## Important Theories in Mathematics.

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## ABSTRACT:

This paper contains some very important theories based on some very basic concepts used in our day to day life. These statements will reduce the calculation and also reduce the thinking time. Concepts like finding exponent of a prime in any number, Triangle, Prism and Geometry are covered in this paper. This paper elaborates how to find the exponent of some special non prime numbers in any number. This paper also shows some interesting things about a prism and glitches of human eye.

## Half Exponential Non Prime Number.

$4=2 \times 2$
$6=2 \times 3$
$8=2 \times 2 \times 2$
$9=3 \times 3$
$10=2 \times 5$
$12=2 \times 2 \times 3$
$14=2 \times 7$
$15=5 \times 3$
$16=2 \times 2 \times 2 \times 2$
$18=2 \times 3 \times 3$
$20=2 \times 2 \times 5$
$21=3 \times 7$
$22=2 \times 11$
$24=2 \times 2 \times 2 \times 3$
$25=5 \times 5$
The above numbers are the non primes from 1 to 25 . Their prime factorisation is also mentioned.
We know that calculating exponent of prime in any number ! (Factorial) is easy. Exponent of prime $=\mathrm{p}$ in n ! Will be.

$$
\left[\frac{n}{p}\right]+\left[\frac{n}{p^{2}}\right]+\left[\frac{n}{p^{3}}\right]+\left[\frac{n}{p^{4}}\right]+\ldots\left[\frac{n}{p^{k}}\right] \text { where } p^{k+1}>n
$$

However this process is not valid for non prime numbers and thus it becomes difficult to calculate the exponent of a non prime in a number!.
Half Exponential Non Prime Number Theory states that when the prime factorisation of a non prime comes in this form:

$$
x=2 \times \frac{x}{2} \text { where } \mathrm{x} \text { is a non prime number. }
$$

If the prime factorisation comes in this way then the exponent of that non prime number can be calculated by calculating the exponent of $\frac{x}{2}$ in the required number by the method used for primes as $\frac{x}{2}$ will be a prime number.

The numbers satisfying this condition from 1 to 25 are $6,10,14$ and 22 . They are underlined above for your reference

Now, let us see some examples:
Find the exponent of 6 in 800 !
$6=2 \times 3$, exponent of 6 in 800 ! will be equal to exponent of 3 in 800 !

$$
\left[\frac{800}{3}\right]+\left[\frac{800}{3^{2}}\right]+\left[\frac{800}{3^{3}}\right]+\left[\frac{800}{3^{4}}\right]+\left[\frac{800}{3^{5}}\right]+\left[\frac{800}{3^{6}}\right]=
$$

$$
266+88+29+9+3+1=396
$$

Thus, exponent of 6 in 800 ! will be 396.

Find the exponent of 10 in 1000!
$10=2 \times 5$, exponent of 10 in 1000 ! will be equal to exponent of 5 in 1000

$$
\left[\frac{1000}{5}\right]+\left[\frac{1000}{5^{2}}\right]+\left[\frac{1000}{5^{3}}\right]+\left[\frac{1000}{5^{4}}\right]=
$$

$$
200+40+8+1=249
$$

Thus, exponent of 10 in 1000 ! will be 249 .
Find the exponent of 14 in 500 !
$14=2 \times 7$, exponent of 14 in 500 ! will be equal to exponent of 7 in 500 !

$$
\left[\frac{500}{7}\right]+\left[\frac{500}{7^{2}}\right]+\left[\frac{500}{7^{3}}\right]=
$$

$$
71+10+1=82
$$

Thus, the exponent of 14 in 500 ! will be 82 .
Find the exponent of 22 in 1500 !
$22=2 \times 11$, exponent of 22 in 1500 ! will be equal to exponent of 11 in 1500 !

$$
\left[\frac{1500}{11}\right]+\left[\frac{1500}{11^{2}}\right]+\left[\frac{1500}{11^{3}}\right]=
$$

$$
136+12+1=149
$$

Thus, the exponent of 22 in 1500 ! will be 149 .

## Triangle Prims Area Theorem.

Let us suppose that we have 4 triangles; $\triangle \mathrm{ABC}, \triangle \mathrm{GBA}, \Delta \mathrm{GCB}$ and $\triangle \mathrm{GAC}$.


Let us suppose that all the 4 triangles have equal base. $\operatorname{Seg} \mathrm{BC}=\operatorname{seg} \mathrm{BA}=\operatorname{seg} \mathrm{CB}=\operatorname{seg} \mathrm{AC}$.
Here, $\triangle \mathrm{ABC}$ is an equilateral triangle. As seg BC is equal to the bases of the other triangles, $\mathrm{AB}=\mathrm{AC}=\mathrm{BC}=\mathrm{BA}=\mathrm{CB}=\mathrm{AC}$.
Now, a prism is constructed with the help of these four triangles by arranging them in this way.


Note: (1) ' $x$ ' denotes the angle by which the prism is been constructed.
(2) $\mathrm{x} 3^{\circ}>\mathrm{x} 2^{\circ}>\mathrm{x} 1^{\circ}$

When we see this prism from above (top view), it looks like a triangle with 3 separations.
(3) $1(\mathrm{AC})=1(\mathrm{BC})=1(\mathrm{AB})$
(4) V1, V2 and V3 are the three vertexes of this triangle.

Now, by taking V1 vertex on the upper side.
A


C
By keeping V1 on the upper side, we can see that all areas are equal, means,
In $\triangle \mathrm{ABC}$,
Area of $\triangle \mathrm{AGB}=$ area of $\triangle \mathrm{AGC}=$ area of $\triangle \mathrm{BGC}$.
Let area of each triangle be a.
Now, let's keep V2 vertex on the upper side


In (V2) second vertex on upper side,

In $\triangle \mathrm{ABC}$,
Area of $\Delta \mathrm{GAB}=$ area of $\Delta \mathrm{GAC}$
$\Delta \mathrm{GAB}>\Delta \mathrm{GBC}$ similarly,
$\Delta \mathrm{GAC}>\Delta \mathrm{GBC}$
Here, area of $\Delta \mathrm{GAB}$ is twice the area of $\Delta \mathrm{GBC}$.
Similarly, area of $\Delta \mathrm{GAC}$ is twice the area of $\Delta \mathrm{GBC}$.

So, $\Delta \mathrm{GAB}=\Delta \mathrm{GAC}=2 \Delta \mathrm{GBC}$.

Now, let the area of $\Delta \mathrm{GBC}$ be a and the area of $\Delta \mathrm{GAB}$ and $\Delta \mathrm{GAC}$ be 2 a .

Now, let us keep V3 vertex on the upper side.
A

B
C

In $\triangle \mathrm{ABC}$,
Area of $\Delta \mathrm{GBC}<$ area of $\Delta \mathrm{GAB}$

And
Area of $\Delta \mathrm{GBC}=$ area of $\Delta \mathrm{GAC}$.
Here, area of $\Delta \mathrm{GAB}>$ area of $\Delta \mathrm{GAC}$.
$\Delta \mathrm{GAB}=\Delta \mathrm{GBC}+\Delta \mathrm{GAC}$
$\Delta \mathrm{GAB}=\Delta \mathrm{GAC}+\Delta \mathrm{GAC}$
$\Delta \mathrm{GAB}=2 \Delta \mathrm{GAC}$.

Similarly, $\triangle \mathrm{GAB}=2 \Delta \mathrm{GBC}$.
Now, let the area of $\Delta \mathrm{GBC}$ and $\Delta \mathrm{GAC}$ be a and the area of $\Delta \mathrm{GAB}$ be 2 a .
Now, Adding all the areas
When V1 vertex was on the upper side.
$\Delta \mathrm{GBC}=\mathrm{a}$
$\Delta \mathrm{GAB}=\mathrm{a}$
$\Delta \mathrm{GAC}=\mathrm{a}$
When V2 vertex was on the upper side.
$\Delta \mathrm{GBC}=\mathrm{a}$
$\Delta \mathrm{GAB}=2 \mathrm{a}$
$\Delta \mathrm{GAC}=2 \mathrm{a}$
When V3 vertex was on the upper side.
$\Delta \mathrm{GBC}=\mathrm{a}$
$\Delta \mathrm{GAB}=2 \mathrm{a}$
$\Delta \mathrm{GAC}=\mathrm{a}$
Adding areas.
$\Delta \mathrm{GAB}=\mathrm{a}+2 \mathrm{a}+2 \mathrm{a}=5 \mathrm{a}$
$\Delta \mathrm{GAC}=\mathrm{a}+2 \mathrm{a}+\mathrm{a}=4 \mathrm{a}$
$\Delta \mathrm{GBC}=\mathrm{a}+\mathrm{a}+\mathrm{a}=3 \mathrm{a}$
$\Delta \mathrm{GAB}>\Delta \mathrm{GAC}>\Delta \mathrm{GBC}$
And
$\mathrm{X} 3^{\circ}>\mathrm{x} 2^{\circ}>\mathrm{x} 1^{\circ}$.
It might be an eye glitch that we see all the three different sections while observing through keeping different vertexes on the upper side.
From this we can say that,
"The angle made by the triangles for the construction of a prism is indirectly proportional to the areas of triangles on the plan (Top View)."

## Reference:

1) Maharashtra State Board, Navneet, std 10thMathematics Digest (part 1) book (Year of publication $=2022$ )-by Navneet.
2) Numerical Methods. (Year of publication = 2006) -by Dr P. Kandasamy, Dr K. Thilagavathy and Dr K. Gunavathi.
3) Encyclopaedia of Mathematics.
