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A Computational Approach for Analysis of Scattering Characteristics for Strips of Resistive Material

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A B S T R A C T

This article proposes a computational approach for analysis of scattering characteristics for strips of resistive material. For this purpose, the integral equation technique is used for modeling the problem. In order to solve the model, an effective approach is formulated based on using a set of basis functions that reduces the integral equation model to a linear system of algebraic equations. Solving this system gives an approximate solution for the integral equation. Numerical results obtained from implementation of the proposed approach are given for better illustration.

Keywords: Resistive material; Scattering analysis; Computational approach.

1. Introduction

One of the interesting and powerful techniques that has a wide variety of applications in material science is integral equation method. Various forms of such equations are used for mathematical modeling of many problems in engineering studies. Some instances of the related applications are reviewed here.

A numerical method for solving Hallén’s integral equation is proposed in [1] for analysis of electromagnetic radiation from a diploe antenna of perfectly conducting material. The authors use a special representation of orthogonal triangular basis functions to implement their method. Reference [2] studies the stress intensity factors and their correlations with bimaterial parameters for the interface planar cracks by hypersingular integral equations. A new bimaterial parameter γ is defined, and the correlations of the stress intensity factors of interface planar crack with two parameters ε and γ are proved. The relative displacement fundamental function is proposed based on the crack’s peripheral equation. In [3], a combined Laplace transform and boundary element method is used to find numerical solutions to problems of another class of anisotropic functionally graded materials which are governed by a variable coefficients parabolic equation. A transformation is used to reduce the variable coefficients equation to a constant coefficients equation, which is then transformed into a boundary-only integral equation. In [4], the new system of hypersingular integral equations (HSIEs) for the thermally insulated inclined cracks and thermally insulated circular arc cracks subjected to remote shear stress in bonded dissimilar materials is formulated by using the modified complex potentials (MCPs) function method with the continuity conditions of the resultant force, displacement and heat conduction functions. This new system of HSIEs is derived from the elasticity problem and heat conduction problem by using crack opening displacement (COD) function and temperature jump along the crack faces. In [5], the convergence of an iterative method called Adomian decomposition method is analyzed to solve the Fuzzy Volterra Integral Equations (FVIE) with time lag, the uniqueness and convergence of the method are equivalent. In [6], a spectral formulation of the boundary integral equation method for antiplane problems is presented. The boundary integral equation method relates the slip and the shear stress at an interface between two half-planes. It involves evaluating a space-time convolution of the shear stress or the slip at the interface. In the spectral formulation, the convolution with respect to the spatial coordinate is performed in the spectral domain. This leads to greater numerical efficiency. In [7], the static interaction between an eccentricly loaded rectangular rigid foundation and layered transversely isotropic soils is investigated. Firstly, the solution of the layered transversely isotropic soils is derived using the analytical layer element method. After decomposing the deformation forms of the rigid rectangular foundation under eccentric loads, the dual integral equations are established through mixed boundary conditions. Then, the Jacobi orthogonal polynomial and the Bessel function are employed to solve the above dual integral equations, and the static response solution of the rigid rectangular foundation subjected to eccentric loads is obtained through superposition.

This article proposes a computational approach for analysis of scattering characteristics for strips of resistive material. For this purpose, we firstly survey the mathematical modeling of the problem in detail. We will see that this modeling leads to a Fredholm integral equation of the first kind. A computational approach is necessary to solve the related integral equation because its analytical solution is not easily available. Therefore, we will formulate an effective numerical approach for solution of the integral equation arising in the problem based on using the truncated cosines as a set of basis functions. Using this approach, the integral equation reduces to a linear system of algebraic equations. Solving this system gives an approximate solution for the problem. In fact, Solution of the integral equation gives the current distribution induced on the surface of the resistive strip. Moreover, once the current distribution is obtained, we will be able to calculate the related radar cross section (RCS) of the strip too.

The organization of this article is as follows. First of all, we review the mathematical modeling of the problem based on the integral equation form to obtain a model for computing the strip current density and calculating its RCS. Then, we review the truncated cosines as a set of orthogonal basis functions. After that, we formulate our proposed computational approach by using the set of truncated cosines in order to solve the integral equation model of the problem. Finally, we present the numerical results obtained from implementation of the proposed approach. The results are given for both the current density and the RCS.

2. Modeling the problem of scattering from strips of resistive material

In Fig. 1, there is a strip of resistive material that is very long in the $\pm z$ direction. This strip is encountered by an incoming plane wave E^{inc} that has a polarization with its electric field parallel to the z-axis. The magnetic field of this wave is entirely in the x-y plane, and is therefore transverse to the z-axis. It is called transverse magnetic (TM) polarized wave. This polarization therefore produces a current on the strip that flows along the z-axis.

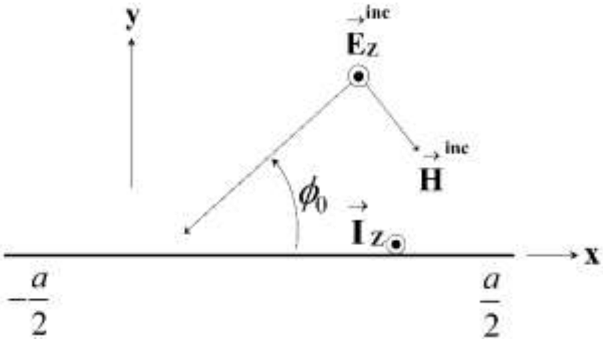

Fig. 1. A strip of resistive material of width a is encountered by an incoming TM-polarized plane wave.

The magnetic vector potential A_z of the current flowing along the strip is given by [8, 9]

$$A_z = \frac{\mu_0}{4j}\int_{-a/2}^{a/2} I_z(x')H_0^{(2)}(k|x-x'|)\,\mathrm{d}x', \qquad (1)$$

where,

$k = \frac{2\pi}{\lambda}$, is free space wave number;

λ, is the wave length;

$\mu_0 = 4\pi \times 10^{-7}$ H/m, is free space permeability;

$G(x,x') = \frac{1}{4j}H_0^{(2)}(k|x-x'|)$, is 2D free space Green's function;

$H_0^{(2)}(x)$, is a Hankel function of the second kind of zero order.

So, the electric field is given by

$$E_z(x) = j\omega A_z(x), \qquad (2)$$

or

$$E(x) = \frac{\omega\mu_0}{4}\int_{-a/2}^{a/2} I_z(x')H_0^{(2)}(k|x-x'|)\,\mathrm{d}x'. \qquad (3)$$

Assume that $R_s(x)$ is the surface resistance of the strip and note that the units of surface resistance are in Ω/m^2. The boundary condition at the surface of a thin resistive strip is given by the following equation [8, 9]:

$$-E^{inc} = E^{scat} + R_s(x)J(x), \qquad (4)$$

where,

J(x), is the surface current of the strip;

E^{scat}, is the scattered electric field produced by the surface current.

Assuming $E^{inc} = e^{jkx\cos\phi_0}$, from Eq. (3) and Eq. (4) it follows:

$$R_s(x)I(x) + \frac{\omega\mu_0}{4}\int_{-a/2}^{a/2} I(x')H_0^{(2)}(k|x-x'|)\,\mathrm{d}x' = -e^{jkx\cos\phi_0}, \qquad (5)$$

where, $I(x)$ is the current of the strip.

We can also rewrite Eq. (5) as

$h(x) + \int_a^b G(x,x')h(x')\,\mathrm{d}x' = g(x)$, (6)

where,

$h(x) = I(x)$;

$G(x,x') = \frac{\omega\mu_0}{4}\frac{1}{R_s(x)}H_0^{(2)}(k|x-x'|)$;

$g(x) = -\frac{1}{R_s(x)}e^{jkx\cos\phi_0}$.

Equations (5) and (6) are in the form of Fredholm integral equation of the second kind and can be solved by a computational approach. However, from Eq. (5) we can obtain the induced current $I(x)$ on the strip. Moreover, after computing $I(x)$, we can calculate the related RCS of the strip.

RCS in two dimensions is defined mathematically as [8, 9]

$\sigma(\phi) = \lim_{r\to\infty} 2\pi r \frac{|E^{scat}|^2}{|E^{inc}|^2}$. (7)

In two dimensions, the free space Green's function is

$G(r,r') = \frac{1}{4j}H_0^{(2)}(k|r-r'|)$. (8)

The magnetic vector potential in two-dimensional space is

$A(r) = \mu \int\int J(r')G(r,r')\,\mathrm{d}s'$. (9)

The electric field is given by

$E = j\omega A$. (10)

Combining (8), (9), and (10) we obtain

$E(r) = \frac{\omega\mu}{4}\int\int J(r')H_0^{(2)}(k|r-r'|)\,\mathrm{d}s'$. (11)

In the TM situation, the incident electric field along the strip is 1 V/m ($|E^{inc}|^2 = 1$). So, the denominator of Eq. (7) is unity. This allows us to turn our attention to the numerator. To evaluate (11), we note that as $r \to \infty$, we can use the large argument approximation for the Hankel function [8, 9]

$H_0^{(2)}(r) \approx \sqrt{\frac{2}{\pi r}}e^{-j(r-\frac{\pi}{4})}$. (12)

Substituting this into (11) and implementing Eq. (7) for the TM case, we obtain

$\sigma(\phi) = \frac{k\eta^2}{4}\left|\int_{strip} I(x',y')e^{jk(x'\cos\phi + y'\sin\phi)}\,\mathrm{d}l'\right|^2$. (13)

where, $\eta = 376.73 \quad \Omega$.

In the presented case, the strip is restricted to the x-axis, which simplifies Eq. (13)

$\sigma(\phi) = \frac{k\eta^2}{4}\left|\int_{-a/2}^{a/2} I(x')e^{jkx'\cos\phi}\,\mathrm{d}x'\right|^2$. (14)

Also, it is possible to define a logarithmic quantity with respect to the RCS, so that

$\sigma_{dBlm} = 10log_{10}\ \sigma$. (15)

3. Computational approach for analysis of scattering from resistive material strips

For determining the current density induced on the strip, we have to find an approximate solution for integral equation (5). But, firstly, we need to survey an appropriate set of basis functions in order to use it to formulate our propose computational approach. The desired set of basis functions is surveyed in the following subsection.

3.1 Basis functions

Let us consider an m-set of truncated cosines for any positive integer m over real interval $[a,b)$ as [10]

$$\mathcal{T}_i(t) = \begin{cases} \cos(\gamma(t-a-ih-\frac{h}{2})), & a+ih \le t < a+(i+1)h, \\ 0, & \text{otherwise}, \end{cases} \quad (16)$$

where $h = \frac{b-a}{m}$, $i = 0,1,\dots,m-1$, and γ has a real value and may be considered as a regularization factor.

The above definition can obviously make a set of disjoint and orthogonal basis functions. For arbitrary i and j, such that $i = 0,1,\dots,m-1$ and $j = 0,1,\dots,m-1$, we have

$$< \mathcal{T}_i,\mathcal{T}_j > = \int_a^b \mathcal{T}_i(t)\mathcal{T}_j(t)\,\mathrm{d}t = \begin{cases} \frac{h}{2} + \frac{1}{2\gamma}\sin(\gamma h), & i = j, \\ 0, & i \ne j, \end{cases} \quad (17)$$

in which $<.,.>$ indicates the inner product.

Moreover, it is clear that function $\mathcal{T}_i$ may be considered as follows:

$$\mathcal{T}_i(\mathrm{t}) = \varphi_i(t)\cos(\gamma(t - a - ih - \tfrac{h}{2})), \qquad (18)$$

where φ_i is ith block-pulse function (BPF) defined as

$$\varphi_i(t) = \begin{cases} 1, & a + ih \le t < a + (i+1)h, \\ 0, & \text{otherwise.} \end{cases} \qquad (19)$$

The disjointness and orthogonality properties of $\mathcal{T}_i$'s can make them very efficient for approximation of functions. The expansion of a function f over $[a, b)$ with respect to $\mathcal{T}_i$, $i = 0,1, \dots, m-1$, may be compactly written as [10]

$$f(t) \simeq \sum_{i=0}^{m-1} f_i \mathcal{T}_i(t), \ (20)$$

where f_i's, the expansion coefficients, may be computed by

$$\begin{aligned} f_i &= < f, \mathcal{T}_i > \\ &= \int_a^b f(t)\mathcal{T}_i(t)\,\mathrm{d}t. \end{aligned} \qquad (21)$$

In the next subsection, we will propose a computational approach based on these functions for analysis of scattering from resistive material strips.

3.2 Implementation of computational approach

Let us consider again the second kind Fredholm integral equation (6) as follows:

$$h(x) + \int_a^b G(x, x')h(x')\,\mathrm{d}x' = g(x), \qquad a \le x < b, \qquad (22)$$

where the functions G and g are known but h is the unknown function to be determined.

Approximating the unknown function h with respect to the truncated cosines using (20) gives

$$h(x) \simeq \sum_{i=0}^{m-1} h_i \mathcal{T}_i(x), \qquad (23)$$

and

$$h(x') \simeq \sum_{i=0}^{m-1} h_i \mathcal{T}_i(x'), \qquad (24)$$

where h_i's are defined as in (21). Note that we have replaced variable t with variable x or variable x'. Now, substituting (23) and (24) into (22) results in

$$\sum_{i=0}^{m-1} h_i \mathcal{T}_i(x) + \int_a^b G(x, x')(\sum_{i=0}^{m-1} h_i \mathcal{T}_i(x'))\,\mathrm{d}x' = g(x), \qquad (25)$$

or

$$\sum_{i=0}^{m-1} h_i \mathcal{T}_i(x) + \sum_{i=0}^{m-1} h_i \int_a^b G(x, x')\mathcal{T}_i(x')\,\mathrm{d}x' = g(x), (26)$$

and consequently

$$\sum_{i=0}^{m-1} h_i \left(\mathcal{T}_i(x) + \int_a^b G(x, x')\mathcal{T}_i(x')\,\mathrm{d}x'\right) = g(x). \qquad (27)$$

Now, choosing m appropriate points x_j, $j = 0,1, \dots, m-1$, such that $x_j \in [a, b)$, we obtain

$$\sum_{i=0}^{m-1} h_i \left(\mathcal{T}_i(x_j) + \int_a^b G(x_j, x')\mathcal{T}_i(x')\,\mathrm{d}x'\right) = g(x_j), \ \ j = 0,1, \dots, m-1. \qquad (28)$$

(28) is a linear system of m algebraic equations in terms of m unknown coefficients h_i. Solution of this systems gives h_i's, and then we obtain an approximate solution $h(x) \simeq \sum_{i=0}^{m-1} h_i \mathcal{T}_i(x)$ for (22).

4. Numerical results of the proposed computational approach's implementation

Now, we have the required tools for analysis of scattering characteristics for strips of resistive material.

By applying the proposed approach in solution of integral equation (6), we can compute the induced current density $I(x)$ on the surface of strip. After that, we will be able to calculate its related RCS using (14) and (15).

Figures 2–5 give the current density on the strip for $R_s = 0{,}500{,}1000(\Omega/\mathrm{m}^2)$, $\phi_0 = \frac{\pi}{2}$, $a = 6\lambda(\mathrm{m})$, and $f = 0.3$GHz. Moreover, the related RCSs are shown in Fig. 6.

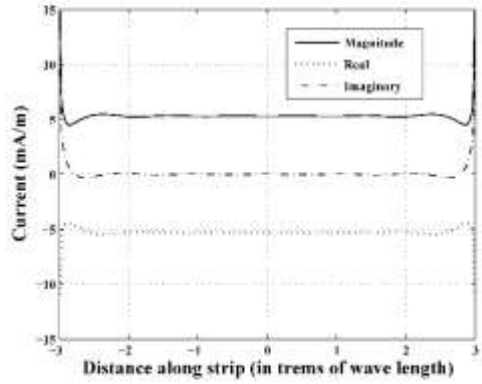

Fig. 2. Current distribution across a $6-\lambda$ strip for $R_s = 0$ and $f = 0.3$ GHz.

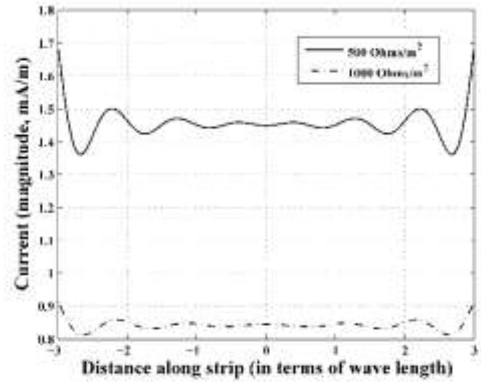

Fig. 3. Current magnitude across the $6-\lambda$ resistive strip for R_s of 500 and 1000 (Ω/m^2), and $f = 0.3$ GHz.

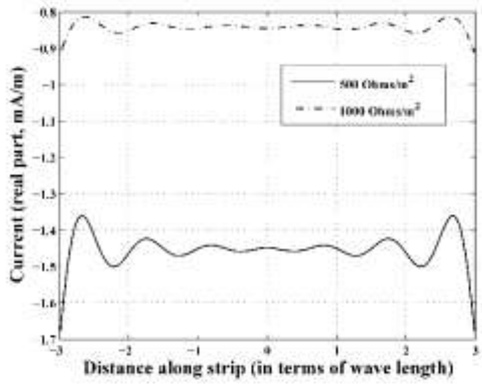

Fig. 4. The real part of current across the $6-\lambda$ resistive strip for R_s of 500 and 1000 (Ω/m^2), and $f = 0.3$ GHz.

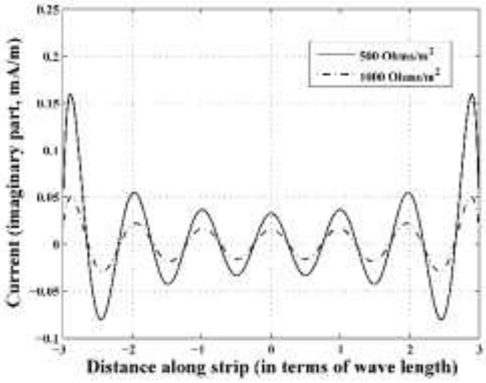

Fig. 5. The imaginary part of current across the $6-\lambda$ resistive strip for R_s of 500 and 1000 (Ω/m^2), and $f = 0.3$ GHz.

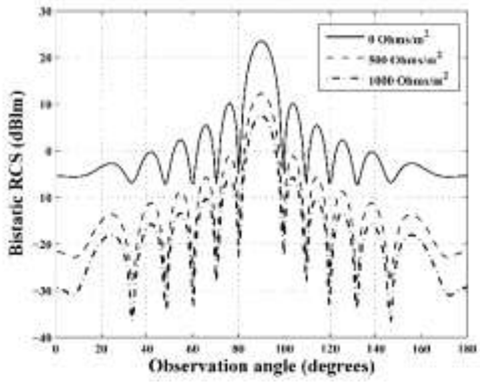

Fig. 6. The bistatic RCS of the $6-\lambda$ resistive strip for R_s of 0, 500, 1000 (Ω/m^2), and $\phi_0 = \frac{\pi}{2}$.

5. Conclusion

A computational approach was proposed in this article for analysis of scattering characteristics for strips of resistive material. We saw that the mathematical modeling of this problem led to a second kind Fredholm integral equation. An efficient computational method was formulated in order to solve the mentioned integral equation because its analytical solution is not easily available. The proposed method was implemented based on using the truncated cosines as a set of basis functions to reduce the integral equation model to a linear system of algebraic equations. Solving this system gave an approximate solution for the problem. Finally, the numerical results obtained from implementation of the proposed approach were are given for both current density and RCS of the strip. The advantages of the proposed computational approach are as follows. The first advantage is its flexibility to be generalized for applying in analysis of other structures with different geometries. As the second advantage it must be mentioned that the numerical results obviously confirm the computational efficiency of the proposed method as an integral equation technique which is well-known to be a very accurate approach. The third advantage is that a similar formulation can be applied in solution of other scattering bodies made of materials with arbitrary electrical conductivity or resistance.

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