International Journal of Research Publication and Reviews, Vol 4, no 11, pp 1785-1792 November 2023

International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Transition Study of Isotropic S-S-F-F Structural Plate Element Under Buckling Load Considering the Polynomial Case of Odd Order.

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ABSTRACT

The considered the impact of buckling load on at structural plate element which is support on two main edges. The y component rest on simply and clamped supported edge while the x axis rests on also Simple and clamped edges. The orientation form SSFF shape orientation. The research was carried out using 3rd energy Functional. The simple simple fixed fixed plate was considered as the direct independent plate. This is a situation where the material properties are the same round about the shape of the object. Such properties includes the flexural rigidity, poison ratio and young elastic modulus of elasticity. Due to the shape orientation which form bases for the entire study, the shape functions were first formulated, after which the various integral values of the differentiated shape functions, of the various boundary conditions were all derived. From the results gotten, the stiffness coefficients of the various boundary cases were also formulated. Further resolutions of these, produced the Third order strain energy equation and further expansion Third order strain energy equation gave rise to the Third Order Overall Potential Energy Functional. The Third Order Overall Potential Energy Functional, with respect to the amplitude was further integrated and this gave a result known as the Lead equation. Further minimization of the derived Lead equation gave rise to the formulation of the vital buckling load equations, together with its coefficients. Next to this was the formulation of the non-dimensional buckling load parameters and upon substitution of the aspect ratios m/n ranges from one to two and considering it at the interval of one-tenth. The behavior of the non-buckling load parameters against the various aspect ratios shown from the final values of the analysis

Key words:

- Sm-Sm-Fx-Fx Plate
- Overall Potential Energy Functional,
- Non-buckling load parameters
- Lead Equation,
- Vital Buckling load

1. Introduction

Structural plate elements are widely used Engineering materials. Their relevance in Civil, Structural, Mechanical and Aeronautic Engineering, can not be over emphasized. The impact of the use of these materials cannot be over look due to their high importance in our everyday life. Due to their importance and wide application, several researches have been conducted with the target of maximizing their high values for wider structural applications. Such researches dates as far back as 1776, when Euler carried out a free vibration analysis of plate problems. His work stimulated Chladni (a German physicist) into research which led resulted to the discovery of the various modes of free vibration of plates This plate element can be considered as a structural element which is either straight or curved, and also possessing three dimensions referred to as the primary, secondary and tertiary dimensions. The smallest of the dimensions is the tertiary dimension also known as the plate thickness. They are usually very small when compared to other dimensions. The isotropic rectangular SmSmFxFx plate have all their material properties in all directions as the same and so they classified as direction independent element. Stability analysis sometimes is also referred to as the plate buckling. The subject has been a subject of study in solid structural mechanics for a long time now. Although the buckling analysis of rectangular plates has received the attention of many researchers for several centuries Prior to this time, other researchers have gotten solution using even order energy functional for Buckling of plate. so the resolution of the buckling tendency of SIMPLE SIMPLE FIXED FIXED isotropic plate using odd order energy functional is the gap the work tends to fill. The plate arrangement can be as shown

1.1 Formulation of The Buckling Load Equation.

The summation of Strain energy, Є and External Work, E_w , gives overall potential energy, O_p . Expressed mathematically as

$$O_p = E_w + Є \qquad \text{i}$$

Where as the strain energy, is formulated from the product of normal stress and normal strain, both in the x components as

$$§_x ð_x = \frac{Ez^2}{1-μ^2}\left(\left[\frac{\partial^2 fu}{\partial x^2}\right]^2 + μ\left[\frac{\partial^2 fu}{\partial x \partial y}\right]^2\right) \qquad \text{ii}$$

while also in vertical direction (Y axis) as shown in the Equation iii

$$§_y ð_y = \frac{Ez^2}{1-μ^2}\left(\left[\frac{\partial^2 fu}{\partial y^2}\right]^2 + μ\left[\frac{\partial^2 fu}{\partial x \partial y}\right]^2\right) \qquad \text{iii}$$

Considering the parallel effect of the stress and strain on the plate surface, gives the product of the in-plane shear stress and in-plane shear strain as

$$\tau_{xy}γ_{xy} = 2\,\frac{Ez^2(1-μ)}{(1-μ^2)}\left[\frac{\partial^2 fu}{\partial x \partial y}\right]^2 \qquad \text{iv}$$

Bringing Equations ii, iii and iv together gives

$$§_x ð_x + §_y ð_y + \tau_{xy}γ_{xy} = \frac{Ez^2}{1-μ^2}\left(\left[\frac{\partial^2 fu}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 fu}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 fu}{\partial y^2}\right]^2\right) \qquad \text{v}$$

The strain Energy is given mathematically as $Є = \frac{1}{2}\iint_{xy} \overline{Є}\, dxdy$ vi

$$\text{Where}\quad \overline{Є} = \frac{Ez^2}{1-μ^2}\int\left(\left[\frac{\partial^2 fu}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 fu}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 fu}{\partial y^2}\right]^2\right) \qquad \text{vii}$$

Further rearrangement of Equation vii, gives the third order strain energy equation

$$\text{as}\quad Є = \frac{G}{2}\int_0^n\int_0^m\left(\frac{\partial^3 fu}{\partial x^3}.\frac{\partial fu}{\partial x} + 2\frac{\partial^3 fu}{\partial x\,\partial y^2}.\frac{\partial fu}{\partial x} + \frac{\partial^3 fu}{\partial y^3}.\frac{\partial fu}{\partial y}\right)dxdy \qquad \text{viii}$$

with the external load as $v = -\frac{bkl_x}{2}\int_0^n\int_0^m\left(\frac{\partial fu}{\partial x}\right)^2 dxdy$ ix

The third order total potential energy functional is expressed mathematically as

$$O_p = \frac{G}{2}\int\int\left(\frac{\partial^3 fu}{\partial x^3}.\frac{\partial fu}{\partial x} + 2\frac{\partial^3 fu}{\partial x^2\,\partial y}.\frac{\partial fu}{\partial y} + \frac{\partial^3 fu}{\partial y^3}.\frac{\partial fu}{\partial y}\right)dxdy - \frac{bkl_x}{2}\int\int\frac{\partial^2 fu}{\partial x^2}dxdy \qquad \text{x}$$

Rearranging the total potential energy equation in terms of non-dimensional parameters I, J the buckling load equation is gotten as

$$bkl_{top} = \frac{G}{a^2}\int_0^1\int_0^1.\left(\left[\frac{\partial^3 fu}{\partial J^3}\right].\frac{\partial fu}{\partial J} + 2\frac{1}{p^2}\left[\frac{\partial^3 fu}{\partial J \partial I^2}\right].\frac{\partial fu}{\partial J} + \frac{1}{p^4}\left[\frac{\partial^3 fu}{\partial I^3}\right].\frac{\partial fu}{\partial I}\right)dJdI \qquad \text{xi}$$

$$bkl_{bottom} = \int_0^1\int_0^1\left(\frac{\partial fu}{\partial J}\right)^2 dJdI \qquad \text{xii}$$

$$bkl_x = \frac{bkl_{top}}{bkl_{bottom}} \qquad \text{xiii}$$

1.2 Shape Function formulation and analysis

Two major support conditions were considered, in the derivation of the shape functions and they namely Fixed support which was denoted as Fx and Simple support which is denoted as Sm. For Simple support condition, the deflection equation *fw* and the 2nd order derivative of the deflection equation fw^2, were equated to zero and simultaneous equations were formed by considering I = 0 at the left hand support in the case of the horizontal component (X axis) and I = 1 at the right side of the same component. Also considering the top as J = 0 and J = 1 at the bottom support for the Y axis. These equations were solved simultaneously to obtain the various values of the primary and secondary dimensions (n_1, m_1, n_2, m_2, n_3, m_3, n_4 and m_4) for the SmSmFxFx plate element. Where I and J are non-dimensional parameters parallel to X and Y axis respectively as earlier explained.

1.3 Formulation of the Deflection Equation

Figure i Isotropic Rectangular SmSmFxFx Plate

The case of horizontal Direction (X- X axis)

Figure ii Horizontal Support

Considering the X- X axis

But $fw_x = m_o + m_1I + m_2 I^2 + m_3I^3 + m_4I^4 + m_5I^5$ 1

The first derivation of Equation 1 gives

$fw_x^1 = m_1 + 2m_2I + 3m_3I^2 + 4m_4I^3 + 5m_5I^4$ 2

also the second derivative of the Equation 1 gives

$fw_x^2 = 2m_2 + 6m_3I + 12m_4I^2 + 20m_5I^3$ 3

and finally the third derivative the same Equation gives

$fw_x^3 = 6m_3 + 24m_4I + 60m_5I^2$ 4

1.3.1 Analysis of the Horizontal component

Introducing the boundary conditions on the horizontal component

At the left support, I = 0

When $fw_x = 0$

$fw_x = 0 = m_o + 0 + 0 + 0 + 0$ 5

$m_o = 0$

Also when $fw_x^2 = 0$ 6

$fw_x^2 = 0 = 2m_2 + 0 + 0 + 0$ 7

$2m_2 = 0$ 8

$m_2 = 0$ 9

at the right support, I =1

$fw_x^1 = m_1 + 0 + 3m_3 + 4m_4 + 5m_5 = -\frac{2m_5}{3}$ 10

Further simplifying Equation 10 gives

$m_1 = -3m_3 - 4m_4 - 5m_5 - \frac{2m_5}{3}$ 11

Also for the second derivative of the deflection on the X axis,

$fw_x^2 = 0 = 0 + 6m_3 + 12m_4 + 20m_5$ 12

rearranging the equation and making n_3 the subject gives

$m_3 = \frac{-12m_4 - 20m_5}{6}$ 13

in simpler form as

$m_3 = \frac{-10m_5}{3} - 2m_4$ 14

Solving for the third derivative of the deflection on the horizontal component gives

$fw_x^3 = 0 = 6m_3 + 24m_4 + 60m_5$ 15

That is

$fw_x^3 = 0 = m_3 + 4m_4 + 10m_5$ 16

$n_3 = \frac{-10m_5 - 4m_4}{1}$ 17

Resolving Equation 14 and 17 together gives

$\frac{-10m_5}{3} - 2m_4 = \frac{-60m_5 - 24m_4}{6}$ 18

and further simplifying gives

$m_4 = \frac{-10m_5}{3}$ 19

But substituting Equation 19 into Equation 17 gives

$m_3 = \frac{-10m_5 - 4(\frac{-10m_5}{3})}{1}$ 20

Further simplification gives

$m_3 = \frac{10m_5}{3}$ 21

Putting Equations 19 and 21 into Equation 11 gives

$m_1 = -\frac{2m_5}{3} - 3(\frac{10m_5}{3}) - 4(\frac{-10m_5}{3}) - 5m_5$ 22

and finally

$m_1 = -\frac{7m_5}{3}$ 23

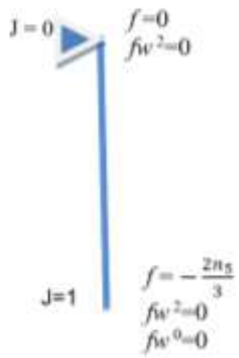
Recall that $fw_x^2 = m_o + m_1 I + m_2 I^2 + m_3 I^3 + m_4 I^4 + m_5 I^5$ 24

Putting the derived values into Equation 1 gives

$fw_x = n_5 \left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right)$ 25

1.3.2 Analysis of the Vertical Component

The case of horizontal Direction (Y- Y axis)



Also introducing the boundary conditions on the vertical component

At the top support, I = 0

When $fw_y = 0$

$fw_y = 0 = n_o + 0 + 0 + 0 + 0$ 26

$m_o = 0$

Also when $fw_y^2 = 0$ 27

$fw_y^2 = 0 = 2n_2 + 0 + 0 + 0$ 28

$2n_2 = 0$ 29

$n_2 = 0$ 30

at the right support, J =1

$fw_y^1 = n_1 + 0 + 3n_3 + 4n_4 + 5n_5 = -\frac{2n_5}{3}$ 31

Further simplifying Equation 31 gives

$n_1 = -3n_3 - 4n_4 - 5n_5 - \frac{2n_5}{3}$ 32

Also for the second derivative of the deflection on the Y axis,

$fw_y^2 = 0 = 0 + 6n_3 + 12n_4 + 20n_5$ 33

Rearranging the equation and making n_3 the subject gives

$n_3 = \frac{-12n_4 - 20n_5}{6}$ 34

in simpler form as

$n_3 = \frac{-10n_5}{3} - 2n_4$ 35

Solving for the third derivative of the deflection on the vertical component gives

$fw_y^3 = 0 = 6n_3 + 24n_4 + 60n_5$ 36

That is

$fw_y^3 = 0 = n_3 + 4n_4 + 10n_5$ 37

$n_3 = \frac{-10n_5 - 4n_4}{1}$ 38

Resolving Equation 35 and 38 together gives

$\frac{-10n_5}{3} - 2n_4 = \frac{-60n_5 - 24n_4}{6}$ 39

and further simplifying gives

$n_4 = \frac{-10n_5}{3}$ 40

But substituting Equation 19 into Equation 17 gives

$n_3 = \frac{-10n_5 - 4(\frac{-10n_5}{3})}{1}$ 41

Further simplification gives

$n_3 = \frac{10n_5}{3}$ 42

Putting Equations 19 and 21 into Equation 11 gives

$n_1 = -\frac{2n_5}{3} - 3(\frac{10n_5}{3}) - 4(\frac{-10n_5}{3}) - 5n_5$ 43

and finally

$n_1 = -\frac{7n_5}{3}$ 44

Recall that $fw_y^2 = n_o + n_1J + n_2J^2 + n_3J^3 + n_4J^4 + n_5J^5$ 45

Putting the derived values into Equation 1 gives

$fw_y = n_5(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 46

That means

$fw = fw_x * fw_y = m_4(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5) * n_5(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 47

Factorizing further gives the Amplitude and the shape function

$= m_4n_5(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 48

The shape function f is give as $(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 49

1.4 Derivation of The Stiffness Coefficients

Equation 49 is further differentiated at different stages, from where the stiffness coefficients were derived. These includes

$\frac{\partial f}{\partial I} = (-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 50

$\frac{\partial^2 f}{\partial I\, \partial J} = (-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4)(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4)$ 52

$\frac{\partial f}{\partial I\, \partial J^2} = (-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4)(\frac{60J^1}{3} - \frac{120J^2}{3} + 20J^3)$ 53

$\frac{\partial^2 k}{\partial I^2} = (\frac{60I^1}{3} - \frac{120I^2}{3} + 20I^3)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 54

$\frac{\partial^3 f}{\partial I^3} = (\frac{60}{3} - \frac{240I}{3} + 60I^2)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 55

also

$\frac{\partial f}{\partial J} = (-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5)(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4)$ 56

$\frac{\partial^2 f}{\partial J^2} = (-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5)(\frac{60J^1}{3} - \frac{120J^2}{3} + 20J^3)$ 57

$\frac{\partial^3 f}{\partial J^3} = (-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5)(\frac{60}{3} - \frac{240J}{3} + 60J^2)$ 58

Integrating the product of the Equation 55 by 50 gives the first stiffness coefficient.

That is

$k_{ssff1} = \int_0^1 \int_0^1 \frac{\partial^3 f}{\partial I^3} * \frac{\partial f}{\partial I} dIdJ$ 59

$k_{ssff1} = \int_0^1 \int_0^1 \left[\left(\frac{60}{3} - \frac{240I}{3} + 60I^2\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) * (-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)\right] dIdJ$ 60

bringing the like terms together gives

$= \int_0^1 \int_0^1 \left[\left(\frac{60}{3} - \frac{240I}{3} + 60I^2\right)\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) * \left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)\right] dIdJ$ 60a

multiplying them gives

$= \int_0^1 \int_0^1 \left[\left(\frac{60}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) - \frac{240I}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) + 60I^2\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\right) * \left(-\frac{7J}{3}(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) + \frac{10J^3}{3}(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) - \frac{10J^4}{3}(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) + J^5(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)\right)\right] dIdJ$ 60b

further minimization yields

$k_{ssff1} = *$

$=$

also integrating the product Equation 53 by 51 give the second stiffness coefficient.

That is

$k_{ssff2} = \int_0^1 \int_0^1 \frac{\partial^3 f}{\partial I \, \partial J^2} * \frac{\partial f}{\partial I} dIdJ$ 61

$k_{ssff2} = \int_0^1 \int_0^1 \left[(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4)(\frac{60J^1}{3} - \frac{120J^2}{3} + 20J^3) * (-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)\right] dIdJ$ 62

Bring the like terms together gives

$\int_0^1 \int_0^1 \left[\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) * (\frac{60J^1}{3} - \frac{120J^2}{3} + 20J^3)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)\right] dIdJ$ 62a

Multiplying the like terms gives

$= \int_0^1 \int_0^1 \left[\left(-\frac{7}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) + \frac{30I^2}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) - \frac{40I^3}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) + 5I^4\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\right) * (\frac{60J^1}{3}(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) - \frac{120J^2}{3}(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) + 20J^3(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5))\right] dIdJ$ 62b

$k_{ssff2} = * \quad =$

Furthermore integrating the product Equation 58 by 56 give the third stiffness coefficient. That is

$k_{ssff3} = \int_0^1 \int_0^1 \frac{\partial^3 f}{\partial J^3} * \frac{\partial f}{\partial J} dIdJ$ 63

$k_{ssff3} = \int_0^1 \int_0^1 \left[\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right)\left(\frac{60}{3} - \frac{240J}{3} + 60J^2\right) * (-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5)(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4)\right] dIdJ$ 64

$k_{ssff3} = \int_0^1 \int_0^1 \left[\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right)\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right) * \left(\frac{60}{3} - \frac{240J}{3} + 60J^2\right)(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4)\right] dIdJ$

$$= \int_0^1 \int_0^1 \left[\left(-\frac{7I}{3}\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right) + \frac{10I^3}{3}\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right) - \frac{10I^4}{3}\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right) + I^5\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right)\right) * \left(\frac{60}{3}(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4) - \frac{240J}{3}(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4) + 60J^2(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4)\right)\right] dIdJ$$

And finally integrating the product Equation 51 by 51 give the sixth stiffness coefficient.

That is

$$k_{ssff\,6} = \int_0^1 \int_0^1 (\frac{\partial f}{\partial I} * \frac{\partial f}{\partial I}) dIdJ \qquad 63$$

$$k_{ssff6} = \int_0^1 \int_0^1 \left[\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) * (-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)\right] dIdJ \qquad 64$$

Collecting the like terms together gives

$$= \int_0^1 \int_0^1 \left[\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4) * \left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)\right] dIdJ \qquad 65$$

Opening the brackets gives

$$= [[\left(-\frac{7}{3}(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4) + \frac{30I^2}{3}(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4) - \frac{40I^3}{3}(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4) + 5I^4(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4)\right) * \left(-\frac{7J}{3}(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) + \frac{10J^3}{3}(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) - \frac{10J^4}{3}(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) + J^5(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)\right)]^1]^1 \qquad 65b$$

Putting the upper and lower limit values gives

$k_{ssff\,6} = *$

=

Reducing Equation xiii in terms of the stiffness coefficients gives

$$bkl_x = \frac{D(kssff_1 + 2\frac{1}{p^2}kssff_2 + \frac{1}{p^4}kssff_3)}{kssff_6 m^2} \qquad 65$$

Substituting the real values in to Equation 65 gives

$$bkl_x = \frac{D(....+2\frac{1}{p^2}...... + \frac{1}{p^4}......)}{......m^2} \qquad 66$$

RESULTS AND DISCUSSION.

The results for the stiffness coefficients and the critical buckling load coefficients were derived. The critical buckling load coefficients were considered at different aspect ratios. The first table represents the values of the stiffness coefficients while the other contains the critical buckling coefficients for the aspect ratio of m/n, both for the previous and present study. The values of the aspect Ratios ranges from 2.0 to 1.0 with arithmetic increase of 0.1. From the values generated in the tables, it was observed that as the aspect ratio increases from 1.0 to 2.0, the critical buckling load decreases. This occurred both in the present and previous results.

Table 1.1 Stiffness Coefficients from Previous researchers

Stiffness coefficients, sc	Derived values
k_{ssff1}	0.67096
k_{ssff2}	0.04043
k_{ssff3}	0.006047
k_{ssff6}	0.0159444

Table 1.2 Stiffness Coefficients from Present Work

Stiffness coefficients, k	Derived values
k_{ssff1}	0.65455
k_{ssff2}	0.0400137
k_{ssff3}	0.0059651
k_{ssff6}	0.0153545

Table 1.3 Critical buckling load values for CSCF Plate from Previous/Present.

m/n		2	1.9	1.8	1.7	1.6
B		43.9346	44.0759	44.2415	44.4373	44.6711
B_x	Previous	43.3728	43.5151	43.6826	43.8814	44.1201
	Present	43.9346	44.0759	44.2415	44.4373	44.6711

Table 1.3 cont'd.

m/n		1.5	1.4	1.3	1.2	1.1	1
B		44.9533	45.2985	45.7268	46.2674	46.9632	47.88
$B_{x\frac{G}{n^2}}$	Previous	44.4101	44.7674	45.2148	45.7859	46.5315	47.5319
	Present	44.9533	45.2985	45.7268	46.2674	46.9632	47.88

REFERENCES

[1] An-Chien W., Pao-Chun L. and Keh-Chynan, T. (2013)." High – Mode Buckling-restrained Brace Core Plates".Journals of the International Association for Earthquake Engineering.

[2] Ali Reza Pouladkhan (2011). "Numerical Study of Buckling of Thin Plate".International Conference on Sustainable Design and Construction Engineering. Vol. 78,Issue: 1, Pp. 152 – 157.

[3] Shinde, B.M., Sayyad, A.S. & Kowade, A.B. (2013). Thermal Analysis Of Isotropic Plates Using Hyperbolic Shear Deformation Theory. *Journal of Applied and Computational Mechanics* 7 (2013) 193-204

[4] Singh, S.K & Chakrabarti, A. (2012). Buckling Analysis of Laminated Composite Plates Using an Efficient C^0 FE Model. *Latin American Journal of Solids and Structures*. Soni, S.R. (1975), *Vibrations of Elastic Plates and Shells of Variables Thickness*. Ph.D. Thesis. University of Roarkee.

[5] Srinivasa,C.V., Suresh, Y.J. and Prema, W.P. (2012). "Buckling Studies on Laminated Composite Skew Plate". *International Journal of ComputerApplications*. Vol. 37, Issue:1, Pp. 35-47.

[6] AydinKomur and Mustafa Sonmez (2008). "Elastic Buckling of Rectangular Plates Under Linearly Varying In-plane Normal Load with a Circular Cutout". International Journal of Mechanical Sciences. Vol. 35, Pp. 361 – 371.

[7] Ahmed Al-Rajihy (2008). "The Axisymmetric Dynamics of Isotropic Circular Plates with Variable Thickness Under the Effect of Large Amplitudes". Journal of Engineering, Vol. 14, Issue :1, Pp. 2302 – 2313.

[8] Audoly, B., Roman, B. and Pocheau, A. (2002). Secondary Buckling Patterns of a Thin Plate under In-plane Compression. The European Physical Journal BCondensed Matter and Complex Systems, Vol. 27, No. 1 (May).

[9] Azhari, M, Shahidi, A.R, Saadatpour, M.M (2004) "Post Local Buckling of Skew and Trapezoidal Plate". Journal of Advances in Structural Engineering, Vol. 7, Pp 61 - 70.

[10] Azhari, M. and Bradford, M.A. (2005), "The Use of Bubble Functions for the Post-Local Buckling of Plate Assemblies by the Finite Strip Method", International Journal for Numerical Methods in Engineering, Vol.38, Issue 6.

[11] Bhaskara, L.R. and Kameswara, C.T. (2013)." Buckling of Annular Plate with Elastically Restrained External and Internal Edges". Journal of Mechanics Based Design of Structures and Machines. Vol. 41, Issue 2. Pp. 222 - 235.

[12] Da-Guang Zhang (2014). "Nonlinear Bending Analysis of FGM Rectangular Plates with Various Supported Boundaries Resting on Two-Parameter Elastic ". Archive of Applied Mechanics. Vol. 84, Issue: 1, Pp.1 -20.