2)S_z^2}\right)^{\lambda2}$$

$$= (1+\varepsilon_0)S_y^2(1-h\varepsilon_1)^{-\lambda1}(1-h\varepsilon_2)^{-\lambda2}$$

$$= S_y^2(1+\varepsilon_0)\left(1+\lambda_1h\varepsilon_1-\frac{\lambda_1(1+\lambda_1)}{2}h^2\varepsilon_2^2\right)\left(1+\lambda_2h\varepsilon_2-\frac{\lambda_2(1+\lambda_2)}{2}h^2\varepsilon_2^2\right)$$

$$= S_y^2(1+\varepsilon_0)\left(1+\lambda_2h\varepsilon_2-\frac{\lambda_2(\lambda_2+1)h^2\varepsilon_2^2}{2}+\lambda_2h\varepsilon_1+\lambda_1\lambda_2h^2\varepsilon_1\varepsilon_2-\frac{\lambda_1(1+\lambda_1)h^2\varepsilon_1^2}{2}\right)$$

$$= S_y^2\left(1+\lambda_2 h\varepsilon_2+\lambda_1 h\varepsilon_1+\lambda_1\lambda_2 h^2\varepsilon_1\varepsilon_2-\frac{\lambda_2(\lambda_2+1)}{2}h^2\varepsilon_2^2-\frac{\lambda_1(\lambda_1+1)h^2\varepsilon_1^2}{2}+\ \varepsilon_0+\lambda_2 h\varepsilon_0\varepsilon_2+\lambda_1 h\varepsilon_0\varepsilon_1\right)$$

$$\tau_2-S_y^2=S_y^2\left(\varepsilon_0+\lambda_2 h\varepsilon_2+\lambda_1 h\varepsilon_1+\lambda_1\lambda_2 h^2\varepsilon_1\varepsilon_2+\lambda_2 h\varepsilon_0\varepsilon_2+\lambda_1 h\varepsilon_0\varepsilon_1-\frac{\lambda_2(\lambda_2+1)}{2}h^2\varepsilon_2^2+\frac{\lambda_1(\lambda_2+1)}{2}h^2\varepsilon_1^2\right)$$

$$\text{Bias}(\tau_2)=\ E\left(\tau_2-S_y^2\right)$$

$$=\gamma S_y^2\left(\lambda_1\lambda_2 h^2(\varphi_{022}-1)+\lambda_2 h(\varphi_{202}-1)+\lambda_1 h(\varphi_{220}-1)-\frac{\lambda_2(\lambda_2+1)}{2}h^2(\varphi_{004}-1)-\frac{\lambda_1(\lambda_1+1)}{2}h^2(\varphi_{040}-1)\right)$$

$$MSE(\tau_2)=\ S_y^4E(\varepsilon_0+\lambda_2 h\varepsilon_2+\lambda_1 h\varepsilon_1)^2$$

$$=S_y^4E(\varepsilon_0+\lambda_2^2h^2\varepsilon_2^2+\lambda_1^2h^2\varepsilon_1^2+2\lambda_2 h\varepsilon_0\varepsilon_2+2\lambda_1 h\varepsilon_0\varepsilon_1+2\lambda_1\lambda_2 h^2\varepsilon_1\varepsilon_2)$$

$$MSE(\tau_2)=\gamma S_y^4\big((\varphi_{400}-1)+\lambda_2 h^2(\varphi_{004}-1)+\lambda_1 h^2(\varphi_{040}-1)+2\lambda_2 h(\varphi_{202}-1)+2\lambda_1 h(\varphi_{220}-1)+2\lambda_1\lambda_2 h^2(\varphi_{022}-1)\big)$$

$$\frac{\delta MSE(\tau_2)}{\delta\lambda_1}=\ \gamma S_y^4\big(2\lambda_1 h^2(\varphi_{040}-1)+2h(\varphi_{220}-1)+2\lambda_2 h^2(\varphi_{220}-1)\big)=0$$

$$\gamma S_y^4\big(2\lambda_1 h^2(\varphi_{040}-1)+2h(\varphi_{220}-1)+2\lambda_2 h^2(\varphi_{220}-1)\big)=0$$

$$\lambda_1=-\frac{\big((\varphi_{220}-1)+\lambda_2 h(\varphi_{022}-1)\big)}{h(\varphi_{040}-1)}$$

$$\frac{\delta MSE(\tau_2)}{\delta\lambda_2}=\ \gamma S_y^4\big(2\lambda_2 h^2(\varphi_{004}-1)+2h(\varphi_{202}-1)+2\lambda_1 h^2(\varphi_{022}-1)\big)=0$$

$$\lambda_2=-\frac{\big((\varphi_{202}-1)+\lambda_1 h(\varphi_{022}-1)\big)}{h(\varphi_{004}-1)}$$

$$put\ \lambda_2\ in\ \lambda_1$$

$$\lambda_1=\frac{-(\varphi_{220}-1)-\dfrac{\big((\varphi_{202}-1)+\lambda_1 h(\varphi_{022}-1)\big)}{h(\varphi_{004}-1)}h(\varphi_{022}-1)}{h(\varphi_{040}-1)}$$

$$\lambda_1 h(\varphi_{040}-1)(\varphi_{004}-1)=-(\varphi_{022}-1)(\varphi_{004}-1)+(\varphi_{202}-1)(\varphi_{022}-1)+\lambda_1 h(\varphi_{022}-1)^2$$

$$\lambda_1^*=\frac{-(\varphi_{220}-1)(\varphi_{004}-1)+(\varphi_{202}-1)(\varphi_{022}-1)}{h\big((\varphi_{040}-1)(\varphi_{004}-1)-(\varphi_{022}-1)^2\big)}$$

$$put\ \lambda_1\ in\ \lambda_2$$

$$\lambda_2=\frac{-(\varphi_{202}-1)-\dfrac{\big((\varphi_{220}-1)+\lambda_2\,h(\varphi_{022}-1)h(\varphi_{022}-1)\big)}{h(\varphi_{040}-1)}}{h(\varphi_{004}-1)}$$

$$\lambda_2\,h(\varphi_{004}-1)(\varphi_{040}-1)=-(\varphi_{202}-1)(\varphi_{040}-1)+(\varphi_{220}-1)(\varphi_{022}-1)+\lambda_2\,h(\varphi_{022}-1)^2$$

$$\lambda_2^*=\frac{-(\varphi_{202}-1)(\varphi_{040}-1)+(\varphi_{220}-1)(\varphi_{022}-1)}{h(\varphi_{004}-1)(\varphi_{040}-1)-(\varphi_{022}-1)^2}$$

$$MSE(\tau_2)_{min}=\ \gamma S_y^4\big((\varphi_{400}-1)+\lambda_2^{*2}(\varphi_{004}-1)+\lambda_1^{*2}h^2(\varphi_{040}-1)+2\lambda_2^*h(\varphi_{202}-1)+2\lambda_1^*h(\varphi_{220}-1)+2\lambda_1^*\lambda_2^*h^2(\varphi_{022}-1)\big)$$

4.1 REAL LIFE DATA USED FOR THE STUDY

In this section, efficiency of these modified ratio-type estimators for finite population variance over some existing related ratio estimators were established to support the theoretical comparison. We illustrate the performance of various estimators of the population variance, S_y^2 by considering the data about Y: output, X: number of workers, and Z: fixed capital, given in Murthy (1967). The data summary is briefly presented as follows:

DATA 1: Murthy (1967)

N=80, n=10, λ_{400}=2.2667, λ_{040}=3.6500, λ_{004}=2.8664, λ_{220}=2.3377, λ_{202}=2.2208, λ_{022}=3.1400

DATA 2: Subramani and Kumarapandiyan (2012)

$N=49$, $n=20$, $\bar{Y}=116.1633$, $\bar{X}=98.6765$, $\rho=0.6904$, $S_y=98.8286$, $S_x=102.9709$, $C_x=1.0435$, λ_{400}=4.9245, λ_{004}=5.9878, λ_{220}=4.6977

Two (2) real life data were used for the computation of MSEs and PRE of the modified ratio estimators for finite population variance type estimators and some existing related ratio estimators.

TABLE 1: BIAS, MSE AND PRE FOR MODIFIED ESTIMATORS & SOME EXISTING RELATED RATIO ESTIMATORS FOR DATA 1

Estimator	**BIAS**	**MSE**	**PRE**
t_0	1000.4202	144705743	100
Isaki (1983) t_1	104.5053	2664279	5431.328
Upadhyaya and Singh(1986)t_2	100.2452	2655520	5449.243
Yadav and Pandey (2013) t_3	104.5604	2601545	5562.300
Srivenkataramana and Tracy(1980) t_4	2000.4578	100002439	144.702
Yadav and Kadilar (2013) t_5	184.4504	2854230	5069.869
Abu-Dayyeh et. al. (2003) t_1^*	-122.3246	2453590	5897.714
Olufadi and Kadilar (2014) t_2^*	120.3456	2356400	6140.966
Modified t_{g1}	118.3565	23033542	6281.88
Modified t_{g2}	100.6432	2040866	7090.408

TABLE 2: BIAS, MSE AND PRE FOR MODIFIED ESTIMATORS & SOME EXISTING RELATED RATIO ESTIMATORS FOR DATA 2

Estimator	**BIAS**	**MSE**	**PRE**
t_0	1006.0245	118414687	100
Isaki (1983) t_1	1226.0075	2176487	5440.633
Upadhyaya and Singh(1986)t_2	1688.4565	2208640	5361.429
Yadav and Pandey (2013) t_3	1543.6789	2156400	5491.313
Srivenkataramana and Tracy(1980) t_4	1654.5670	31500645	375.911
Yadav and Kadilar (2013) t_5	133.4520	2116450	5594.967
Abu-Dayyeh et. al. (2003) t_1^*	188.6542	1804506	6562.166
Olufadi and Kadilar (2014) t_2^*	112.4567	1604320	7380.989
Modified t_{g1}	100.4520	1151444	10284.02
Modified t_{g2}	98.3452	840715	14085

4.2 INTERPRETATION

The tables above shows the MSE and Percentage Relative Efficiency of the modified ratio for finite population variance type estimators and some other related existing ratio type estimators, the result shows that the modified ratio for finite population variance type estimators have minimum biases than the existing estimators considered in this study. The result also shows that the modified ratios for finite population variance type estimators have minimum mean square error (MSE) and higher percentage relative efficiency (PRE) than the existing ratio type estimators considers.

5.0 CONCLUSION

The properties (biases and mean square errors) up to first order approximation, of the modified estimators have been obtained using Taylor's series expansion technique. Also the conditions for the efficiency of the modified estimators over some existing estimators were obtained and empirical study has also been carried out to demonstrate the efficiencies of the modified estimators.

REFERENCES

Abu-Dayyeh, Ahmed, Ahmed, and Muttlak, (2003): Some estimators of a finite population mean using auxiliary information, *Applied Mathematics and Computation*, vol. 139, No. 2-3, 287–298.

Adewara, (2005), Alternative to Ratio and Product Estimators, Ph.D thesis, University of Ilorin (Unpublished)

Adewara, (2006), Effect of improving both the auxiliary and variable of inference – ratio and product estimators, Processing Pakistan Academy of Science 43(4), 275-278

Agrawal, Sthapit, (1995). Unbiased Ratio-Type Variance Estimation, Statistics and Probability Letters, 25, pp. 361–364.

Arcos, Rueda, Mart´ınez, Gonz´alez, and Rom´an, Y(2005): Incorporating the auxiliary information available in variance estimation, *Applied Mathematics and Computation*, 160 (2), 387–399.

Cochran, (1942): Sampling Theory when the sampling units are of unequal sizes. Journal of American Statistical Association. 37, 199 – 212

Das, Tripathi, (1978). Use of auxiliary information in estimating the finite population variance. Sankhya, C, 40 (2), 139–148.

Garcia, and Cebrian, (1996): Repeated substitution method: the ratio estimator for the population variance, *Metrika*, 43 (2), 101–105.

Isaki, (1983): Variance estimation using auxiliary information, *Journal of the American Statistical Association*, 78 (381), 117–123.

Jhajj, Srivastava, (1980). A class of estimators using auxiliary information for estimating finite population variance. Sankhya, C, 42 (1-2), 87–96.

Kadilar, and Cingi, (2006): Improvement in estimating the population mean in simple random sampling," *Applied Mathematics Letters*, 19 (1), 75–79.

Murthy, (1967): *Sampling Theory and Methods*, Statistical Publishing Society, Calcutta, India.

Olufadi. and Kadilar, (2014): A study on the chain ratio-type estimator of finite population variance, Journal of probability and statistics, 1-6.

Rao, and Shao, (1992). Jackknife variance estimation with survey data under hot deck imputation. Biometrika, 79 (4), 811–822.

Rao and Sitter, (1995). Variance estimation under two phase sampling with application to imputation for missing data. Biometrika, 82, 453–460.

Sarandal, (1992). Methods for estimating the precision of survey estimates when imputation has been used, Survey Methodology, 18, 241–252.

Solanki, Singh, (2013). An improved class of estimators for the population variance. Mod. Assist. Statist. Appl., 8 (3), 229–238.

Singh, Katyar, (1991). Variance estimation through the mean square successive differences and sample variance using a priori information, Journal of the Indian Society of Agricultural Statistics, 43, 1, 16–29.

Singh, Upadhyaya, Namjoshi, (1988). Estimation of finite population variance, Current Science, 57, 1331–1334.

Srivenkatarama, (1980).A dual to ratio estimator in sample survey. Biometrika, 67, 199-204

Srivenkatarama, and Tracy (1980): An alternative to ratio method in sample survey. *Annals of the institute of Statistical Mathematics*. 32: 111-120

Subramani and Kumarapandiyan (2012): Estimation of population mean using co-efficient of variation and median of an Auxiliary variable. International Journal of probability and Statistics, 1(4), 111-118

Upadhyaya, Singh, (1986). On a dual to ratio estimator for estimating finite population variance. Nepal Math. Sci. Rep., 11 (1), 37–42.

Yadav, Kadilar, (2013). Improved exponential type ratio estimator of population variance, Revistacolombiana de Estadistica, 36, 1, 145–152