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On Generalised Hyperperfect Numbers

Bhabesh Das

Department of Mathematics
Arya Vidyapeeth College (Autonomous)
Guwahati, Assam,India- 781016
Email: mtbdas99@gmail.com

ABSTRACT:

It is well-known that a natural number is n is called k −hyperperfect number if $n = 1 + k[\sigma(n) - n - 1]$. In this paper, we introduce the notion of r near k −hyperperfect number and r deficient k −hyperperfect number, where r and k are positive integers.

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1. Introduction

A positive integer n is called classical perfect number [1] if sum of proper divisors of n is equal to the number n itself . Proper divisors of n are all positive divisors of n other than n itself. The sum of all divisors of n is denoted by $\sigma(n)$. If $n$is a perfect number, then using the divisor function σ, we can write $\sigma(n) = 2n$. All known perfect numbers are even. One possible generalization of perfect numbers is the hyperperfect numbers. D. Minoli and R.Bear [4] introduced the notion of hyperperfect number. A natural number is n is called k −hyperperfect number if $n = 1 + k[\sigma(n) - n - 1]$ and it can be rewrite as $\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k}$. Perfect numbers are 1 −Hyperperfect number. A. Bege and K.Fogarasi [2] have given list of k −hyper perfect numbers and some conjecture related to k −Hyperperfect numbers. If n is a 2 −hyperperfect number, then n is a solution of the equation $\sigma(n) = \frac{3}{2}n + \frac{1}{2}$.

Near k −hyperperfect and deficient k −hyperperfect numbers are generalized notion of k −hyperperfect numbers [3].

A positive integer n is called near k −hyperperfect number [3] with proper divisor d (the divisor d is termed as redundant divisor of n) if $\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k} + d$.

For any prime q, if $k = q - 1$, then for near $(q-1) -$ hyperperfect numbers,

$$\sigma(n) = \frac{q}{q-1}n + \frac{q-2}{q-1} + d.$$

In particular$q = 2$, near 1−hyperperfect numbers are the solution of the equation $\sigma(n) = 2n + d$. Solutions of this equation are well-known near perfect numbers [5].

For $q = 3$, near 2 −hyperperfect numbers are solution of the equation

$$\sigma(n) = \frac{3}{2}n + \frac{1}{2} + d.$$

A positive integer n is called deficient k −hyperperfect number [3] with proper divisor d (the divisor d is termed as redundant divisor of n) if

$$\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k} - d$$

For any prime q, if $k = q-1$ then for deficient $(q-1)-$ hyperperfect numbers, we obtain

$$\sigma(n) = \frac{q}{q-1}n + \frac{q-2}{q-1} - d$$

In particular $q = 2$, deficient 1−hyperperfect numbers are the solution of the equation

$$\sigma(n) = 2n - d.$$

Solutions of this equation are well-known deficient perfect numbers [5].

For $q = 3$, deficient 2 −hyperperfect numbers are solution of the equation

$$\sigma(n) = \frac{3}{2}n + \frac{1}{2} - d.$$

and $q = 5$, deficient $4-$hyperperfect numbers are solution of the equation

$$\sigma(n) = \frac{5}{4}n + \frac{3}{4} - d, \text{ etc.}$$

2.Main Result:

In this section, we introduce the notion of r near $k-$hyperperfect number and r deficient $k-$hyperperfect numbers.

Definition 2.1. We call a positive integer n is a r near $k-$hyperperfect number if there exist r proper positive divisors $d_1, d_2, \dots. d_r$ of n such that

$$\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k} + d_1 + d_2 + \dots. + d_r.$$

If $r = 1$, then $\sigma(n) = \frac{k+1}{k}n + \frac{k-1}{k} + d_1$ and therefore Near $k-$hyper perfect numbers are 1 near $k-$hyper perfect numbers .

For any prime$q$, if $k = q - 1$, then for r near $(q-1)-$ hyperperfect numbers,

$$\sigma(n) = \frac{q}{q-1}n + \frac{q-2}{q-1} + d_1 + d_2 + \dots. + d_r.$$

Following proposition gives a form of $r$near $(q-1)-$ hyperperfect numbers.

Proposition 2.1. If $n = q^{l-1}[q^l - (q-1)q^{l_1} - (q-1)q^{l_2} - \dots. - (q-1)$, where $q^l - (q-1)q^{l_1} - (q-1)q^{l_2} - \dots. - (q-1)q^{l_r} - (q-1)$ is a prime and $l > l_1 \geq l_2 \geq \dots. \geq l_r$, then n is a r near $(q-1)$ hyperperfect number .

Proof. If q and $q^l - (q-1)q^{l_1} - (q-1)q^{l_2} - \dots. - (q-1)q^{l_r} - (q-1)$ are distinct primes, then by definition of σ we can write

$$\sigma(n) = \sigma(q^{l-1})\sigma[q^l - (q-1)q^{l_1} - (q-1)q^{l_2} - \dots. - (q-1)q^{l_r} - (q-1)]$$
$$= \frac{q^l - 1}{q-1}[q^l - (q-1)q^{l_1} - (q-1)q^{l_2} - \dots. - (q-1)q^{l_r} - (q-1) + 1]$$
$$= \frac{q^l}{q-1}[q^l - (q-1)q^{l_1} - (q-1)q^{l_2} - \dots. - (q-1)q^{l_r} - (q-1)] +$$
$$\frac{1}{q-1}[q^l - q^l + (q-1)q^{l_1} + (q-1)q^{l_2} + \dots. + (q-1)q^{l_r} + (q-2)]$$
$$= \frac{qn}{q-1} + \frac{q-2}{q-1} + q^{l_1} + q^{l_2} + \dots. + q^{l_r}.$$

Since $l > l_1 \geq l_2 \geq \dots. \geq l_r$, so $q^{l_1}, q^{l_2}, \dots. q^{l_r}$are redundant divisors of n.

From the proposition 2.1., we have the following corollary.

Corollary 2.1. If $n = 3^l(3^{l+1} - 2.3^{l_1} - \dots - 2.3^{l_r} - 2)$, where $3^{l+1} - 2.3^{l_1} - \dots - 2.3^{l_r} - 2$ is a prime and, then n is a r near 2 $-$hyperperfect number .

Corollary 2.2. If $n = 5^l(5^{l+1} - 4.5^{l_1} - \dots - 4.5^{l_r} - 4)$, where $5^{l+1} - 4.5^{d_1} - \dots - 4.5^{l_r} - 4$ is an odd prime , then n is a r near 4 $-$hyperperfect number .

Corollary 2.3. If $n = 7^l(7^{l+1} - 6.7^{l_1} - \dots - 6.7^{l_r} - 6)$, where $7^{l+1} - 6.7^t - 6$ is an odd prime , then n is a r near 6 $-$hyperperfect number .

Proposition 2.2. Suppose that $p = q^l - (q-1)$ is an odd prime, then $n = q^{l-1}p^3$is a 2 near $(q-1)-$ hyperperfect number with redundant divisors $p^2$and p.

Proof. Clearly $p^2$and p are proper divisor of $n = q^{l-1}p^3$.

For $n = q^{l-1}p^3$, we get

$$\sigma(n) = \sigma(q^{l-1})\sigma(p^3) = \frac{q^l - 1}{q-1}(p^3 + p^2 + p + 1)$$
$$= \frac{q^l p^3 + (q^l - 1)(p^2 + p + 1) - p^3}{q-1}$$
$$= \frac{q^l p^3 + (q^l - q + 1)(p^2 + p + 1) - p^3 + (q-2)(p^2 + p + 1)}{q-1}$$
$$= \frac{q^l p^3 + p^3 + p^2 + p - p^3 + (q-2)(p^2 + p + 1)}{q-1}$$
$$= \frac{q^l p^3 + (q-1)(p^2 + p) + (q-2)}{q-1}$$
$$= \frac{qn}{q-1} + \frac{q-2}{q-1} + p^2 + p.$$

Corollary 2.4. If $3^l - 2$ is an odd prime, then $n = 3^{l-1}(3^l - 2)^3$ is a 2 near 2 $-$hyperperfect number .

Corollary 2.5. If $5^l - 4$ is an odd prime, then $n = 5^{l-1}(5^l - 4)^2$is a 2 near 4 $-$hyperperfect number.

Corollary 2.6. If $7^l - 6$ is an odd prime, then $n = 7^{l-1}(5^l - 4)^2$is a 2 near 6 $-$hyperperfect number.

Definition 2.2. We call a positive integer n is r deficient k −hyperperfect number if there exist r positive divisors $d_1, d_2, \dots. d_r$ of n such that

$$\sigma(n) = \frac{k+1}{k} n + \frac{k-1}{k} - d_1 - d_2 - \dots. - d_r.$$

If $r = 1$, then $\sigma(n) = \frac{k+1}{k} n + \frac{k-1}{k} - d_1$ and therefore deficient k −hyper perfect numbers are 1 deficient k −hyper perfect numbers .

For any prime q, if $k = q - 1$, then for r deficient $(q-1)$ − hyperperfect numbers,

$$\sigma(n) = \frac{q}{q-1} n + \frac{q-2}{q-1} - d_1 - d_2 - \dots. - d_r.$$

Following proposition gives a form of r deficient $(q-1)$ − hyperperfect numbers.

Proposition 2.3. If $n = q^l[q^{l+1} + (q-1)q^{l_1} + (q-1)q^{l_2} + \dots. + (q-1)q^{l_r} - (q-1)]$, where $q^{l+1} + (q-1)q^{l_1} + (q-1)q^{l_2} + \dots. + (q-1)q^{l_r} - (q-1)$ is a prime and $l > l_1 \geq l_2 \geq \dots. \geq l_r$, then n is a r deficient $(q-1)$ − hyperperfect number.

Proof. $\sigma(n) = \sigma(q^l)\sigma[q^{l+1} + (q-1)q^{l_1} + (q-1)q^{l_2} + \dots. + (q-1)q^{l_r} - (q-1)]$

$$= \frac{q^{l+1}-1}{q-1}[q^{l+1} + (q-1)q^{l_1} + (q-1)q^{l_2} + \dots. + (q-1)q^{l_r} - (q-1) + 1]$$

$$= \frac{q^{l+1}}{q-1}[q^{l+1} + (q-1)q^{l_1} + (q-1)q^{l_2} + \dots. + (q-1)q^{l_r} - (q-1)] + \frac{q^{l+1}}{q-1}$$

$$- \frac{1}{q-1}[q^{l+1} + (q-1)q^{l_1} + (q-1)q^{l_2} + \dots. + (q-1)q^{l_r} - (q-1) + 1]$$

$$= \frac{qn}{q-1} + \frac{q-2}{q-1} - q^{l_1} - q^{l_2} - \dots. - q^{l_r}.$$

Since $l > l_1 \geq l_2 \geq \dots. \geq l_r$, so $q^{l_1}, q^{l_2}, \dots. q^{l_r}$ are redundant divisors of n.

Corollary 2.7. If $n = 3^l(3^{l+1} + 2.3^{l_1} + \dots + 2.3^{l_r} - 2)$, where $3^{l+1} + 2.3^{l_1} + \dots + 2.3^{l_r} - 2$ is a prime and, then n is a r deficient 2 −hyperperfect number .

Corollary 2.8. If $n = 5^l(5^{l+1} + 4.5^{l_1} + \dots + 4.5^{l_r} - 4)$, where $5^{l+1} + 4.5^{l_1} + \dots + 4.5^{l_r} - 4$ is an odd prime , then n is a r deficient 4 −hyperperfect number .

Corollary 2.9. If $n = 7^l(7^{l+1} + 6.7^{l_1} + \dots + 6.7^{l_r} - 6)$, where $7^{l+1} + 6.7^{l_1} + \dots + 6.7^{l_r} - 6$ is an odd prime, then n is a r deficient 6 −hyperperfect number .

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