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Unsteady Flow Past an Accelerated Vertical Plate through Porous Medium in The Presence of Magneto-Hydro Dynamic (MHD) and Thermal Radiation with Variable Temperature and Uniform Mass Diffusion

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ABSTRACT

Analytical solution of an unsteady flow past an accelerated vertical plate with the variable temperature and uniform mass diffusion has been carried out through the porous medium in the presence of thermal radiation and MHD. The dimensional governing equations were solved using Laplace transform technique and computed for different parameters using MATLAB. The effect of different parameters like Schmidt number, prandtl number, mass Grashof number, thermal Grashof number, were obtained and presented graphically. The velocity profiles for the different values of the Modify Grashof Number Gc increases with the increasing of the Gc, The velocity profiles of Thermal Grashof Number Gr increases with the decreasing of Gc. The velocity profiles of Permiability K increase with the increase in K, The velocity profiles of Magneto-Hydrodynamic M increase with the increase in M, The velocity profiles of Thermal Radiation R decrease with the increase in thermal Radiation R, effect of velocity for the different values of time t has a slite increase of velocity at result of slite increa of time t. The temperature profiles for the different values of Pradtl Number is noted that temperature decreases with the increasing of the prandtl number Pr. The temperature profiles for the different values of time t is noted that temperature decreases with the increasing of time t. The effect of Concentration for the different values of Thermal Radiation R is noted that the concentration decreases with the increasing of Thermal Radiatio R.

Keywords: Heat Transfer, MHD, Porous Medium, Thermal Radiation, Unsteady Flow.

Introduction

The effect of thermal radiation is significant in some industrial application such as glass productionand furnace design and in space technology application such as cosmical flight, aerodynamics rocket, propulsion system, plasma physics which operate at high temperature. Effect of heat and mass transfer plays play a vital role in manufacturing industries for space craft design in cooling of the liquid metal of the nuclear reaction, the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, solar energy collectors, design of chemical processing equipment,satellites and space vehicles. Flow through the porous medium has vital role in engineering problems such as purification of crude oil, the moment of oil and natural gas through oil sand stone reservoirs, underground water resources, flow of blood, paper and pulps industry membrane separation process, are examples of such engineering applications. Palani & Abbas, (2009) free convection MHD flow with thermal radiation from an impulsive started vertical plate, Muthucumaraswamy, (2010) stdied the effects of rotation on MHD flow past an accelerated isothermal vertical plate with heat and mass diffusion, Vijayalakshmi, (2012) combined effects of thermal radiation and MHD of flow past an accelerated vertical plate in a rotating fluid, Ahmed & Kalita, (2012) analyzed the Laplace technique on magnetohydrodynamic radiating and chemically reacting fluid over an infinite vertical surface, Idowu, etal(2013) considered heat and mass transfer of magnetohydrodynamic (MHD) and dissipative fluid flow past a moving vertical porous plate with variable suction, Ravikumar, etal (2013) analysed the magnetic field effect on transient free convection flow through porous medium past an impulsively started vertical plate with fluctuating temperature and mass diffusion, Uwanta & Yale, (2014) considered the analytical solution of unsteady flow past an accelerated vertical plate with constant temperature and variable mass transfer, Hemamalini & Suresh, (2015) analysed the unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion through porous medium, Deka, etal (2015) studied porous medium with exponentially decaying wall temperature, A R Vijayalakshmi, (2016) considered analytical solution of vertical plate in a rotating fluid with variable temperature , uniform mass diffusion in the presence of MHD and thermal, Muthucumaraswamy & Sivakumar, (2016) considered MHD flow past a parabolic flow past an infinite isothermal vertical plate in the presence of thermal radiation and chemical reaction.

Problems Formulation

Considering an unsteady flow of an incompressible viscous fluid initially at rest past of an infinite vertical plate with variable temperature through the paorous medium in the presence of radiation is considered. The flow is assumed to be in x- direction which takes vertical plate in the upward direction.The y-axis is taken to be normal to the plate. Initially the plate and the fluid are in same temperature with the same concentration level C' at all points. At time $t' > 0$ the plate accelerated with velocity $u' = \frac{t' u_0^2}{v}$ in its own plane. The plate temperature is raised to T_w' and the level of concentration near the plate is raised to C_w'linearly with the time t.

The governing equation

Then under the usual Boussinesq's approximation the unsteady flow is governed by the following equation.

$$\frac{\partial u'}{\partial t'} = g\beta\left(T' - T_\infty'\right) + g\beta^*\left(C' - C_\infty'\right) + V\frac{\partial^2 u'}{\partial y^2} - V\frac{u'}{K'} + \frac{\sigma B_0^2 u' u_0^2}{\rho V} \quad (1)$$

$$\rho Cp\frac{\partial T'}{\partial t'} = K\frac{\partial^2 T'}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \quad (3)$$

The initial and boundary condition are given as;

$$u' = 0,\ T' = T_\infty' \quad C_w' = C_\infty' \quad t' \le 0 \text{ for all y}$$

$$u' = \frac{u_0^3 t'}{v} \quad T' = T_\infty' + \theta\left(T_w' - T_\infty'\right)At' \quad C' = C_\infty' + C\left(C_w' - C_\infty'\right)At', t' > 0 \text{ , } at \quad y = 0 \quad (4)$$

$$u' = 0, \quad T_w' \to T_\infty' \quad C_w' \to C_\infty' \quad \text{as } y \to \infty$$

where $A = \frac{u_0^2}{v}$

The non-dimensional quantities are;

$$U = \frac{u'}{u_0} \quad \Rightarrow u' = Uu_o \quad Y = \frac{yu_0}{v} \Rightarrow y = \frac{Yv}{u_o} \quad t = \frac{t' u_0^2}{v} \Rightarrow t' = \frac{tv}{u_0^2}$$

$$\theta = \frac{T' - T_\infty'}{T_w' - T_\infty'} \quad \Rightarrow T' = T_\infty' + \theta\left(T_w' - T_\infty'\right) \quad C = \frac{C' - C_\infty'}{C_w' - C_\infty'} \quad \Rightarrow C' = C_\infty' + C\left(C_w' - C_\infty'\right) \quad (5)$$

$$Gr = \frac{g\beta V\left[\left(T_w' - T_\infty'\right)\right]}{u_0^3} \quad Gc = \frac{g\beta^* V\left[\left(C_w' - C_\infty'\right)\right]}{u_0^3} \quad Sc = \frac{V}{D} \quad M = \frac{\sigma B_0^2 u}{\rho}$$

$$\frac{1}{K} = \frac{u_o K'}{V^2} \quad \Rightarrow K' = \frac{V^2}{Ku_0} \quad Pr = \frac{\mu c_p}{K} \quad R = \frac{16a^*\delta V^2 T_\infty^3 \theta}{Ku_0^2}$$

We arrived at

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} - KU - MU \quad (6)$$

$$\frac{\partial^2\theta}{\partial y^2} - Pr\frac{\partial\theta}{\partial t} + R\theta = 0 \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{8}$$

The initial and boundary conditions are reduced to;

$$\begin{aligned} &U = 0,\ \ \theta = 0,\ \ C = 0,\ \ \text{for all } y, t \le 0 \\ &U = 1,\ \ \theta = 1,\ \ C = 1,\ \ \text{at } y = 0,\ t > 0 \\ &U = 0,\ \ \theta \to 0,\ C \to 0,\ \ \text{as } y \to \infty \end{aligned} \tag{9}$$

Method of Solution

Equations (6) – (8), are solved subjected to the boundary conditions of (9), and the solutions are obtained for concentration, temperature and velocity flow in terms of exponential and complementary error function using the Laplace- transform technique as follows;

$$C = erfc\ \frac{y\sqrt{Sc}}{\sqrt[2]{t}} \tag{10}$$

$$\theta = \frac{1}{2}\left\{e^{2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr} + \sqrt{Rt}\right) + e^{-2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr} - \sqrt{Rt}\right)\right\}$$

$$\text{where } \eta = \frac{y}{2\sqrt{t}} \tag{11}$$

$$\begin{aligned} U =& \frac{1}{2}\left\{e^{y\sqrt{K}} erfc\left(\sqrt{qt} + \eta\sqrt{q}\right) + e^{-y\sqrt{q}} erfc\left(-\sqrt{qt} + \eta\sqrt{q}\right)\right\} \\ &- \frac{Gr}{2a}\left\{\begin{matrix} e^{y\sqrt{q}} erfc\left(\sqrt{qt} + \eta\sqrt{q}\right) + \\ e^{-y\sqrt{q}} erfc\left(-\sqrt{qt} + \eta\sqrt{q}\right) \end{matrix}\right\} - \frac{Gr}{b}\left[erfc(\eta) e^{-y\sqrt{q}}\right] \\ &+ \frac{Gc}{2f}\left\{\begin{matrix} e^{y\sqrt{q}} erfc\left(\sqrt{qt} + \eta\sqrt{q}\right) + \\ e^{-y\sqrt{q}} erfc\left(-\sqrt{qt} + \eta\sqrt{q}\right) \end{matrix}\right\} - \frac{Gc}{q}\left[erfc(\eta) e^{-y\sqrt{q}}\right] \\ &+ \frac{Gr}{2a}\left\{\begin{matrix} e^{y\sqrt{\Pr}} erfc\left(\sqrt{Rt} + \eta\sqrt{\Pr}\right) \\ + e^{-y\sqrt{\Pr}} erfc\left(-\sqrt{Rt} + \eta\sqrt{\Pr}\right) \end{matrix}\right\} - \frac{Gre^{-y\sqrt{R}}}{2b}\left\{\begin{matrix} e^{y\sqrt{\Pr R}} erfc\left(\sqrt{Rt} + \eta\sqrt{\Pr}\right) \\ + e^{-y\sqrt{\Pr R}} erfc\left(-\sqrt{Rt} + \eta\sqrt{\Pr}\right) \end{matrix}\right\} \\ &- \frac{Gct}{f}\left\{\begin{matrix} \left(1 + 2\eta^2 Sc\right) erfc\left(\eta\sqrt{Sc}\right) \\ - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}} e^{-\eta^2 Sc} \end{matrix}\right\} - \frac{Gc}{q}\left[erfc(\eta)\sqrt{Sc}\right] \end{aligned} \tag{12}$$

$$a = (\Pr - 1) \quad b = (K - R) \quad f = (Sc - 1) \quad \eta = \frac{y}{2\sqrt{t}} \quad q = (K + M)$$

Results and discussion

The velocity profiles for the different values of the Modify Grashof Number (Gc = 2,3, 4) is shown in the Fig:4.2.1.It is shown that the velocity increases with the increasing of the Gc. The velocity profiles for the different values of Thermal Grashof Number (Gr = -2,-3,-4) is shown in Figure 4.2.2: It is observed that the velocity increases with the decreasing of the Gc. The velocity profiles for the different values of Permiability (K = 2, 4, 6) is shown in Figure 4.2.3: It is ovserved that the velocity increase with the increase in K. The velocity profiles for the different values of Magneto-Hydrodynamic (M = 2, 2.5, 3,) is shown in Figure 4.2.4: It is ovserved that the velocity increase with the increase in M.The velocity profiles for the different values of Thermal Radiation (R = 2.4, 3, 3.4) is shown in Figure 4.2.5: It is ovserved that the velocity decrease with the increase n thermal Radiation R.The effect of velocity for the different values of the time (t = 0.2, 0.4, 0.6) is shown in Figure 4.2.6: It is notice the slite increase of velocity at result of slite increa of time t. The temperature profiles for the different values of Pradtl Number (Pr =2, 5, 7.1) is shown in Figure:4.2.7: is noted that temperature decreases with the increasing of the prandtl number Pr. The temperature profiles for the different values of time (t = 5, 10, 15) is shown in Figure:4.2.8: is noted that temperature decreases with the increasing of time t. The effect of Concentration for the different values of Thermal Radiation (R = 10, 15, 20) is seen in the Figure:4.2.9: It is noted that the concentration decreases with the increasing of Thermal Radiatio R.

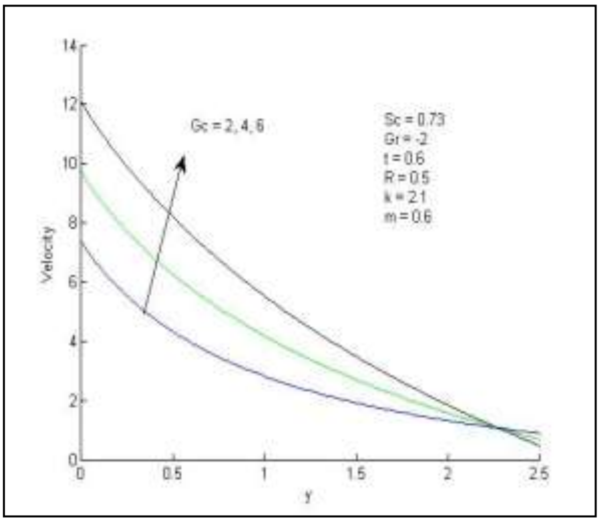

Figure 1: The velocity profiles different value of Gc

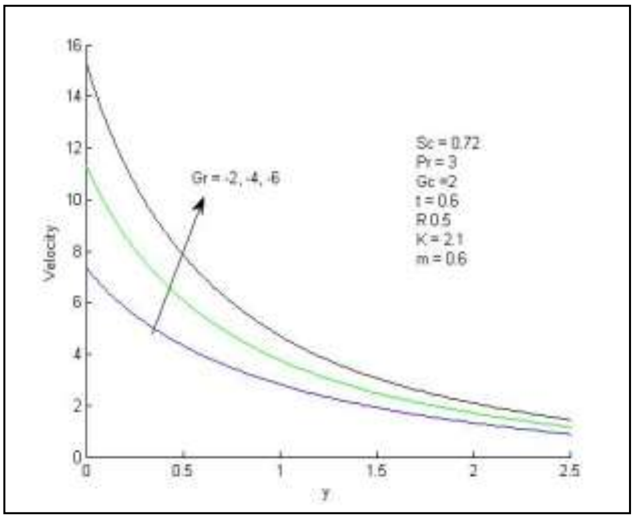

Figure 2: The velocity profiles different value of Gr

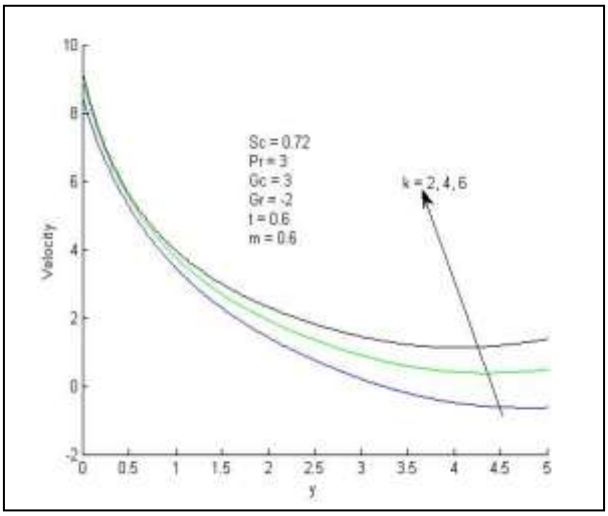

Figure 3: The velocity profiles different value of K

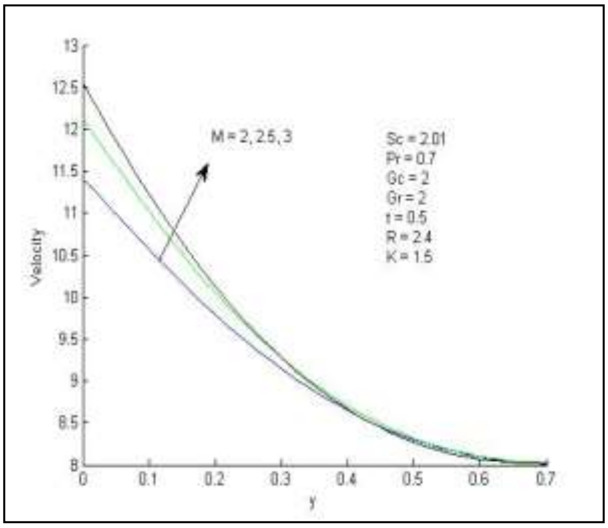

Figure 4: The Temperature Profiles for different value of M

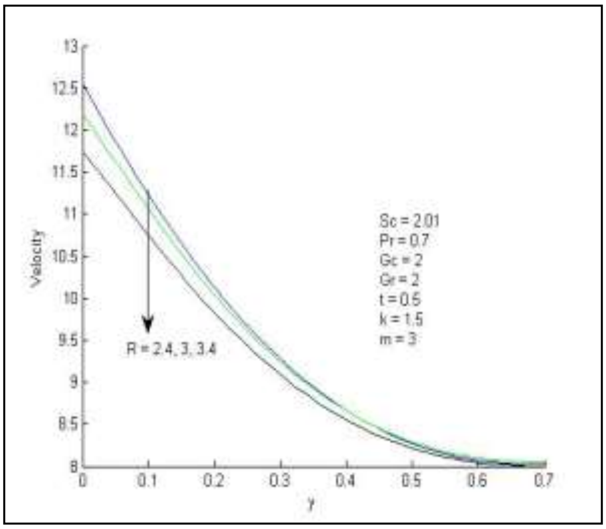

Figure 5: The velocity profiles different value of R

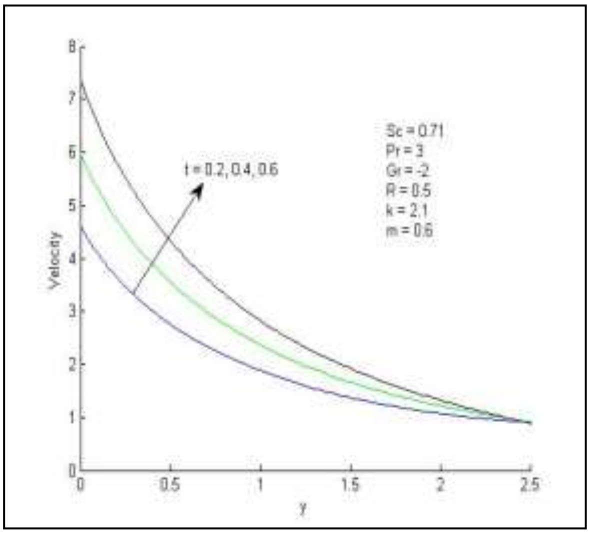

Figure 6: The velocity profiles different value of t

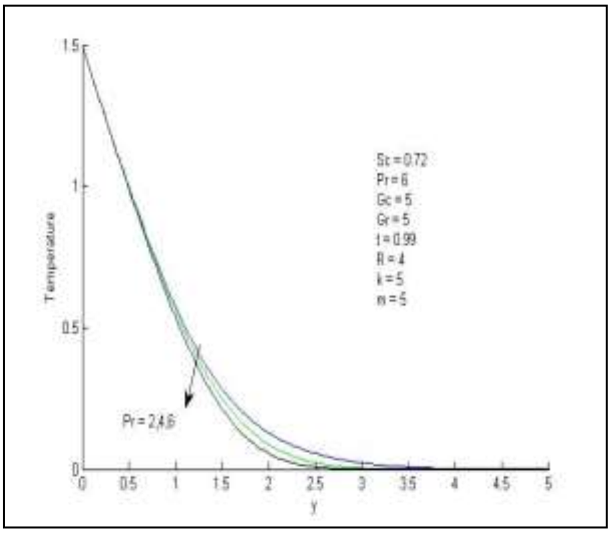

Figure 7: The Temperature Profiles for different value of Pr

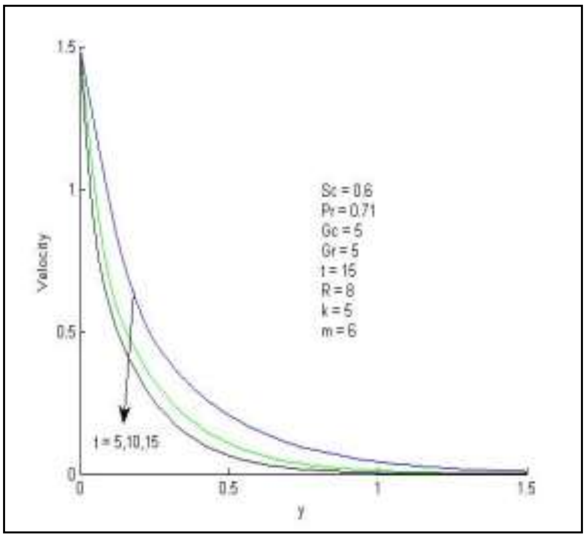

Figure 8: The Temperature Profiles for different value of t

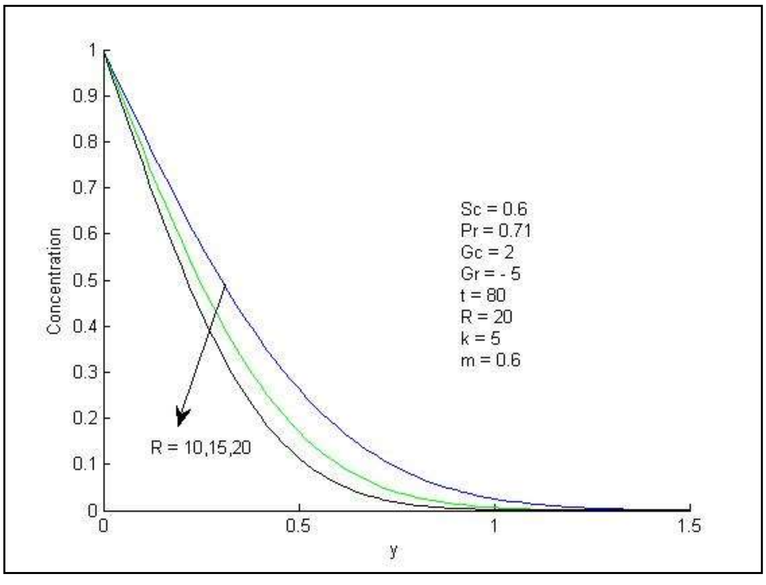

Figure 9: The Concentration Profiles for different value of R

Conclusion

Analytical solution of an unsteady flow past an accelerated vertical plate with the variable temperature and uniform mass diffusion has been carried out through the porous medium in the presence of thermal radiation, and effect of magneto-hydro dynamic (MHD). In this problem the dimensional governing equations was analysed and solved by Laplace transform technique and compute for different parameters using MATLAB. The effect of different parameters like Schmidt number, Prandtl number, mass Grashof number, thermal Grashof number, and time are studied the results for the flow profiles of velocity, temperature and concentration where obtained.

Recomendations

In this research paper, we solved fluid flow problem using analytical method of solution namely Laplace transforms, it is therefore recomended that these problem can be further solve using Pertubation Techniques.

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