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Intensity Duration Frequency of Maximum Rainfall using Gumbel Distribution for Tarnab Peshawar

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A B S T R A C T

The aim of this work is the generation of intensity duration frequency relation for Tarnab (a rural settlement in the suburbs of Peshawar Pakistan). A 33 years of rainfall data from Agriculture Research Institute Tarnab is employed for this purpose with specified duration of (0.08, 0.5, 1, 2, 12, 24) h and return periods (2, 10, 25, 50, 75, 100) years on the basis of yearly daily maximum. The observed data is plotted via the Gumbel distribution with respect to all the duration and return periods. For the derivation of empirical equation, the standard Bernard equation is applied for the region of Tarnab. The constants in the empirical relation which are subject to the region are also derived using linear regression. The values obtained from the empirical (estimated) relation are negligibly variant than those obtained from Gumbel distribution. The empirical equation for each return period is then plotted against the one obtained from Gumbel method showing a good fit of data. Additionally the standard error of estimates σ_s and correlation coefficient R^2 for each return period is calculated in support of the argument of agreement between the observed and estimated results.

Keywords: Intensity Duration Frequency, Rainfall Intensity, Gumbel Distribution, Rainfall Frequency, Empirical Model

1. Introduction:

In order to evaluate the vulnerability of water resource structures as well as planning, construction, and operation, one of the most crucial techniques in water resources engineering is the rainfall Intensity-Duration-Frequency (IDF) study. When estimating rainfall depth at a site with a certain exceedance probability and duration, hourly rainfall data can be subjected to Extreme Value Analysis (EVA), which yields the IDF Relationship (**Burlando and Rosso 1996**). As the name suggests EVA is done by taking the peak values of rainfall for a specified duration at a single station or from multiple stations; in the latter case the study becomes regional (**Bougadis, J., and Adamowski, K. 2006**). An IDF curve displays the probability of rainstorms occurring at a given length and intensity as well as the expected time interval between storms with comparable properties. IDF curves are an effective tool for displaying the extreme rainfall that is predicted in a region of interest since they display the average intensity of rainfall at each return period (RP) for all rainfall duration's. IDF curves give useful information for forecasting future flooding events and the frequency of particular rainfall or flow amounts. To predict the desired rainfall intensity for a certain length and frequency, a set of statistically produced IDF curves suitable for a given region is usually employed. (**Basumatary, V., and Sil, B. 2017**).

Nomenclature

P_t = depth of precipitation at time t
P_{24} = precipitation for 24 hours
y_n = Gumbel variate
S_n = standard deviation of Gumbel's variate
Y_T = reduced variate
T = return period
K_T = frequency factor
X_T = required rainfall depth
I = rainfall intensity
P_{avg} = average precipitation for specific duration
S = standard deviation of the precipitation for specific duration
d = specific duration
C, m, e = parameters of the empirical equation subject to the region under study
σ_s = standard error of estimates
R^2 = correlation coefficient

2. Review of Literature

Since its set up in 1932, IDF relations are still somewhat missing in most of the developing countries and especially for most of the regions in Pakistan, there is a minute amount of work seen in this regard such as **Umair, R et.al, 2023**; **Zubair, K 2007** and **Ahmed, R., and Ali, S. 2016.** On the other hand the literature based on IDF calculations seems to be quite rich; **Hamaamin, 2017** derived the IDF curves for Sulaimani City in Iraq for variable duration's and return periods. He found that the values of rainfall intensities for each of the selected RP is in good agreement with the estimated values derived from empirical relation of the region. **Dorneles et al. 2019** employed both the empirical as well as pluviographic methods for calculations of rainfall intensity by using different distributions such as (Normal, Log normal and Gamma) in addition to Gumbel distribution. While for the empirical case, they applied the Weibull conceptualisation for RP 2, 5, 10, 20 and 25 while for the RP of 50 and 100, the customary equation. They concluded a little to mention difference between the results obtained from both the models (Weibull and customary method). **Sabino 2019** and coworkers constructed IDF curves by employing pluviographic investigation and empirical modelling for various duration's and RP; displaying a high value of correlation ranging from 86.65% to 95.96% among the equations. They have found a large amount of variance in the results obtained from different stations located in Mato Grosso State, Brazil and suggested installation of additional number of weather stations . **Cardoso et. al. 2014**, proposed a pulviographic and disaggregation models for comparing the rainfall intensities, concluding a separation between the values of both models as the duration increases. They came up with a new regional equation for Sages, Brazil which was more sophisticated than the previous one in the sense that it made the use of hydrology more feasible. **Nyamathi and Kumar, 2020,** studied the generation of IDF curves using different probability distribution functions and chi square test to find the good fit among the distributions for various stations in the study. **Shukor et. al. 2020** projected a recent bias correction approach known as Quantile mapping into the Community Climate System Model version 3 (CCSM3) for the state of Kelantan, Malaysia. They insisted that the IDF curves generated by their model are more accurate than the pre-existing historical curves and those generated by CCSM3 (without deviation correction) due to methodological deviation from the true values.

3. Physical model

In the next subsections, we will employ Gumbel distribution to the observed rainfall data and plot its results. Then we will work out the empirical equation proposed by Bernard and plot its results as well in relation to the ones obtained by Gumbel's technique.

3.1 Mo\deling via Gumbel Distribution

The Meteorological Observatory is located at Tarnab which is nearly 16 Km away from the center of District Peshawar. The geographical location of the observatory are 34.012603º N and 71.7035023º E having an elevation of 309 m above the sea level with a hot semi-arid climate.

In the calculation phase, peak annual rainfall data for a 24 hour duration is used which is collected at the site mentioned for the years 1990-2022. Precipitation of Short duration (**Namitha 2019)** of 0.08-hr, 0.5-hr, 1-hr, 2-hr, 12-hr and 24-hr being calculated by the following equation.

$$P_t = P_{24}\left(\frac{t}{24}\right)^{1/3} \quad (1)$$

where P_t is the depth of precipitation at time t and P_{24} is the precipitation for 24 hours. Table 1 shows the use of equation 1 for estimating short time rainfall with the average and standard deviations for each rainfall duration.

For a sample size of 33, the values of Gumbel variate (y_n) and its average standard deviation (S_n) are 0.5388 and 1.1226 respectively (**Selaman 2007**). For different return periods/recurrence intervals, the values of Y_T are calculated by

$$Y_T = ln[ln(\frac{T}{T-1})] \quad (2)$$

Where Y_T and T gives the reduced variate for a given T and time period such as 2 years, 10 years etc respectively.

Table 1: Short duration estimation of rainfall by equation 1

Year	Peak daily rainfall (mm)/year (P_{24})	Precipitation $P_t = P_{24}\left(\frac{t}{24}\right)^{1/3}$					
		Hour					
		0.08	0.5	1	2	12	24
1990	46.3	0.0535	0.3215	0.6430	1.2861	7.7166	15.433
1991	30	0.347	0.2083	0.4166	0.833	5	10
1992	51.04	0.0590	0.3544	0.7088	1.4177	8.5066	17.013
1993	61	0.0706	0.4236	0.8472	1.6944	10.1666	20.333

1994	37	0.0428	0.2569	0.5138	1.0277	6.1666	12.33
1995	36	0.0416	0.25	0.5	1	6	12
1996	48	0.0555	0.3333	0.6666	1.333	8	16
1997	48.4	0.0560	0.3361	0.6722	1.3444	8.06666	16.133
1998	51.2	0.0592	0.3555	0.7111	1.4222	8.5333	17.066
1999	36	0.0416	0.25	0.5	1	6	12
2000	38	0.0439	0.2638	0.5277	1.055	6.333	12.666
2001	35.6	0.0412	0.2472	0.4944	0.9888	5.9333	11.866
2002	38	0.0439	O.2638	0.5277	1.0555	6.333	12.666
2003	45	0.0520	0.3125	0.625	1.25	7.5	15
2004	47.4	0.0548	0.3291	0.6583	1.3166	7.9	15.8
2005	32	0.0370	0.2222	0.444	0.888	5.333	10.666
2006	42	0.0486	0.2916	0.5833	1.1666	7	14
2007	50	0.0578	0.3472	0.6944	1.3888	8.333	16.666
2008	78	0.0902	0.5416	1.0833	2.166	13	26
2009	40	0.0462	0.2777	0.555	1.111	6.666	13.333
2010	95	0.1099	0.6597	1.3194	2.6388	15.833	31.666
2011	50.2	0.0581	0.3486	0.6972	1.3944	8.366	16.733
2012	41.2	0.0476	0.2861	0.5722	1.1444	6.8667	13.733
2013	54.6	0.0631	0.3791	0.7583	1.5166	9.1	18.2
2014	54.4	0.0629	0.3777	0.7555	1.5111	9.066	18.133
2015	105	0.1215	0.7292	1.4583	2.9167	17.5	35
2016	35.8	0.0414	0.2486	0.4972	0.9944	5.967	11.933
2017	60.8	0.0703	0.4222	0.8444	1.6889	10.133	20.2667
2018	116	0.1342	0.8055	1.6111	3.222	19.33	38.667
2019	64	0.0740	0.444	0.889	1.778	10.667	21.333
2020	50	0.0578	0.3472	0.6944	1.389	8.333	16.666
2021	60	0.0694	0.4166	0.8333	1.1667	10	20
2022	42.4	0.0490	0.2944	0.5888	1.178	7.066	14.133
P_{avg}		**0.0603**	**0.3620**	**0.7240**	**1.4480**	**8.6885**	**17.3771**
S		**0.0232**	**0.1397**	**0.2794**	**0.5588**	**3.3528**	**6.7056**

Equation 2 with the use of the values of y_n and S_n delivers the following equation.

$$K_T = \frac{Y_T - y_n}{S_n} \tag{3}$$

here K_T is known as the frequency factor. Its values are given in table 2 for each RP. Using equation 3, we can find rainfall depth and intensity by equations 3 and 4 respectively for each duration and RP as

$$X_T = P_{avg} + K_T S \tag{4}$$

$$I = {X_T}/{t} \tag{5}$$

where X_T and I are the required rainfall depth and intensity respectively, P_{avg} is the average precipitation (rainfall) for specific duration and S is the standard deviation. For the values of P_{avg} and S, refer to table 1.

Table 2: Rainfall depth and intensities for each RP by Gumbel method

Duration t		Return Period T					
hour	min	2	10	25	50	75	100
		Frequency Factor K_T for each Return Period					
		-0.153	1.524	2.369	2.995	3.360	3.617
		Rainfall depth (mm), using equation 4					
0.08	5	0.0567	0.0956	0.1152	0.1298	0.1382	0.1442
0.5	30	0.3405	0.5749	0.6929	0.7805	0.8313	0.8674
1	60	0.6811	1.1499	1.3859	1.5610	1.6627	1.7348
2	120	1.3622	2.2999	2.7719	3.1220	3.3255	3.4696
12	720	8.1739	13.8003	16.6321	18.7329	19.9540	20.8182
24	1440	16.3479	27.6007	33.2644	37.4660	39.9082	41.6366
hour	min	Rainfall Intensity (mm/hr), using equation 5					
0.08	5	8.1770	13.7877	16.6117	18.7066	19.9243	20.7861
0.5	30	1.3622	2.2999	2.7719	3.1220	3.3255	3.4696
1	60	0.6811	1.1499	1.3859	1.5610	1.6627	1.7348
2	120	0.3405	0.5749	0.6929	0.7805	0.8313	0.8674
12	720	0.0567	0.0958	0.1155	0.1300	0.1385	0.1445
24	1440	0.0283	0.0479	0.0577	0.0650	0.0692	0.0722

3.2 Modeling via empirical equation

After Gumbel statistical approach, we turn to empirical modelling of our data to check how its fit with the previous method. For this purpose, the following empirical equation (**Mahdi, 2020)** is employed which is applicable to every region

$$I = C\frac{T^m}{d^e} \tag{6}$$

where the parameters C, m and e are constants subject to the region under study. T and d represent the return period and duration respectively. For the determination of above parameters, equation 6 is converted into logarithmic form

$$log\ I = log\ K - e\ log\ d \tag{7}$$

here

$$K = CT^m \tag{8}$$

By using linear regression, where plotting $log\ I$ on y-axis and $log\ d$ on x-axis for each of the RP gives the slope of the straight line gives the value of parameter e and the y-intercept yields the value of $log\ K$. In our case, the value of e varies for each RP. Therefore the average of e is $e_{av} = \frac{\sum e}{n}$. Now taking log of equation 8

$$\log K = \log C + m \log T \tag{9}$$

By plotting $\log K$ on y-axis and $\log T$ on the x-axis, the values of m and C are obtained where the first one is the slope of the straight line and the latter is the anti-log of the y-intercept. Table 6 shows the values of parameter e and intensities for each of the return period and duration by equation 6, whereas the values of parameters m and C are 0.2349 and 0.6176 respectively. Table 7 shows the empirical IDF relations with respect to each RP.

Table 3: Rainfall intensities for each RP by empirical method

Duration t		Return Period T					
hour	min	2	10	25	50	75	100
		Values of parameter *e* for each Return Period					
		8.31×10^{-6}	1.84×10^{-4}	2.28×10^{-4}	2.52×10^{-4}	2.64×10^{-4}	2.71×10^{-4}
		Rainfall intensity (mm/hr), using equation (6)					
0.08	5	9.0865	13.2685	16.4577	19.3699	21.3066	22.797
0.5	30	1.4538	2.1222	2.6321	3.0977	3.4074	3.6457
1	60	0.7269	1.0609	1.3158	1.5486	1.7033	1.8225
2	120	0.3634	0.5304	0.6578	0.7741	0.8515	0.9110
12	720	0.0605	0.0883	0.1095	0.1289	0.1418	0.1517
24	1440	0.0302	0.0441	0.0547	0.0644	0.0709	0.0758

Table 4: Derived IDF relation for each RP empirically

Return Period (year)	Empirical IDF relation (equation 6)
2	$I = 0.6176(T^{0.2349}/d^{8.31\times10^{-6}})$
10	$I = 0.6176(T^{0.2349}/d^{1.84\times10^{-4}})$
25	$I = 0.6176(T^{0.2349}/d^{2.287\times10^{-4}})$
50	$I = 0.6176(T^{0.2349}/d^{2.525\times10^{-4}})$
75	$I = 0.6176(T^{0.2349}/d^{2.641\times10^{-4}})$
100	$I = 0.6176(T^{0.2349}/d^{2.715\times10^{-4}})$

4. Results and Discussions

The observed rainfall data of Tarnab from table 2 is plotted in Fig 1 using Gumbel distribution method. Based on the profile of IDF curves, in all RP, rainfall intensities drop-off with rainfall duration and increase as RP grow (**Newton et. al. 2017),** an established trend of IDF curves. As the RP increases, the separation between the curves diminishes for short duration getting quite close for 75 and 100 years RP (Table 1). However, for larger durations rainfall intensities for all RP lie extremely close to each other.

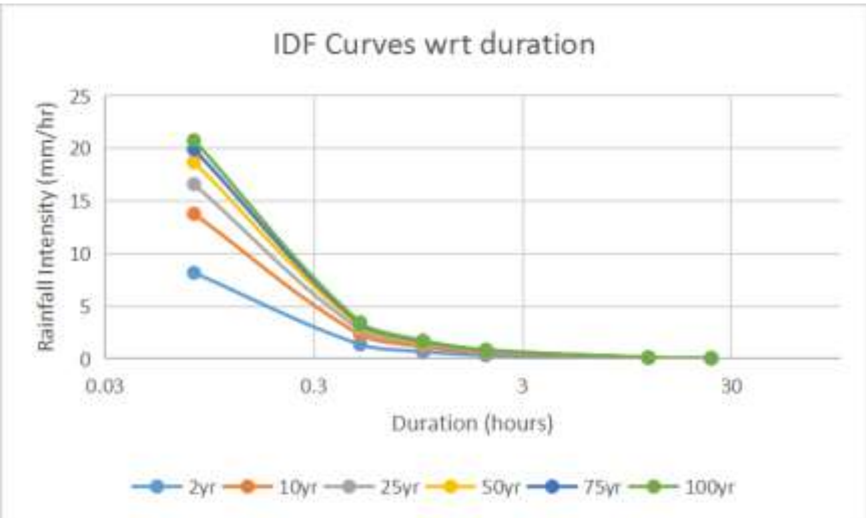


Fig 1: IDF curves obtained by Gumbel distribution

All these results can be attributed to the fact that the region under study receives a very minute amount of rainfall annually, which leads to such conglomeration of the curves for long durations. Fig 2 shows the IDF profile with respect to RP where the RP of shortest duration of 0.08 h exhibiting the maximum rain intensity. However as we increase the duration, the intensity of rain becomes small and small.

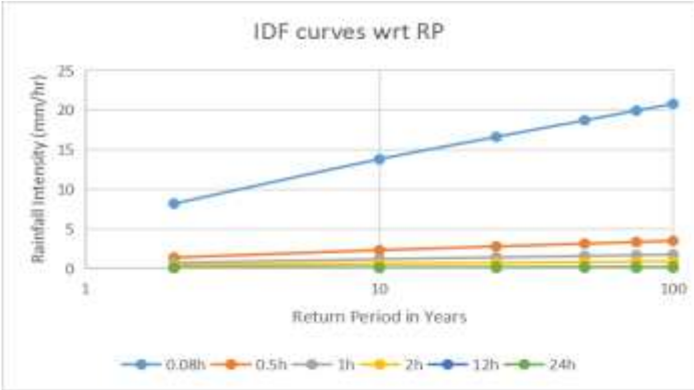
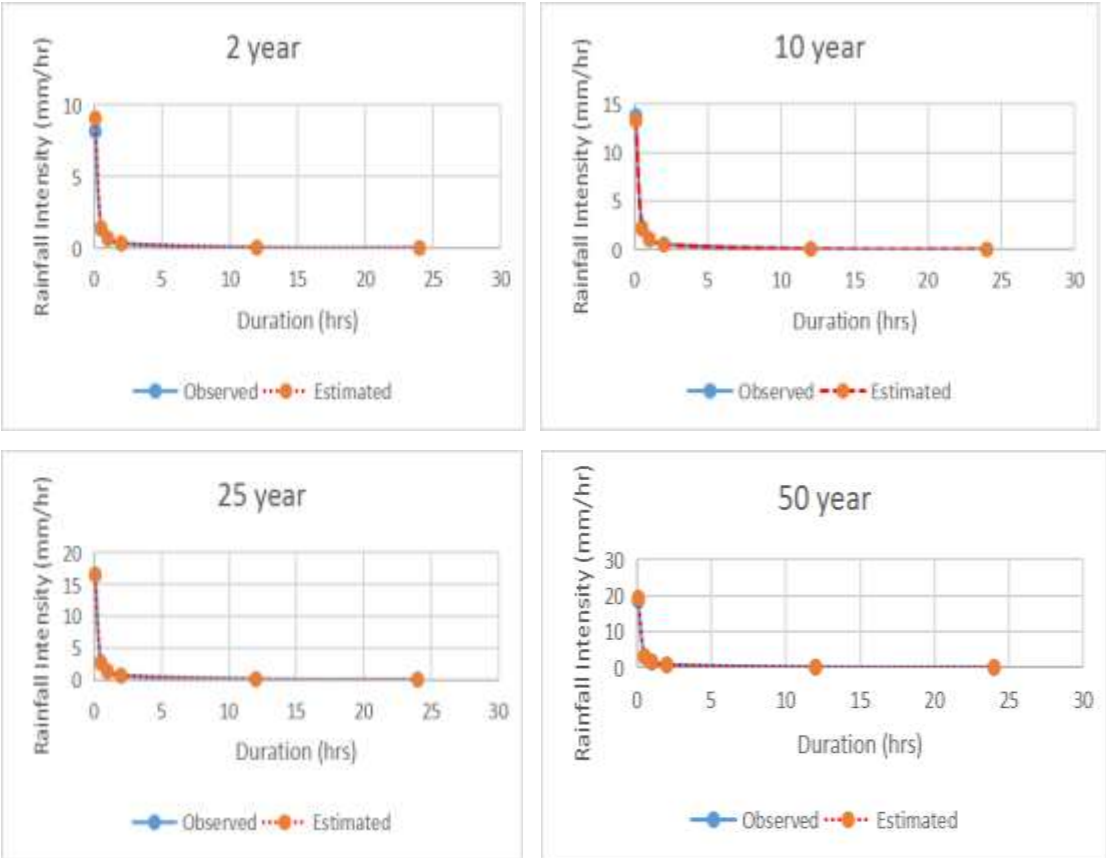


Fig 2: IDF curves obtained by Gumbel distribution in terms of RP

In Fig 3, the intensities of all the RP are plotted separately via both the Gumbel distribution and those obtained by equation 6. All the curves show a good agreement between the observed and estimated values of rainfall intensities. For further elaboration Table 5 show the values of standard error of estimates and R^2 for all the return periods. Fig 3 and Table 5 combined support the argument of good match between the observed and estimated data.



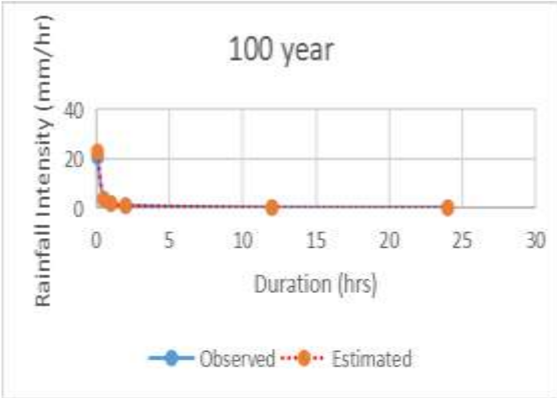


Fig 3: IDF curves for each RP by Gumbel (blue) and empirical (orange)

Table 5: Values of standard error of estimates and R^2 for each RP

Return Period (year)	Standard error of estimates σ_s	R^2
2	0.3737	1
10	0.2277	1
25	0.0907	1
50	0.2710	1
75	0.5656	1
100	0.8250	1

5. Conclusions

In this work we have employed Gumbel distribution method along with empirical derivation to obtain IDF relations from both the techniques for Tarnab ranging for different durations (0.08, 0.5, 1, 2, 12, 24) hours and RP (2, 10, 25, 50, 75 and 100) years. The results acquired via both the methods has shown a good agreement as clear form Fig 3 and Table 5 with a correlation $R^2 = 1$ and $\sigma_s < 0$ for each of the RP.

The results obtained predict the rainfall analysis of Tarnab accurately and in alignment with the globally accepted Bernard equation with a negligible error. However, the authors emphasize on applying other statistical distributions (Agarwal et. al.2021) and encroaching towards the correction of this work to probabilistic flood analysis (Breinl et. al. 2021); deeming it rather more useful in predicting the future flood discharges subject to changing climatic conditions. For instance, on 29th of July 2010, the central region of Peshawar received a 274 mm of rainfall which was the highest recorded for the region, however, Tarnab has received only 95 mm on the same day. Although accurate real time predictions of natural events is a challenging task but these probabilities still provide a glimpse of the big picture, making it quite useful for the water management departments to take necessary decisions.

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Conflict of interest

No funding were taken for conducting this research work. The authors disclose no conflict of interest whether financial or in the ideas presented in this article.

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