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Stability Study of Thin Rectangular C-S-C-F Plate Under Buckling Load Using 3rd Order Functionals

Uzoukwu C. S, Ibearugbulem O. M., Ikpa P. N., Iroegbu R. E, Obumseli P. C., Chike K. O., Nwokorobia G.C., Anyanwu V. K.

Civil Engineering Department, Federal University of Technology Owerri, Imo State, Nigeria
Email: cskambassadors@gmail.com
Phone: +2348130131114

ABSTRACT

The research centers on the buckling effect of rectangular Clamped-Simple-Clamped- Fixed plate Isotropic plate. This was done using 3rd energy Functional. The CiSiCiFi plate was considered as the direct independent plate. That means the material properties like the flexural rigidity, poison ratio and young elastic modulus of elasticity are the same round about the shape of the object. The shape functions were first derived and then the various integral values of the differentiated shape functions, of the various boundary conditions were all gotten. Based on the derived results, the stiffness coefficients of the various boundary arrangements were also formulated. Upon further minimizations , the Third order strain energy equation was derived and further expansion Third order strain energy equation gave rise to the Third Order Overall Potential Energy Functional. The Third Order Overall Potential Energy Functional, with respect to the amplitude was further integrated and this gave a result known as the Lead equation. Further minimization of the Lead equation gave rise to the Vital buckling load equations. Next to this was the formulation of the non-dimensional buckling load parameters which, m/n ranging from 1.0 to 2.0,and considering it at the interval of 0.1. The relationship of the non-buckling load parameters against the various aspect ratios was shown on the graph.

Key words:

- Overall Potential Energy Functional,
- Lead Equation,
- Vital Buckling load

Introduction

A plate element can be considered as a structural element having straight or curves boundaries, and also possessing three dimensions known as the primary, secondary and tertiary dimension. The tertiary dimension also known as the plate thickness are usually very small compared to other dimensions. The isotropic rectangular CiSiCiFi plate have all their material properties in all directions as the same and so they classified as direction independent element. Stability analysis sometimes is referred to as the plate buckling has been a subject of study in solid structural mechanics for a long time now. Although the buckling analysis of rectangular plates has received the attention of many researchers for several centuries Prior to this time,. other researchers have gotten solution using both the Second and Fourth the Order energy functional for Buckling of plate. None of the scholars have any work on buckling of plate using Third order energy functional and so the resolution of the buckling tendency of CLAMPED SIMPLE CLAMPED FIXED isotropic plate using third order energy functional is the gap the work tends to fill. The plates arrangement can be as shown

1.1 Formulation of The Buckling Load Equation.

Overall potential energy, O_p is the summation of Strain energy, Є and External Work, E_w given as: $O_p = Є + E_w$ 1i

To derive the strain energy,Є the product of normal stress and normal strain in x direction is considered as

$$§_x ð_x = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 fk}{\partial x^2}\right]^2 + \mu\left[\frac{\partial^2 fk}{\partial x \partial y}\right]^2\right) \quad 1ii$$

while their product in y direction is considered as

$$§_y ð_y = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 fk}{\partial y^2}\right]^2 + \mu\left[\frac{\partial^2 fk}{\partial x \partial y}\right]^2\right) \quad 1iii$$

And finally the product of the in-plane shear stress and in-plane shear strain is given as: $\tau_{xy}\gamma_{xy} = 2\ \frac{Ez^2(1-\mu)}{(1-\mu^2)}\left[\frac{\partial^2 fk}{\partial x \partial y}\right]^2$ 1iv

adding all together gives

$\S_x ð_x + \S_y ð_y + \tau_{xy}\gamma_{xy}\ = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 fk}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 fk}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 fk}{\partial y^2}\right]^2\right)$ 1v

But $\epsilon = \frac{1}{2}\iint_{xy}\overline{\epsilon}\,\mathrm{dxdy}$ where $\overline{\epsilon} = \frac{Ez^2}{1-\mu^2}\ \int \left(\left[\frac{\partial^2 fk}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 fk}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 fk}{\partial y^2}\right]^2\right)$ 1vi

Upon minimisation of the expressions above, the third order strain energy equation is given as

$\epsilon = \frac{G}{2}\int_0^n \int_0^m \left(\frac{\partial^3 fk}{\partial x^3}.\frac{\partial fk}{\partial x} + 2\frac{\partial^3 fk}{\partial x\,\partial y^2}.\frac{\partial fk}{\partial x} + \frac{\partial^3 fk}{\partial y^3}.\frac{\partial fk}{\partial y}\right)\mathrm{dxdy}$ 2i

with the external load as $v = -\frac{Bx}{2}\int_0^n \int_0^m \left(\frac{\partial fk}{\partial x}\right)^2 \mathrm{dxdy}$ 2ii

The third order total potential energy functional is expressed mathematically as

$O_p = \frac{G}{2}\int\int \left(\frac{\partial^3 fk}{\partial x^3}.\frac{\partial fk}{\partial x} + 2\frac{\partial^3 fk}{\partial x^2\,\partial y}.\frac{\partial fk}{\partial y} + \frac{\partial^3 fk}{\partial y^3}.\frac{\partial fk}{\partial y}\right)\mathrm{dxdy} - \frac{Bx}{2}\int\int \frac{\partial^2 fk}{\partial x^2}\mathrm{dxdy}$ 2ii

Rearranging the total potential energy equation in terms of non dimensional parameters, the buckling load equation is gotten as

$B_x = \frac{\frac{G}{a^2}\int_0^1\int_0^1.(\left[\frac{\partial^3 fk}{\partial J^3}\right].\frac{\partial fk}{\partial J} + 2\frac{1}{p^2}\left[\frac{\partial^3 fk}{\partial J \partial I^2}\right].\frac{\partial fk}{\partial J} + \frac{1}{p^4}\left[\frac{\partial^3 fk}{\partial I^3}\right].\frac{\partial fk}{\partial I})\mathrm{dJdI}}{\int_0^1\int_0^1\left(\frac{\partial fk}{\partial J}\right)^2 \mathrm{dJdI}}$ 2iii

1.2 Derivation of Shape Function

Three major support conditions were considered, in the derivation of the shape functions and they namely Fixed support which was denoted as Fi, Simple support which is denoted as Si and Clamped support which is denoted as Ci. For Simple support condition, the deflection equation F and the 2nd order derivative of the deflection equation F^2, were equated to zero and simultaneous equations were formed by considering J = 0 at the left hand support for X axis and I = 1 at the right. Also considering the top as J = 1 and I = 1 at the bottom support for the Y axis. For the Clamped support condition, the deflection equation, F and 1st order derivative of the deflection equation, F^1, were equated to zero and simultaneous equations were formed by considering J = 0 at the top support and I = 0 at the bottom support for the Y axis, while at the Right hand support, J = 1 while I = 1 at the left support for X axis. These equations were solved simultaneously to obtain the various values of the primary and secondary dimensions (n_1, m_1, n_2, m_2 n_3, m_3, n_4 andm_4) for the CiSiCFi plate element. Where J and I are non-dimensional axis parallel to X and Y axis respectively as earlier explained.

1.3 Formulation of Shape Function For Clamped-Simple-Clamped-Fixed Plate

Fig 1a Isotropic Rectangular CiSiCiFi Plate

The case of horizontal Direction (X- X axis)

Fig 1b Simple-Fixed Support on x-x axis

Considering the X- X axis

But $F_x = n_o + n_1 J + n_2 J^2 + n_3 J^3 + n_4 J^4 + n_5 J^5$ 6

$F_x^1 = n_1 + 2n_2 J + 3n_3 J^2 + 4n_4 J^3 + 5n_5 J^4$ 7

$F^2 = 2n_2 + 6n_3 J + 12n_4 J^2 + 20n_5 J^3$ 8

$F^3 = 6n_3 + 24n_4 J + 60n_5 J^2$ 9

Introducing the boundary conditions, reduces the Equations 6-9 as

At the left support, J = 0

When $F_x = 0$

$F_x = 0 = n_o + 0 + 0 + 0 + 0$ 10

$n_o = 0$

Also when $f_x^{ii} = 0$ 11

$F^{ii} = 0 = 2n_2 + 0 + 0 + 0$ 12

$2n_2 = 0$ 13

$n_2 = 0$ 14

At the right support, J =1

$F_x^1 = n_1 + 0 + 3n_3 + 4n_4 + 5n_5 = -\frac{2n_5}{3}$ 15

Further simplifying Equation 15 gives

$n_1 = -\frac{2n_5}{3} - 3n_3 - 4n_4 - 5n_5$ 16

Also for the second derivative of the Deflection,

$F^2 = 0 = 0 + 6n_3 + 12n_4 + 20n_5$ 17

Making n_3 the subject gives

$n_3 = \frac{-12n_4 - 20n_5}{6}$ 18

$n_3 = \frac{-10n_5}{3} - 2n_4$ 19

For the third derivative of the Deflection,

$F^3 = 0 = 6n_3 + 24n_4 + 60n_5$ 20

$F^3 = 0 = 6n_3 + 24n_4 + 60n_5$ 21

$n_3 = \frac{-60n_5 - 24n_4}{6}$ 22

Comparing Equation 19 and 22 gives

$\frac{-10n_5}{3} - 2n_4 = \frac{-60n_5 - 24n_4}{6}$ 23

Bringing the like terms together and further simplifying gives

$n_4 = \frac{-10n_5}{3}$ 24

But substituting Equation 24 into Equation 22 gives

$n_3 = \frac{-60n_5 - 24(\frac{-10n_5}{3})}{6}$ 25

Further simplification gives

$n_3 = \frac{10n_5}{3}$ 26

Putting Equations 25 and 26 into Equation 16 gives

$n_1 = -\frac{2n_5}{3} - 3(\frac{10n_5}{3}) - 4(\frac{-10n_5}{3}) - 5n_5$ 27

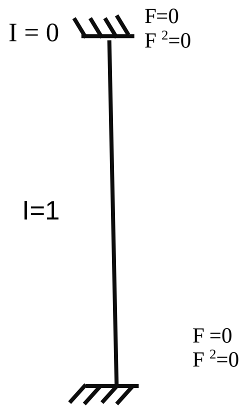
$n_1 = -\frac{7n_5}{3}$ 28

Recall that $F_x = n_o + n_1J + n_2J^2 + n_3J^3 + n_4J^4 + n_5J^5$ 29

Putting the derived values into Equation 29 gives

$F_x = n_5(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 30

The case of horizontal Direction (Y- Y axis)



$F_y = m_o + m_1I + m_2 I^2 + m_3 I^3 + m_4 I^4$ 31

The first derivative on Y axis gives

$F_y^1 = m_1+2 m_2 I + 3m_3I^2 + 4m_4 I^3$ 32

Considering the boundary conditions on the clamped ends gives

At I = 0,

$F_y = 0 = m_o + 0+ 0 + 0 + 0$ 33

Leaving $m_o = 0$ 34

Also

$F_y^1 = m_1+0 + 0 + 0 + 0$ 35

$m_1= 0$ 36

At I = 1,

$F_y = 0 = 0 + 0 + m_2 + m_3 + m_4$ 37

$m_2 + m_3 = - m_4$ 38

$F_y^1 = 0 = 0 + 2m_2 + 3m_3 + 4m_4$ 39

$m_2 = - m_3 - m_4$ 40

Putting Equation 40 into the first derivatives gives

$F_y^1 = 0 = 0 + 2(- m_3 - m_4) + 3m_3 + 4m_4$ 41

Opening the bracket gives

$m_3 + 2m_4 = 0$ 42

That means $m_3 = -2m_4$ 43

Putting it back into Equation 40 gives

$m_2 = - (-2m_4) - m_4$ 44

$m_2 = +m_4$ 45

Substituting the derived values into Equation 30 gives

$F_y = m_4I^2 -2m_4I^3 + m_4I^4$ 46

$F_y = m_4(I^2 -2I^3 + I^4)$ 47

That means $F = F_x*F_y = m_4(I^2 -2I^3 + I^4) * n_5(-\frac{7J}{3}+\frac{10J^3}{3}-\frac{10J^4}{3}+J^5)$ 48

$fk = fk_x*fk_y = m_4n_5 (I^2 -2I^3 + I^4)(-\frac{7J}{3}+\frac{10J^3}{3}-\frac{10J^4}{3}+J^5)$ 49

The shape function is give as $(I^2 - 2I^3 + I^4)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 50

Equation 50 is further differentiated at different stages , from where the stiffness coefficients were derived. These includes

$$\frac{\partial fk}{\partial J} = (I^2 - 2I^3 + I^4)(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4) \quad 51$$

$$\frac{\partial^2 fk}{\partial J\,\partial I} = (-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)(2I - 6I^2 + 4I^3) \quad 52$$

$$\frac{\partial fk}{\partial J\,\partial I^2} = (-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)(2 - 12I + 12I^2) \quad 53$$

$$\frac{\partial^2 fk}{\partial J^2} = (I^2 - 2I^3 + I^4)(-\frac{7}{3} + 60J - 10 * 4J^2 + 20J^3) \quad 54$$

$$\frac{\partial^3 fk}{\partial J^3} = (I^2 - 2I^3 + I^4)(60 - 80J + 60J^2) \quad 55$$

also

$$\frac{\partial fk}{\partial I} = (2I - 6I^2 + 4I^3)\,(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) \quad 56$$

$$\frac{\partial^2 fk}{\partial I^2} = (2 - 12I + 12I^2)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) \quad 57$$

$$\frac{\partial^3 fk}{\partial I^3} = (-12 + 24I)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) \quad 58$$

Integrating the product Equation 55 by 51 give the first stiffness coefficient. That

is

$$sc_1 = \int_0^1 \int_0^1 \frac{\partial^3 fk}{\partial J^3} * \frac{\partial fk}{\partial J}\, dIdJ \quad 59$$

$$sc_1 = \int_0^1 \int_0^1 \left[(I^2 - 2I^3 + I^4)(60 - 80J + 60J^2) * (I^2 - 2I^3 + I^4)\,(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)\right] dIdJ \quad 60$$

bringing the like terms together gives

$$= \int_0^1 \int_0^1 \left[(I^2 - 2I^3 + I^4)(I^2 - 2I^3 + I^4) * (60 - 80J + 60J^2)(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)\right] dIdJ \quad 60a$$

multiplying them gives

$$= \int_0^1 \int_0^1 \left[\left(I^2(I^2 - 2I^3 + I^4) - 2I^3\,(I^2 - 2I^3 + I^4) + I^4(I^2 - 2I^3 + I^4)\right) * \left(60\left(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4\right) - 80J\left(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4\right) + 60J^2\left(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4\right)\right)\right] \quad dIdJ \quad 60b$$

further minimization yields

$sc_1 = 0.789 * 0.8296$

$= 0.65455$

also integrating the product Equation 53 by 51 give the second stiffness coefficient.

That is

$$sc_2 = \int_0^1 \int_0^1 \frac{\partial^3 fk}{\partial J\,\partial I^2} * \frac{\partial fk}{\partial J}\, dIdJ \quad 61$$

$$sc_2 = \int_0^1 \int_0^1 \left[\left(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4\right)(2 - 12I + 12I^2) * (I^2 - 2I^3 + I^4)(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)\right] dIdJ \quad 62$$

Bring the like terms together gives

$$\int_0^1 \int_0^1 \left[(2 - 12I + 12I^2)(I^2 - 2I^3 + I^4) * \left(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4\right)(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)\right] dIdJ \quad 62a$$

Multiplying the like terms gives

$$\int_0^1 \int_0^1 \left[(2\,(I^2 - 2I^3 + I^4) - 12I\,(I^2 - 2I^3 + I^4) + 12I^2(I^2 - 2I^3 + I^4)) * \left(-\frac{7}{3}(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4) + 30J^2(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4) - \frac{40J^3}{3}(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)) + 5J^4\right)\right] dIdJ \quad 62b$$

$sc_2 = 0.01891 * 2.11599$

$= 0.0400137$

Furthermore integrating the product Equation 58 by 56 give the third stiffness coefficient. That is

$$sc_3 = \int_0^1 \int_0^1 \frac{\partial^3 fk}{\partial I^3} * \frac{\partial fk}{\partial I}\, dIdJ \quad 63$$

$$sc_3 = \int_0^1 \int_0^1 \left[(-12 + 24I)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5) * (2I - 6I^2 + 4I^3)\,(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)\right] dIdJ \quad 64$$

$sc_3 = 0.0001579 * 3.77781$

= 0.00059651

and finally integrating the product Equation 51 by 51 give the sixth stiffness coefficient. That is

$$sc_6 = \int_0^1 \int_0^1 (\frac{\partial fk}{\partial J} * \frac{\partial fk}{\partial J}) dIdJ \qquad 63$$

$$sc_6 = \int_0^1 \int_0^1 \left[(I^2 - 2I^3 + I4)(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4) * (I^2 - 2I^3 + I4)(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)\right] dIdJ \qquad 64$$

Collecting the like terms together gives

$$= \int_0^1 \int_0^1 \left[(I^2 - 2I^3 + I4) * (I^2 - 2I^3 + I4)(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)\right] dIdJ \qquad 65$$

Opening the brackets gives

$$= \int_0^1 \int_0^1 \left[(I^2 (I^2 - 2I^3 + I4) - 2I^3(I^2 - 2I^3 + I4) + I4(I^2 - 2I^3 + I4)) * (-\frac{7}{3}(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4) + 30J^2(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4) - \frac{40J^3}{3}(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4) + 5J^4(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4))\right] dIdJ \qquad 65b$$

sc_6 = 0.018111 * 0.8478

= 0.0153545

Reducing Equation 2iii in terms of the stiffness coefficients gives

$$B_x = \frac{D(sc_1 + 2\frac{1}{p^2}sc_2 + \frac{1}{p^4}sc_3)}{sc_6 a^2} \qquad 65$$

Substituting the real values in to Equation 65 gives

$$B_x = \frac{D(0.65455 + 2\frac{1}{p^2}0.0400137 + \frac{1}{p^4}0.00059651)}{0.0153545a^2} \qquad 66$$

RESULTS AND DISCUSSION.

The results for the stiffness coefficients and the critical buckling load coefficients were derived. The critical buckling load coefficients were considered at different aspect ratios. The first table represents the values of the stiffness coefficients while the other contains the critical buckling coefficients for the aspect ratio of m/n, both for the previous and present study. The values of the aspect Ratios ranges from 2.0 to 1.0 with arithmetic increase of 0.1. From the values generated in the tables, it was observed that as the aspect ratio increases from 1.0 to 2.0, the critical buckling load decreases. This occurred both in the present and previous results.

Table 1.1 Stiffness Coefficients from Previous researchers

Stiffness coefficients, sc	Derived values
sc_1	0.67096
sc_2	0.04043
sc_3	0.006047
sc_6	0.0159444

Table 1.2 Stiffness Coefficients from Present Work

Stiffness coefficients, sc	Derived values
sc_1	0.65455
sc_2	0.0400137
sc_3	0.0059651
sc_6	0.0153545

Table 1.3 Critical buckling load values for CSCF Plate from Previous/Present.

m/n		2	1.9	1.8	1.7	1.6
B		43.9346	44.0759	44.2415	44.4373	44.6711
B_x	Previous	43.3728	43.5151	43.6826	43.8814	44.1201
	Present	43.9346	44.0759	44.2415	44.4373	44.6711

Table 1.3 cont'd.

m/n		1.5	1.4	1.3	1.2	1.1	1
B		44.9533	45.2985	45.7268	46.2674	46.9632	47.88
$B_{x}\frac{G}{n^2}$	Previous	44.4101	44.7674	45.2148	45.7859	46.5315	47.5319
	Present	44.9533	45.2985	45.7268	46.2674	46.9632	47.88

REFERENCES

[1] An-Chien W., Pao-Chun L. and Keh-Chynan, T. (2013)." High – Mode Buckling-restrained Brace Core Plates".Journals of the International Association for Earthquake Engineering.

[2] Ali Reza Pouladkhan (2011). "Numerical Study of Buckling of Thin Plate".International Conference on Sustainable Design and Construction Engineering. Vol. 78,Issue: 1, Pp. 152 – 157.

[3] Shinde, B.M., Sayyad, A.S. & Kowade, A.B. (2013). Thermal Analysis Of Isotropic Plates Using Hyperbolic Shear Deformation Theory. *Journal of Applied and Computational Mechanics* 7 (2013) 193-204

[4] Singh, S.K & Chakrabarti, A. (2012). Buckling Analysis of Laminated Composite Plates Using an Efficient C^0 FE Model. *Latin American Journal of Solids and Structures*. Soni, S.R. (1975), *Vibrations of Elastic Plates and Shells of Variables Thickness*. Ph.D. Thesis. University of Roarkee.

[5] Srinivasa,C.V., Suresh, Y.J. and Prema, W.P. (2012). "Buckling Studies onLaminated Composite Skew Plate". *International Journal of ComputerApplications*. Vol. 37, Issue:1, Pp. 35-47.

[6] AydinKomur and Mustafa Sonmez (2008). "Elastic Buckling of Rectangular Plates Under Linearly Varying In-plane Normal Load with a Circular Cutout". International Journal of Mechanical Sciences. Vol. 35, Pp. 361 – 371.

[7] Ahmed Al-Rajihy (2008). "The Axisymmetric Dynamics of Isotropic Circular Plates with Variable Thickness Under the Effect of Large Amplitudes". Journal of Engineering, Vol. 14, Issue :1, Pp. 2302 – 2313.

[8] Audoly, B., Roman, B. and Pocheau, A. (2002). Secondary Buckling Patterns of a Thin Plate under In-plane Compression. The European Physical Journal BCondensed Matter and Complex Systems, Vol. 27, No. 1 (May).

[9] Azhari, M, Shahidi, A.R, Saadatpour, M.M (2004) "Post Local Buckling of Skew and Trapezoidal Plate". Journal of Advances in Structural Engineering, Vol. 7, Pp 61 - 70.

[10] Azhari, M. and Bradford, M.A. (2005), "The Use of Bubble Functions for the Post-Local Buckling of Plate Assemblies by the Finite Strip Method", International Journal for Numerical Methods in Engineering, Vol.38, Issue 6.

[11] Bhaskara, L.R. and Kameswara, C.T. (2013)." Buckling of Annular Plate with Elastically Restrained External and Internal Edges". Journal of Mechanics Based Design of Structures and Machines. Vol. 41, Issue 2. Pp. 222 - 235.

[12] Da-Guang Zhang (2014). "Nonlinear Bending Analysis of FGM Rectangular Plates with Various Supported Boundaries Resting on Two-Parameter Elastic ". Archive of Applied Mechanics. Vol. 84, Issue: 1, Pp.1 -20.