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Exact Solution of Simple Clamped Thin Plate Using Non-Even Order Energy Functional

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ABSTRACT

The research examined the exact behavior of plate whose ratio of small dimension to the thickness is greater than 20. The first edge is considered as simple while the second edge is taken simple. Also both the third and fourth edges are considered as clamped. Unlike in the case of Ritz and Garlekin, odd order energy functional was adopted in the study. The Orientation functions for each plate was first derived, after then, the integral values of the differentiated Orientation functions for the various boundary cases were formulated. From these, the rigidness coefficients of the various boundary cases were formulated. The various odd order energy functional were formulated but Third order strain energy equation was finally applied for this work. The derived value which was further expanded to generate The Third Order Lead Potential Energy Functional. The Third Order Lead Potential Energy Functional was integrated with respect to the amplitude, giving a result known as the Main equation. Further minimization of the Main equation gave rise to the vital buckling load equations. Finally the non-dimensional buckling load values were generated at the different aspect ratio values by simple substitution. This was achieved by considering the ratio, m/n ranging from 1.0 to 2.0, arithmetically at the interval of 0.1. It was observed critically, that the increase in one value brought about the decrease in the other.

Key words and Notaions:

Lead Potential Energy	L_p
Main equation	M_e
Orientation functions	*f*
Rigidness coefficients	*rc*
Flexural Rigidity	*FG*
Out–of–plane displacement (deflection)	*of*

Introduction

A structural element can be referred to as a plate if it possesses three dimensions known as the primary, secondary and tertiary dimension. One of the dimensions, usually called the tertiary dimension is usually very small compared to the rest of the dimensions. The isotropic rectangular Simple Simple Clamped Clamped plate are direction independent element. This is due to the fact that the material properties in all directions are the same. In this work, the plate element is subjected to Stability analysis, which is sometimes referred to as the plate buckling in solid structural mechanics. Although the buckling analysis of rectangular plates has received the attention of many researchers for several centuries Prior to this time, other researchers have gotten solution using even order energy functional for the study of plate Buckling. This work will investigate the buckling tendency of SIMPLE SIMPLE CLAMPED CLAMPED isotropic plate, under the influence of the non-even order energy functional.

1.1 The Stability Load Equation.

The strain Energy which was derived from the first principle forms the base for the formulation potential energy. Lead potential energy, L_p is the summation of Strain energy, S_e and External Work, X_w given as:

$$L_p = S_e + X_w \qquad 1$$

To derive the strain energy, Є the product of normal stress,η and normal strain, ɥ in x direction is considered as

$$\eta_x ɥ_x = \frac{Ez^2}{1-p_r^2}\left(\left[\frac{\partial^2 of}{\partial x^2}\right]^2 + p_r\left[\frac{\partial^2 of}{\partial x \partial y}\right]^2\right) \quad 2$$

while their product in y direction is considered as

$$\eta_y ɥ_y = \frac{Ez^2}{1-p_r^2}\left(\left[\frac{\partial^2 of}{\partial y^2}\right]^2 + p_r\left[\frac{\partial^2 of}{\partial x \partial y}\right]^2\right) \quad 3$$

and finally the product of the in-plane shear stress and in-plane shear

strain is given as: $\vartheta_{xy}\rho_{xy} = 2\,\frac{Ez^2(1-p_r)}{(1-p_r^2)}\left[\frac{\partial^2 of}{\partial x \partial y}\right]^2$ 4

adding all together gives

$$\eta_x ɥ_x + \eta_y ɥ_y + \vartheta_{xy}\rho_{xy} = \frac{Ez^2}{1-p_r^2}\left(\left[\frac{\partial^2 of}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 of}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 of}{\partial y^2}\right]^2\right) \quad 5$$

But $L_p = \frac{1}{2}\iint_{xy} S_e\, dxdy$ 6

where $S_e = \frac{Ez^2}{1-p_r^2}\int\left(\left[\frac{\partial^2 of}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 of}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 of}{\partial y^2}\right]^2\right)$ 7

Upon minimisation of the expressions above, the third order strain energy equation is given as

$$S_e = \frac{FG}{2}\int_0^n\int_0^m\left(\frac{\partial^3 of}{\partial x^3}.\frac{\partial of}{\partial x} + 2\frac{\partial^3 of}{\partial x \partial y^2}.\frac{\partial of}{\partial x} + \frac{\partial^3 of}{\partial y^3}.\frac{\partial of}{\partial y}\right)dxdy \quad 8$$

with the external load as $X_w = -\frac{Bx}{2}\int_0^n\int_0^m\left(\frac{\partial of}{\partial x}\right)^2 dxdy$ 9

The third order Lead potential energy functional is expressed mathematically as

$$L_p = \frac{FG}{2}\int\int\left(\frac{\partial^3 of}{\partial x^3}.\frac{\partial of}{\partial x} + 2\frac{\partial^3 of}{\partial x^2 \partial y}.\frac{\partial of}{\partial y} + \frac{\partial^3 of}{\partial y^3}.\frac{\partial of}{\partial y}\right)dxdy - \frac{Bx}{2}\int\int\frac{\partial^2 of}{\partial x^2}dxdy \quad 10$$

$$L_p = \frac{FG}{2}\int\int\left(\frac{\partial^3 of}{\partial x^3}.\frac{\partial of}{\partial x} + 2\frac{\partial^3 of}{\partial x^2 \partial y}.\frac{\partial of}{\partial y} + \frac{\partial^3 of}{\partial y^3}.\frac{\partial of}{\partial y}\right)dxdy - \frac{S_{load}}{2}\int\int\frac{\partial^2 of}{\partial x^2}dxdy \quad 11$$

But $of = A * f$ 12

where of, S_{load}, A and of are the deflection, stability load amplitude and shape function and on introducing Equation 12 into Equation 11 gives

$$L_p = \frac{A^2.FG}{2}\int_0^m\int_0^n.\left(\frac{\partial^3 of}{\partial x^3}.\frac{\partial of}{\partial x} + 2\frac{\partial^3 of}{\partial x^2 \partial y}.\frac{\partial of}{\partial y} + \frac{\partial^3 of}{\partial y^3}.\frac{\partial of}{\partial y}\right)dxdy - \frac{A^2.S_{load}}{2}\int\int\frac{\partial^2 of}{\partial x^2}.fdxdy \quad 13$$

The Lead potential energy was further differentiated with respect to the Amplitude and orientation function and that gave

$$\frac{dL_p}{dA} = 0 = \frac{2AFG}{2}\int_0^m\int_0^n.\left(\frac{\partial^3 f}{\partial x^3}.\frac{\partial f}{\partial x} + 2\frac{\partial^3 f}{\partial x^2 \partial y}.\frac{\partial f}{\partial y} + \frac{\partial^3 f}{\partial y^3}.\frac{\partial f}{\partial y}\right)dxdy - \frac{2AS_{load}}{2}\int_0^m\int_0^n.\left(\frac{\partial f}{\partial x}\right)^2 dxdy \quad 14$$

Rearranging Equation 14 in terms of non dimensional parameters $I = \frac{x}{m}$ and $J = \frac{y}{n}$ gives

$$0 = \frac{2A.FG}{2}\int_0^m\int_0^n.\left(\frac{\partial^3 f}{\partial I^3}.\frac{\partial f}{\partial I} + 2\frac{\partial^3 f}{\partial I^2 \partial J}.\frac{\partial f}{\partial J} + \frac{\partial^3 f}{\partial J^3}.\frac{\partial f}{\partial J}\right)dxdy - \frac{2AS_{load}}{2}\int_0^m\int_0^n.\left(\frac{\partial f}{\partial I}\right)^2 dxdy \quad 15$$

Making the stability load the formula gives

$$S_{load} = \frac{TopL_p}{LowL_p} \quad 16$$

where

$$TopL_p = \frac{2A.FG}{2}\int_0^1\int_0^1.\left(\left[\frac{\partial^3 f}{\partial I^3}\right].\frac{\partial f}{\partial I} + 2\frac{1}{p^2}\left[\frac{\partial^3 f}{\partial J \partial I^2}\right].\frac{\partial f}{\partial J} + \frac{1}{p^4}\left[\frac{\partial^3 f}{\partial J^3}\right].\frac{\partial f}{\partial J}\right)dIdJ \quad 17$$

and $LowL_p = \frac{2A.FG}{2}\int_0^1\int_0^1.\left(\frac{\partial f}{\partial I}\right)^2 dIdJ$ 18

That means equation 16 can also be expressed as

$$S_{load} = \frac{FG\int_0^1\int_0^1.\left(\left[\frac{\partial^3 f}{\partial I^3}\right]\frac{\partial f}{\partial I} + 2\frac{1}{p^2}\left[\frac{\partial^3 f}{\partial I \partial J^2}\right]\frac{\partial f}{\partial I} + \frac{1}{p^4}\left[\frac{\partial^3 f}{\partial J^3}\right]\frac{\partial f}{\partial J}\right)dIdJ}{\frac{1}{a^2}\int_0^1\int_0^1\left(\frac{\partial f}{\partial I}\right)^2 dIdJ} \quad 19$$

1.2 Formulation of the orientation function

For the derivation of the shape functions, Two major support conditions were considered, namely Simple support which is denoted as Si and Clamped support which is denoted as Ci. For Simple support condition, the deflection equation "*of*" was differentiated twice to get "of^{2}" on the X axis. Both the deflection equation and second derivatives of the deflection equation were equated to zero, giving two equations. The values of I were considered as one at left edge and zero at the right edge. Then on the left hand support, both the deflection equation and first derivative of the deflection equation were equated to zero and simultaneous equations were also formed by considering I = 0 at the left hand support for X axis. The same process was repeated on Y axis since it shows the same edge orientation like the case of X axis. On the vertical axis, J is one at the top but zero at the bottom.

1.3 Orientation Function For Simple Simple Clamped Clamped Plate

Figure 1 Simple Simple Clamped Clamped Plate

The X axis

Figure 2 Simple-Clamped support on x-x axis

Considering the X- X axis

But $f_x = m_o + m_1I + m_2 I^2 + m_3I^3 + m_4I^4$ 20

$f_x^1 = m_1 + 2m_2I + 3m_3I^2 + 4m_4I^3$ 21

$f_x^{11} = 2m_2 + 6m_3I + 12m_4I^2$ 22

Introducing the boundary conditions, reduces the Equations 20-22 as explained below

At the left support, I = 0

When $f_x = 0$

$f_x = 0 = m_o + m_1I + m_2 I^2 + m_3I^3 + m_4I^4$ 23

$m_o = 0$

Also when $f_x^{11} = 0$ 24

$f_x^{ii} = 0 = 2m_2 + 0 + 0 + 0$ 25

$2m_2 = 0$ 26

$m_2 = 0$ 27

At the right support, I =1

$f_x = 0$

$f_x = m_o + m_1I + m_2 I^2 + m_3I^3 + m_4I^4$ 28

$f_x^1 = m_1 + 2m_2 I + 3m_3I^2 + 4m_4I^3$ 29

Substituting the value I, which considered as 1, gives

$f_x = m_1 + 0 + m_3 + m_4$ 30

That means

$0 = m_1 + m_3 + m_4$ where $m_0 = m_2 = 0$ 31 leaving

$m_1 + m_3 = -m_4$ 32

Also when $f_x^1 = 0$ substituting 1 for I gives

$f_x^1 = 0 = 0 = m_1 + 3m_3 + 4m_4$ 33

Recall that $m_2=0$

That implies that

$-4m_4 = m_1 + 3m_3$ 34

Solving Equation 32 and 34 simultaneously gives

$m_1 = 0.5m_4$,

$m_3 = -1.5m_4$

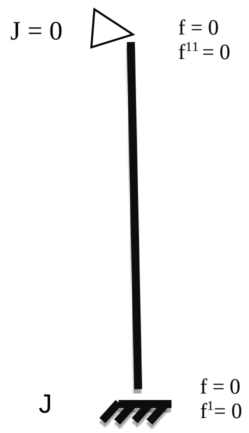
Putting the derived values back to the general Equations gives

$f_x = (0.5m_4)I + 0 + (-1.5m_4)I^3 + m_4 I^4$ 35

That means $f_x = m_4 (0.5I - 1.5I^3 + I^4)$ 36

The case of horizontal Direction (Y- Y axis)

The process remains the same due to the fact that the edges are the same. In both cases, the plate is supported simply on one end and Clamped at the other end.



$f_y = n_o + n_1 J + n_2 J^2 + n_3 J^3 + n_4 J^4$ 37

The first derivative on Y axis gives

$f_y^{11} = 2n_2 J + 6n_3 J + 12n_4 J^2$ 38

Considering the boundary conditions on the clamped ends gives

At J = 0,

$f_y = 0 = n_o + 0 + 0 + 0 + 0$ 39

Leaving $n_o = 0$ 40

Also

$f_y^{11} = n_2 + 0 + 0 + 0 + 0$ 41

$n_2 = 0$ 42

At J = 1,

$f_y = 0 = n_1 + 0 + n_3 + n_4$ 43

$n_1 + n_3 = -n_4$ 44

but differentiating f_y gives

$f_y^1 = n_1 + 2n_2 J + 3n_3 J^2 + 4n_4 J^3$ 45

Recall that $n_2 = 0$, leaving Equation 45 as

$f_y^1 = 0 = 0 = n_1 + 3n_3 + 4n_4$ 46

when the J is substituted as 1

That means $n_1 + 3n_3 = -4n_4$ 47

Solving Equation 44 and 47 together gives

$n_3 = -1.5n_4$ 48

Substituting Equation 48 into 47 gives

$n_1 = 0.5n_4$ 49

Putting them back into the general equation gives

$f_y = (0.5n_4)J + 0 + (-1.5n_4)J^3 + n_4 J^4$ 50

That means

$f_y = n_4(0.5J - 1.5J^3 + J^4)$ 51

Bringing Equation 36 and 51 together gives

But $f = f_y * f_x$

But $f = m_4 * n_4(0.5I - 1.5I^3 + I^4)(0.5J - 1.5J^3 + J^4)$ 52

1.4 Formulation of The Differential values

The Orientation function is give as $(0.5I - 1.5I^3 + I^4)(0.5J - 1.5J^3 + J^4)$ with the Amplitude as

$m_4 * n_4$ and considering the amplitude as 1, Equation 52 was further minimized by differentiating at levels, The formulated values were further integrated to get the various Rigidness coefficients, these includes

$\frac{\partial f}{\partial I} = (0.5 - 4.5I^2 + 4I^3)(0.5J - 1.5J^3 + J^4)$ 53

$\frac{\partial^2 f}{\partial I^2} = (-9I + 12I^2)(0.5J - 1.5J^3 + J^4)$ 54

$\frac{\partial^3 f}{\partial I^3} = (-9 + 24I)(0.5J - 1.5J^3 + J^4)$ 55

$\frac{\partial^2 f}{\partial I \partial J} = (0.5 - 4.5I^2 + 4I^3)(0.5 - 4.5J^2 + 4J^3)$ 56

$\frac{\partial f}{\partial I \partial J^2} = (0.5 - 4.5I^2 + 4I^3)(-9J + 12J^2)$ 57

Also for the vertical components gives

$\frac{\partial f}{\partial J} = (0.5I - 1.5I^3 + I^4)(0.5 - 4.5J^2 + 4J^3)$ 58

$\frac{\partial^2 f}{\partial J^2} = (0.5I - 1.5I^3 + I^4)(-9J + 12J^2)$ 59

$\frac{\partial^3 f}{\partial J^3} = (0.5I - 1.5I^3 + I^4)(-9 + 24J)$ 60

1.5 Formulation of The Rigidness Coefficients

The Rigidness coefficients were derived by further integrating these derived values. That is

$rc_1 = \int_0^1 \int_0^1 \frac{\partial^3 f}{\partial I^3} * \frac{\partial f}{\partial I} dIdJ$ 61

$rc_1 = \int_0^1 \int_0^1 [(-9 + 24I)(0.5J - 1.5J^3 + J^4) * (0.5 - 4.5I^2 + 4I^3)(0.5J - 1.5J^3 + J^4)]\, dIdJ$ 62

bringing the like terms together gives

$= \int_0^1 \int_0^1 [(-9 + 24I)(0.5 - 4.5I^2 + 4I^3) * (0.5J - 1.5J^3 + J^4)(0.5J - 1.5J^3 + J^4)]\, dIdJ$ 63

multiplying them gives

$= \int_0^1\int_0^1\left[-9(0.5 - 4.5I^2 + 4I^3) + 24I(0.5 - 4.5I^2 + 4I^3)) * ((0.5J(0.5J - 1.5J^3 + J^4) - 1.5J^3(0.5J - 1.5J^3 + J^4) + J^4(0.5J - 1.5J^3 + J^4)))\right] dIdJ$ 64

further minimization yields

$rc_1 = (1.8) * (0.007539683)$

$= 0.0135714$

The second rigidness coefficient was derived as follows

$rc_2 = \int_0^1\int_0^1 \frac{\partial^3 f}{\partial I\,\partial J^2} * \frac{\partial f}{\partial I} dIdJ$ 65

$rc_2 = \int_0^1\int_0^1[(0.5 - 4.5I^2 + 4I^3)(-9J + 12J^2) * (0.5 - 4.5I^2 + 4I^3)(0.5J - 1.5J^3 + J^4)]\, dIdJ$ 66

Bring the like terms together gives

$= \int_0^1\int_0^1[(0.5 - 4.5I^2 + 4I^3)(0.5 - 4.5I^2 + 4I^3) * (-9J + 12J^2)(0.5J - 1.5J^3 + J^4)]\, dIdJ$ 67

Multiplying the like terms gives

$\int_0^1\int_0^1\left[\left(0.5\,(0.5 - 4.5I^2 + 4I^3) - 4.5I^2\,(0.5 - 4.5I^2 + 4I^3) + 4I^3(0.5 - 4.5I^2 + 4I^3)\right) * (-9J(0.5J - 1.5J^3 + J^4) + 12J^2(0.5J - 1.5J^3 + J^4))\right] dIdJ$ 68

$rc_2 = (0.085714)*(0.085714)$

$= 0.007347$

Furthermore integrating the product Equation 60 by 58 give the third stiffness coefficient. That is

$rc_3 = \int_0^1\int_0^1 \frac{\partial^3 f}{\partial J^3} * \frac{\partial f}{\partial J} dIdJ$ 69

$rc_3 = \int_0^1\int_0^1[(0.5I - 1.5I^3 + I^4)(-9 + 24J) * (0.5I - 1.5I^3 + I^4)(0.5 - 4.5J^2 + 4J^3)]\, dIdJ$ 70

Bring the like terms together and multiplying them gives

$rc_3 = \int_0^1\int_0^1\left[\left(0.5I(0.5I - 1.5I^3 + I^4) - 1.5I^3(0.5I - 1.5I^3 + I^4) + I^4(0.5I - 1.5I^3 + I^4)\right) * (-9\,(0.5 - 4.5J^2 + 4J^3) + 24J(0.5 - 4.5J^2 + 4J^3))\right] dIdJ$ 71

$rc_3 = (0.0075396) * (1.8)$

$= (0.01357143)$

and finally integrating the product Equation 53 by 53 give the sixth rigidness coefficient.

That is

$rc_6 = \int_0^1\int_0^1\left(\frac{\partial f}{\partial I} * \frac{\partial f}{\partial I}\right) dIdJ$

$rc_6 = \int_0^1\int_0^1((0.5 - 4.5I^2 + 4I^3)(0.5J - 1.5J^3 + J^4) * (0.5 - 4.5I^2 + 4I^3)(0.5J - 1.5J^3 + J^4))\, dIdJ$ 72

Collecting the like terms together and multiplying out gives

$= \int_0^1\int_0^1((0.5\,(0.5 - 4.5I^2 + 4I^3) - 4.5I^2(0.5 - 4.5I^2 + 4I^3) + 4I^3(0.5 - 4.5I^2 + 4I^3))(0.5J - 1.5J^3 + J^4) * (0.5J(0.5J - 1.5J^3 + J^4) - 1.5J^3(0.5J - 1.5J^3 + J^4) + J^4(0.5J - 1.5J^3 + J^4)))\, dIdJ$ 73

Opening the brackets gives

$rc_6 = (0.00754) * (0.00754)$ 74

$= 0.0000568516$

The Stability Equation was finally reduced in terms of the rigidness coefficients and that gives

$S_{load} = \frac{FG(rc_1 + 2\frac{1}{p^2}rc_2 + \frac{1}{p^4}rc_3)}{sc_6 m^2}$ 75

Substituting the real values in to Equation 75 gives

$S_{load} = \frac{D(0.013572 + 2\frac{1}{p^2}0.007374 + \frac{1}{p^4}0.013572)}{0.0006463a^2}$ 76

1.6 Derived Results and Discussions.

The Stability buckling load coefficients were considered at different aspect ratios. Different values for the rigidness coefficients and the critical Stability load coefficients were derived. The rigidness coefficients were shown inthe first table while the other contains the critical stability coefficients for the aspect ratio of m/n, both for the previous and present study. The values of the aspect Ratios ranges from 2.0 to 1.0 and increases at the value 0.1. From the values generated in the tables, it was observed that as the aspect ratio increases from 1.0 to 2.0, the critical buckling load decreases. The observation in both the present and previous work were as presented Table 1i and Table 1ii.

Table 1i Rigidness Coefficients from Present researchers

Rigidness coefficients, rc_i	Results
rc_1	0.013572
rc_2	0.007347
rc_3	0.013572
rc_6	0.0006463

Table 1ii Stiffness Coefficients from Previous Work

Rigidness coefficients, rc_i	Results
rc_1	0.013498
rc_2	0.007433
rc_3	0.01349
rc_6	0.0006455

Table1iii Stability loads for Simple-Simple-Clamped-Clamped Plate from Previous/Present.

m/n		2	1.9	1.8	1.7	1.6
S		27.97463	28.8941	30.0098	31.38204	33.09596
S_{load}	Previous	$27.97463\frac{FG}{m^2}$	$28.8941\frac{FG}{m^2}$	$30.0098\frac{FG}{m^2}$	$31.38204\frac{FG}{m^2}$	$33.09596\frac{FG}{m^2}$
	Present	$27.9959\frac{FG}{m^2}$	$28.90885\frac{FG}{m^2}$	$30.0171\frac{FG}{m^2}$	$31.3808\frac{FG}{m^2}$	$33.08489\frac{FG}{m^2}$

Table 1iii cnt'd

In Conclusion, a simple simple clamped clamped plate order of odd polynomial functional can be resolved using third order energy functional, since the percentage different between the present and previous is very infinitesimal and in some cases the same.

m/n		1.5	1.4	1.3	1.2	1.1	1
S		35.27467	38.10109	41.85542	46.9825	54.21813	64.83966
S_{load}	Previous	$35.27467\frac{FG}{m^2}$	$38.1010\frac{FG}{m^2}$	$41.85542\frac{FG}{m^2}$	$46.9825\frac{FG}{m^2}$	$54.21813\frac{FG}{m^2}$	$64.83966\frac{FG}{m^2}$
	Present	$35.25229\frac{FG}{m^2}$	$38.06567\frac{FG}{m^2}$	$41.80506\frac{FG}{m^2}$	$46.9152\frac{FG}{m^2}$	$54.13223\frac{FG}{m^2}$	$64.73464\frac{FG}{m^2}$

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