International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

An Optimize Decomposition Method for the Solution of System of Integro-Differential Equations

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ABSTRACT

In this seminar work, an optimize decomposition method is introduced based on the Adomian decomposition method to solve nonlinear integro-differential and systems of nonlinear integro-differential equations (IDEs). The introduced technique is simpler and shorter in its computational procedures and yield results faster than the existing method such as the ADM. In addition, it does not require discretization, linearization or any restrictive assumption of any form in providing analytical or approximate solution to linear and nonlinear equations. Also, this technique does not require calculating Adomian's polynomials, Lagrange's multiplier values. These advantages make the OADM it reliable and its efficiency is demonstrated with numerical examples.

Keywords: *optimize decomposition method, integro-differential equation, systems of nonlinear integro-differential equations, Adomian's polynomials, and Lagrange's multiplier values.*

1.0 INTRODUCTION

The true laws of nature cannot be linear, presumably true; mathematicians are continually challenged with the nonlinear problems, particularly in form of partial differential equations, appearing in Physics and engineering. Accordingly, any effort contributing to the world of nonlinear analysis would be of fundamental importance. Among a multitude of previously proposed method to handle nonlinear equations, Adomian Decomposition Method (ADM) which was introduced by the acknowledged mathematician George Adomian in 1984 (Adomian, 1984) has gained astonishing popularity, ADM can provide convenient solutions to a wide range of linear and nonlinear equations [(Odibat, 2019); (Tate & Dinde, 2019);(Manjak *et al*., 2017)]. ADM does not require any linearization, perturbation or discretization and leads to convergent solutions rapidly. To get to know about ADM and its efficiency as well as the further modifications thereof in detail, one is recommended to consult the literature [(Okai *et al*., 2017); (Wazwaz, 2000); (Rach *et al*., 2013); (Wazwaz, 1999)]. Also, many illustrative examples associated with the application of ADM in various areas of science and engineering are available [(Wazwaz, 2001); (Wazwaz, 2005); (Neda *et al*., 2014)]. As it will be discussed, ADM requires a particular series representation called the Adomian polynomials, for the nonlinearities involved in the equation under consideration. Several efforts have been made to derive procedures for computing these kinds of polynomials [(Olayiwola & Kareem, 2022); (Bakodah & Almuhalbedi, 2019); (Odibat, 2019); (Wazwaz, 2000); (Okai *et al*., 2020); (Hemeda, 2018); (Hooman & Hossein, 2011)]. However, some of them are restricted to only special cases of nonlinearity and many of them have been converted into computer codes involving complexity and long programs. Integro-differential equations arise quite frequently as mathematical models in diverse disciplines. The theory and application of the Volterra integro-differential equation play an important role in the mathematical modeling of many fields as: Physics, Biological phenomena and Engineering Sciences in which it is necessary to take into account the effect of real world problems (Wazwaz, 2011). The origins of the study of integro-differential equations may be treated to the work of Abel, Lokato, Fredholm, Malthus, Verhulst and Volterra on problems in mechanics, mathematical biology and economics (Wazwaz, 2011). In this work, we propose an efficient numerical method to effectively handle the system of nonlinear integro-differential equations.

2.0 Classification of Integral Equations

Before beginning to classify a system of integro-differential equation, it will be necessary to make some basic definitions and to introduce a preliminary classification of integral equations.

Definition 1:

An ***integral equation*** is an equation in which the unknown function appears under an integral sign. The general form of non-linear integral equations may be written as follows:

$$\alpha u(x) = f(x) + \lambda \int_a^{b(x)} k(x,t,u(t))dt, \qquad \ldots(1)$$

where the forcing function $f(x)$ and the kernel (or nucleus) $k(x,t)$ are prescribed, while $u(x)$ is the unknown function to be determined. The parameter λ is often omitted; it is, however, of importance in certain theoretical investigations and in the eigenvalue problem.

Definition 2:

The integral eqn. (1) is called ***linear integral equation*** if the kernel $k(x,t,u(t)) = k(x,t)u(t)$, otherwise it is called ***non-linear integral equation***.

Definition 3:

The linear integral eqn. (1) is called ***homogenous***, if $f(x) = 0$ otherwise it is called ***non-homogenous***.

Definition 4:

The integral eqn. (1) is said to be an equation of the ***first kind*** if $\alpha \equiv 0$, i.e.

$$f(x) = -\lambda \int_a^{b(x)} k(x,t,u(t))dt, \qquad \dots(2)$$

Definition 5:

The integral eqn. (1) is said to be an equation of the ***second kind*** if $\alpha = 1$, i.e.

$$u(x) = f(x) + \lambda \int_a^{b(x)} k(x,t,u(t))dt, \qquad \dots(3)$$

Definition 6:

The integral eqn. (1) is called Volterra integral equation (***VIE)***, when $b(x) = x$.

Definition 7:

The integral eqn. (1) is called Fredholm integral equation (***FIE)***, when $b(x) = b$, where b is constant such that $b \geq a$.

Definition 8:

If the kernel $k(x,t)$ in the linear integral eqn. (1) depends only of the difference $(x-t)$, i.e. if the kernel is of the form $k(x,t) = k(x-t)$, such a kernel is called difference kernel, and the linear integral equation is called ***integral equation of the convolution type***.

Definition 9:

The kernel $k(x,t)$ is called separable or degenerate kernel of rank *n* if it is of the form: $k(x,t) = \sum_j^n a_j(x)b_j(t)$ where *n* is finite and the functions $\{a_j\}$ and $\{b_j\}$ are sufficiently smooth functions.

Definition 10:

The integral equations

$$u_i(x) = f_i(x) + \sum_{j=1}^{m} \int_a^x k_{ij}(x,t)u_j(t)dt; \; x \in I = [a,b], \; i = 1,2,\dots,m \qquad \dots(4)$$

$$u_i(x) = f_i(x) + \sum_{j=1}^{m} \int_a^b k_{ij}(x,t)u_j(t)dt\,; \; x \in I = [a,b], \; i = 1,2,\dots,m \qquad \dots(5)$$

where $m \in N$; f_i, $i = 1,2,\dots,m$ are continuous functions on I and k_{ij}, $i = 1,2,\dots,m$ denotes given continuous functions on $\{(t,x): a \leq t \leq x \leq b\}$, while $u_i(x)$, $i = 1,2,\dots,m$ are the unknown functions to be determined are called a **system of linear VIE of second kind** and a **system of linear FIE second kind** respectively.

Definition 11:

An ***integro-differential equation*** is an equation which involves an unknown function *u*(*x*), together with differential and integral operations on *u*(*x*).

3.0 MATERIALS AND METHOD

3.1 Basics of the Adomian Decomposition Method (ADM)

Recall the basic principle ideas of the traditional decomposition method by considering the equation of the form:

$$Lu + Ru + Nu = g \qquad \dots(6)$$

Where L is an invertible operator that can be taken as the highest order differential operator, R is the linear differential operator of lesser order than L, N represents the nonlinear terms and g is the specified analytic function. Applying the inverse operator L^{-1} on both sides of equation Eq. (6) yields

$$u = \varphi + L^{-1}[g] - L^{-1}[Ru] - L^{-1}[Nu] \qquad \dots(7)$$

where φ is determined by the usage of the given initial values. This approach decomposes the results $u(x)$ into a hastily convergent series of solution components, after which decomposes the analytic nonlinearity Nu into the series of the Adomian polynomials, Rach, *et al.*, (2013).

The nonlinear term Nu will be equated to

$$Nu(x) = \sum_{n=0}^{\infty} A_n \qquad \dots(8)$$

Where the A_n are special polynomials called Adomian polynomials and u will be decomposed into

$$u(x) = \sum_{n=0}^{\infty} u_n \qquad \dots(9)$$

Where $A_n = A_n(u_0, u_1, u_2, \dots u_n)$ are the Adomian polynomials, whose definitional formulation

$$A_n(t) = \frac{1}{k!}\frac{d^k}{d\theta^k}\left[N\left(\sum_{i=0}^{k}\theta^i v_i\right)\right]_{\theta=0}. \qquad \dots(10)$$

Was first published by Adomian and Rach in 1983, then the same old Adomian recursion scheme is given by:

$u_0(x) = \varphi + L^{-1}[g]$,

That is,

$$u(x) = \sum_{n=0}^{\infty} u_n = u_0 - L^{-1}\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{\infty} A_n \qquad \dots(11)$$

Consequently, we can write

$u_1 = -L^{-1}Ru_0 - L^{-1}RA_0$

$u_2 = -L^{-1}Ru_1 - L^{-1}RA_1$

$u_3 = -L^{-1}Ru_2 - L^{-1}RA_2$

$\vdots$

$$u_{n+1}(x) = -L^{-1}[Ru_n + A_n] \qquad \dots(12)$$

(Wazwaz, 2011), provided more insight, details, properties, modifications and algorithms for the determination of the Adomian polynomials of the decomposition method for handling the nonlinear components.

3.2 The Optimize ADM for non-linear system of VIDEs

In this section, the method will be applied for system of non-linear VIDEs of the second kind. The strategy present here can also be applied to linear system in the same manner. To demonstrate the efficiency of the proposed method, we applied the optimize ADM to system of non-linear VIDEs for which an analytical solution is available.

Consider the system of non-linear VIDEs of the second kind (Hemeda, 2018):

$u''_1(x) = f_1(x) + \int_0^x [k_{11}(x,t)F_1(u_1(t)) + k_{12}(x,t)F_2(u_2(t))]dt$

$u''_2(x) = f_2(x) + \int_0^x [k_{21}(x,t)F_1(u_1(t)) + k_{22}(x,t)F_2(u_2(t))]dt$

With initial condition

$u_1(0) = c_0, u'_1(0) = c_1.$

$u_2(0) = d_0, u'_2(0) = d_1. \qquad \dots(13)$

By integrating both sides of Eq. (13) twice from 0 to *x* and use the initial conditions, we get

$$u_1(x) = c_0 + c_1x + \frac{1}{2!}\int_0^x (x-t)f_1(t)dt + \frac{1}{2!}\int_0^x (x-t)[k_{11}(x,t)F_1(u_1(t)) + k_{12}(x,t)F_2(u_2(t))]dt$$

$$u_2(x) = d_0 + d_1x + \frac{1}{2!}\int_0^x (x-t)f_2(t)dt + \frac{1}{2!}\int_0^x (x-t)[k_{21}(x,t)F_1(u_1(t)) + k_{22}(x,t)F_2(u_2(t))]dt \qquad \dots(14)$$

To use the optimize ADM; let

$$u_1(x) = \sum_{n=0}^{\infty} u_{1,n}(x), \ u_2(x) = \sum_{n=0}^{\infty} u_{2,n}(x) \qquad \dots(15)$$

Using the ADM recurrence relation, we obtain

$$u_{1,0}(x) = c_0 + c_1x + \frac{1}{2!}\int_0^x (x-t)f_1(t)dt$$

$$u_{2,0}(x) = d_0 + d_1 x + \frac{1}{2!}\int_0^x (x-t) f_2(t)dt$$

$$u_{1,1}(x) = \frac{1}{2!}\int_0^x (x-t)\big[k_{11}(x,t)F_1(u_{1,0}(t)) + k_{12}(x,t)F_2(u_{2,0}(t))\big]dt$$

$$u_{2,1}(x) = \frac{1}{2!}\int_0^x (x-t)\big[k_{21}(x,t)F_1(u_{1,0}(t)) + k_{22}(x,t)F_2(u_{2,0}(t))\big]dt$$

$$u_{1,2}(x) = \frac{1}{2!}\int_0^x (x-t)\big[k_{11}(x,t)F_1(u_{1,0}(t)+u_{1,1}(t)) + k_{12}(x,t)F_2(u_{2,0}(t)+u_{2,1}(t))\big]dt - \big(u_{1,0}(x)+u_{2,0}(x)\big)$$

$$u_{2,2}(x) = \frac{1}{2!}\int_0^x (x-t)\big[k_{21}(x,t)F_1(u_{1,0}(t)+u_{1,1}(t)) + k_{22}(x,t)F_2(u_{2,0}(t)+u_{2,1}(t))\big]dt - \big(u_{1,0}(x)+u_{2,0}(x)\big)$$

and so on. Continuing in this manner, the $(n+1)^{th}$ approximation of the exact solutions for the unknown functions $u_1(x)$ and $u_2(x)$ can be achieved as

$$u_{1,n+1}(x) = \frac{1}{2!}\int_0^x (x-t)^{k-1} k_{11}(x,t)F_1\left[\left(\sum_{m=0}^{n} u_{1,m}(t)\right) + k_{12}(x,t)F_2\left(\sum_{m=0}^{n} u_{2,m}(t)\right)\right]dt$$

$$-\frac{1}{2!}\int_0^x (x-t)^{k-1} k_{11}(x,t)F_1\left[\left(\sum_{m=0}^{n} u_{2,m}(t)\right) + k_{12}(x,t)F_2\left(\sum_{m=0}^{n} u_{2,m}(t)\right)\right]dt$$

$$u_{2,n+1}(x) = \frac{1}{2!}\int_0^x (x-t)^{k-1} k_{21}(x,t)F_1\left[\left(\sum_{m=0}^{n} u_{1,m}(t)\right) + k_{22}(x,t)F_2\left(\sum_{m=0}^{n} u_{2,m}(t)\right)\right]dt$$

$$-\frac{1}{2!}\int_0^x (x-t)^{k-1} k_{21}(x,t)F_1\left[\left(\sum_{m=0}^{n} u_{1,m}(t)\right) + k_{22}(x,t)F_2\left(\sum_{m=0}^{n} u_{2,m}(t)\right)\right]dt \quad \ldots(16)$$

Therefore, the approximate solutions

$$u_1(x) = \sum_{m=0}^{n+1} u_{1,m}(x), \quad \ldots(17)$$

$$u_2(x) = \sum_{m=0}^{n+1} u_{2,m}(x). \quad \ldots(18)$$

The optimize ADM will be illustrated by discussing some example on the system of the non-linear VIDEs.

4.0 NUMERICAL RESULTS

Example 1:

Consider the following system of nonlinear second-order IDEs (Hemeda, 2018)

$$u''(x) = x + u(x) + \int_0^x (-u^2(t) + v^2(t))dt, \quad u(0) = 1, \quad u'(0) = 0 \quad \ldots(19a)$$

$$u''(x) = -x + v(x) + \int_0^x (u^2(t) - v^2(t))dt, \quad v(0) = 0, \quad v'(0) = 1 \quad \ldots(19b)$$

With exact solution $u(x) = cosh(x)$ and $v(x) = sinh(x)$

Applying $L^{-1}(.) = \int_0^x \int_0^x (.)dtdt$ to both sides of Eqn. (19a) and Eqn. (19b), we get

$$u(x) = 1 + \frac{x^3}{6} + \int_0^x (x-t)\,u(t)dt + \frac{1}{2!}\int_0^x (x-t)^2(-u^2(t) + v^2(t))dt, \quad \ldots(20a)$$

$$v(x) = x - \frac{x^3}{6} + \int_0^x (x-t)\,v(t)dt + \frac{1}{2!}\int_0^x (x-t)^2(u^2(t) - v^2(t))dt, \quad \ldots(20b)$$

Thus, to evaluate the above system of equation, we go by applying the recurrence relation as defined in section 3.2,

$u_0(x) = 1,$

$v_0(x) = x,$

$$u_1(x) = \frac{x^3}{6} + \int_0^x (x-t)\,u_0(t)dt + \frac{1}{2!}\int_0^x (x-t)^2(-u_0^2(t) + v_0^2(t))dt = \frac{1}{2}x^2 + \frac{1}{60}x^5$$

$$v_1(x) = -\frac{x^3}{6} + \int_0^x (x-t)\,v_0(t)dt + \frac{1}{2!}\int_0^x (x-t)^2(u_0^2(t) - v_0^2(t))dt = -\frac{1}{60}x^5 + \frac{1}{6}x^3$$

Therefore, according to section 3.2, we have the other components of the OADM for Eqn. (19a and 19b) as follows using the above recursive scheme:

$$u_{n+1}(x) = \frac{x^3}{6} + \int_0^x (x-t)\left(\sum_{m=1}^{n-1} u_m(t)\right)dt + \frac{1}{2!}\int_0^x (x-t)^2\left(-\left(\sum_{m=1}^{n-1} u_m(t)\right)^2 + \left(\sum_{m=1}^{n-1} v_m(t)\right)^2\right)dt - \left(\sum_{m=1}^{n-1} u_m(t)\right)$$

$$v_{n+1}(x) = -\frac{x^3}{6} + \int_0^x (x-t)\left(\sum_{m=1}^{n-1} v_m(t)\right) dt + \frac{1}{2!}\int_0^x (x-t)^2\left(\left(\sum_{m=1}^{n-1} u_m(t)\right)^2 - \left(\sum_{m=1}^{n-1} v_m(t)\right)^2\right)dt - \left(\sum_{m=1}^{n-1} v_m(t)\right)$$

For $n \geq 1$

$$u_2(x) = \frac{1}{1260}x^7 + \frac{1}{24}x^4 - \frac{1}{178200}x^{11} - \frac{1}{43200}x^{10} - \frac{1}{90720}x^9 - \frac{1}{10080}x^8 - \frac{1}{60}x^5$$

$$v_2(x) = -\frac{1}{1260}x^7 + \frac{1}{40}x^5 + \frac{1}{178200}x^{11} + \frac{1}{43200}x^{10} + \frac{1}{90720}x^9 + \frac{1}{10080}x^8$$

$$\begin{aligned} u_3(x) = &-\frac{1}{19219200}x^{13} - \frac{73}{119750400}x^{12} + \frac{1}{720}x^6 + \frac{1}{62163288000}x^{19} \\ &+ \frac{73}{418784256000}x^{18} + \frac{53}{54286848000}x^{17} + \frac{17}{3592512000}x^{16} + \frac{283}{23351328000}x^{15} \\ &+ \frac{59}{3632428800}x^{14} + \frac{103}{19958400}x^{11} + \frac{1}{50400}x^{10} + \frac{1}{36288}x^9 + \frac{1}{10080}x^8 \\ &- \frac{1}{1260}x^7 \end{aligned}$$

$$\begin{aligned} v_3(x) = &\frac{1}{19219200}x^{13} + \frac{73}{119750400}x^{12} - \frac{1}{62163288000}x^{19} - \frac{73}{418784256000}x^{18} \\ &- \frac{53}{54286848000}x^{17} - \frac{17}{3592512000}x^{16} - \frac{283}{23351328000}x^{15} - \frac{59}{3632428800}x^{14} \\ &- \frac{103}{19958400}x^{11} - \frac{1}{50400}x^{10} - \frac{1}{36288}x^9 - \frac{1}{10080}x^8 + \frac{1}{1008}x^7 \end{aligned}$$

The series solution is then obtain by summing the above iterations,

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \ldots$$

$$v(x) = v_0(x) + v_1(x) + v_2(x) + v_3(x) + \ldots$$

$$\begin{aligned} u(x) = &\frac{1}{24}x^4 - \frac{1}{2217600}x^{11} - \frac{1}{302400}x^{10} + \frac{1}{60480}x^9 + 1 + \frac{1}{2}x^2 - \frac{1}{19219200}x^{13} \\ &- \frac{73}{119750400}x^{12} + \frac{1}{720}x^6 + \frac{1}{62163288000}x^{19} + \frac{73}{418784256000}x^{18} \\ &+ \frac{53}{54286848000}x^{17} + \frac{17}{3592512000}x^{16} + \frac{283}{23351328000}x^{15} + \frac{59}{3632428800}x^{14} \end{aligned} \quad \ldots(21a)$$

$$\begin{aligned} v(x) = &x + \frac{1}{120}x^5 + \frac{1}{6}x^3 + \frac{1}{5040}x^7 + \frac{1}{2217600}x^{11} + \frac{1}{302400}x^{10} - \frac{1}{60480}x^9 \\ &+ \frac{1}{19219200}x^{13} + \frac{73}{119750400}x^{12} - \frac{1}{62163288000}x^{19} - \frac{73}{418784256000}x^{18} \\ &- \frac{53}{54286848000}x^{17} - \frac{17}{3592512000}x^{16} - \frac{283}{23351328000}x^{15} - \frac{59}{3632428800}x^{14} \end{aligned} \quad \ldots(21b)$$

Table 1: The comparison between exact solutions $u(x)$ and the approximate solution using OADM

x	EXACT	OADM	ABSOLUTE ERROR
0	1	1	0
0.1	1.0050042	1.0050042	2.32037E-13
0.2	1.0200668	1.0200668	5.5405E-11
0.3	1.0453385	1.0453385	1.32407E-09
0.4	1.0810724	1.0810724	1.23247E-08
0.5	1.127626	1.1276259	6.84604E-08
0.6	1.1854652	1.1854649	2.7462E-07
0.7	1.255169	1.2551681	8.81436E-07
0.8	1.3374349	1.3374325	2.4088E-06

0.9	1.4330864	1.4330805	5.83985E-06
1	1.5430806	1.5430677	1.29301E-05

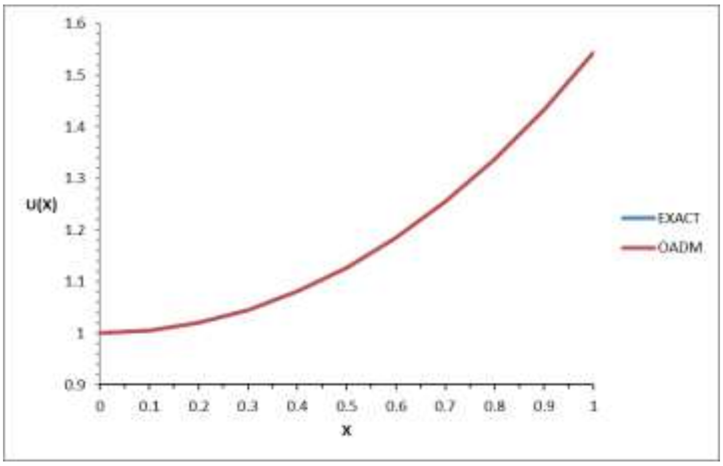

Figure 1: Graphs of the exact solution $u(x)$ and the approximate solution using OADM.

Table 2: The comparison between exact solutions $v(x)$ and the approximate solution using OADM

x	EXACT (V(X))	OADM (V(X))	ABSOLUTE ERROR
0	0	0	0
0.1	0.10016675	0.10016675	1.90264E-14
0.2	0.201336003	0.201336003	9.52666E-12
0.3	0.304520293	0.304520293	3.59075E-10
0.4	0.410752326	0.410752321	4.68166E-09
0.5	0.521095305	0.521095271	3.4085E-08
0.6	0.636653582	0.636653411	1.71485E-07
0.7	0.758583702	0.758583034	6.6784E-07
0.8	0.888105982	0.888103828	2.15398E-06
0.9	1.026516726	1.026510717	6.00863E-06
1	1.175201194	1.175186263	1.49301E-05

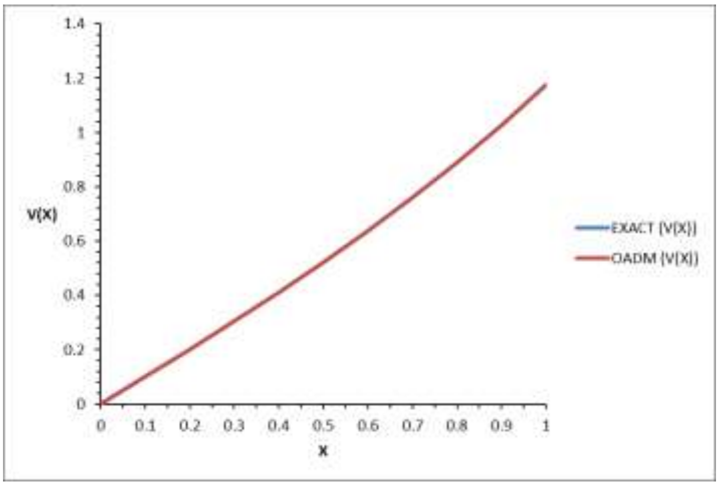

Figure 2: Graphs of the exact solution $v(x)$ and the approximate solution using OADM.

Example 2:

Consider the system of nonlinear Fredholm integro-differential equation (Bakodah & Almuhalbedi, 2019)

$$u''(x) = 2 + \frac{12}{5}x - \int_0^1 x(u^2(t) + v^2(t))dt, \quad u(0) = 1, \quad u'(0) = 0 \qquad \dots(22a)$$

$$v''(x) = -2 + \frac{4}{3}x - \int_0^1 x(u^2(t) - v^2(t))dt, \quad v(0) = 1, \quad v'(0) = 0 \qquad \dots(22b)$$

With exact solution

$$(u(x), v(x)) = (1 + x^2, 1 - x^2)$$

Applying $L^{-1}(.) = \int_0^x \int_0^x (.)dtdt$ to both sides of Eqn. (22a) and Eqn. (22b), we get

$$u(x) = 1 + x^2 + \frac{12}{30}x^3 - \frac{1}{3!}x^3 \int_0^1 (u^2(t) + v^2(t))dt, \qquad \dots(23a)$$

$$v(x) = 1 - x^2 + \frac{4}{18}x^3 - \frac{1}{3!}x^3 \int_0^1 (u^2(t) - v^2(t))dt, \qquad \dots(23b)$$

Thus, to evaluate the above system of equation, we apply the recurrence relation as defined in section 3.2,

$$u_0(x) = 1,$$

$$v_0(x) = 1,$$

$$u_1(x) = x^2 + \frac{12}{30}x^3 - \frac{1}{3!}x^3 \int_0^1 \left(u_0{}^2(t) + v_0{}^2(t)\right)dt = x^2 + \frac{1}{15}x^3$$

$$v_1(x) = -x^2 + \frac{4}{18}x^3 - \frac{1}{3!}x^3 \int_0^1 \left(u_0{}^2(t) - v_0{}^2(t)\right)dt = -x^2 + \frac{2}{9}x^3$$

Therefore, according to section 3.2, we have the other components of the OADM for Eqn. (22a and 22b) as follows using the above recursive scheme:

$$u_{n+1}(x) = x^2 + \frac{12}{30}x^3 - \frac{1}{3!}x^3 \int_0^1 \left(\left(\sum_{m=1}^{n-1} u_m(t) \right)^2 + \left(\sum_{m=1}^{n-1} v_m(t) \right)^2 \right) dt,$$

$$v_{n+1}(x) = -x^2 + \frac{4}{18}x^3 - \frac{1}{3!}x^3 \int_0^1 \left(\left(\sum_{m=1}^{n-1} u_m(t) \right)^2 - \left(\sum_{m=1}^{n-1} v_m(t) \right)^2 \right) dt$$

For $n \geq 1$

$$u_2(x) = -\frac{14183}{170100}x^3$$

$$v_2(x) = -\frac{5449}{24300}x^3$$

$$u_3(x) = \frac{11595897701}{607614210000}x^3$$

$$v_3(x) = \frac{96211}{22504230}x^3$$

$$u_4(x) = -\frac{4284147533147135002980 1}{155061911841448122000000000}x^3$$

$$v_4(x) = -\frac{1630894736467666740 1}{6458222067532200000000}x^3$$

$$\dots$$

The series solution is then obtain by summing the above iterations,

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + u_4(x) + \dots$$

$$v(x) = v_0(x) + v_1(x) + v_2(x) + v_3(x) + v_4(x) + \dots$$

$$u(x) = 1 + x^2 - \frac{6082315921958885209801}{15506191184144812200000000} x^3 \quad \ldots(24a)$$

$$v(x) = 1 - x^2 - \frac{999010084579847401}{6458222067532200000000} x^3 \quad \ldots(24b)$$

Table 3: The comparison between exact solutions $u(x)$ and the approximate solution using OADM

x	EXACT U(X)	OADM U(X)	ABSOLUTE ERROR
0	1	1	0
0.1	1.01	1.009999608	3.92251E-07
0.2	1.04	1.039996862	3.13801E-06
0.3	1.09	1.089989409	1.05908E-05
0.4	1.16	1.159974896	2.51041E-05
0.5	1.25	1.249950969	4.90314E-05
0.6	1.36	1.359915274	8.47262E-05
0.7	1.49	1.489865458	0.000134542
0.8	1.64	1.639799168	0.000200832
0.9	1.81	1.809714049	0.000285951
1	2	1.999607749	0.000392251

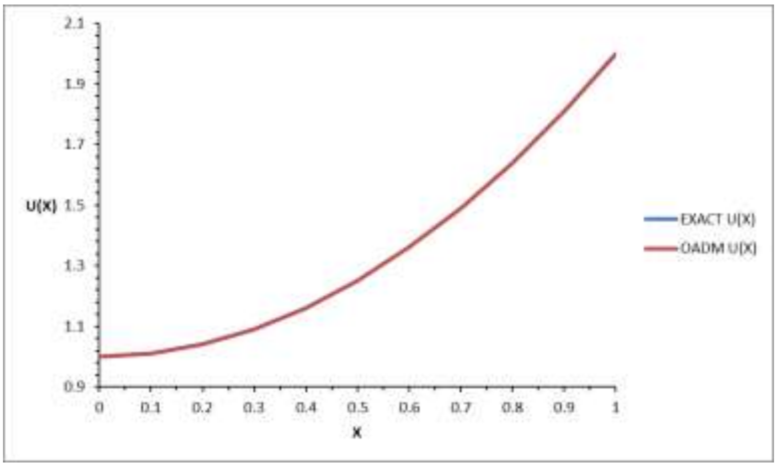

Figure 3: Graphs of the exact solution $u(x)$ and the approximate solution using OADM.

Table 4: The comparison between exact solutions $u(x)$, the approximate solution using OADM and DADM

x	EXACT	OADM (n=4)	DADM (n=6)
0	1	1	1
0.1	1.01	1.009999608	1.009612278
0.2	1.04	1.039996862	1.036898224
0.3	1.09	1.089989409	1.079531505
0.4	1.16	1.159974896	1.135185791
0.5	1.25	1.249950969	1.201534748
0.6	1.36	1.359915274	1.276252044
0.7	1.49	1.489865458	1.357011347
0.8	1.64	1.639799168	1.441486326
0.9	1.81	1.809714049	1.527350647
1	2	1.999607749	1.61227798

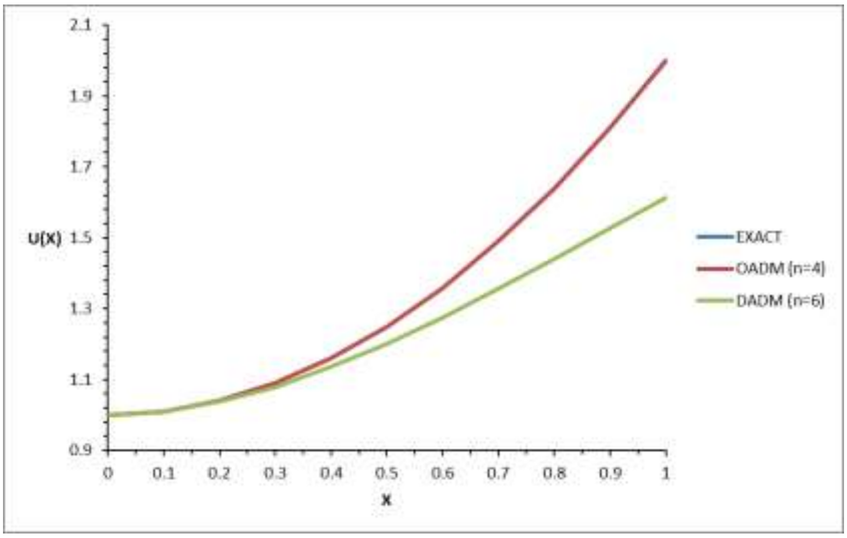

Figure 4: Graphs of the exact solution$u(x)$, the approximate solution Using OADM and DADM

Table 5: The comparison between exact solutions $v(x)$ and the approximate solution using OADM

x	EXACT	OADM	ABSOLUTE ERROR
0	1	1	0
0.1	0.99	0.99	1.54688E-07
0.2	0.96	0.959999	1.2375E-06
0.3	0.91	0.909996	4.17658E-06
0.4	0.84	0.83999	9.90004E-06
0.5	0.75	0.749981	1.9336E-05
0.6	0.64	0.639967	3.34126E-05
0.7	0.51	0.509947	5.3058E-05
0.8	0.36	0.359921	7.92003E-05
0.9	0.19	0.189887	0.000112768
1	0	-0.00015	0.000154688

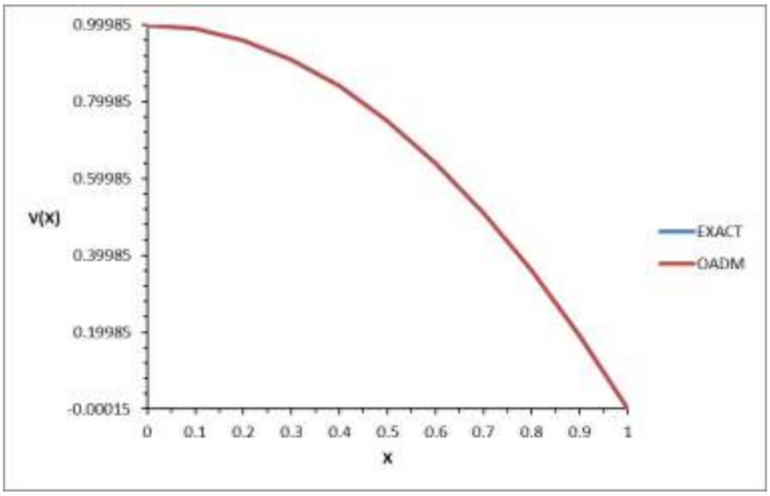

Figure 5: Graphs of the exact solution $v(x)$ and the approximate solution using OADM.

Table 6: The comparison between exact solutions $v(x)$, the approximate solution using OADM and DADM

x	EXACT (V(X))	OADM (V(X))	DADM (V(X))
0	1	1	1
0.1	0.99	0.989999845	0.990186726
0.2	0.96	0.959998762	0.961493812
0.3	0.91	0.909995823	0.915041615
0.4	0.84	0.8399901	0.851950495
0.5	0.75	0.749980664	0.773340811
0.6	0.64	0.639966587	0.680332922
0.7	0.51	0.509946942	0.574047186
0.8	0.36	0.3599208	0.455603963
0.9	0.19	0.189887232	0.326123611
1	0	-0.00015469	0.18672649

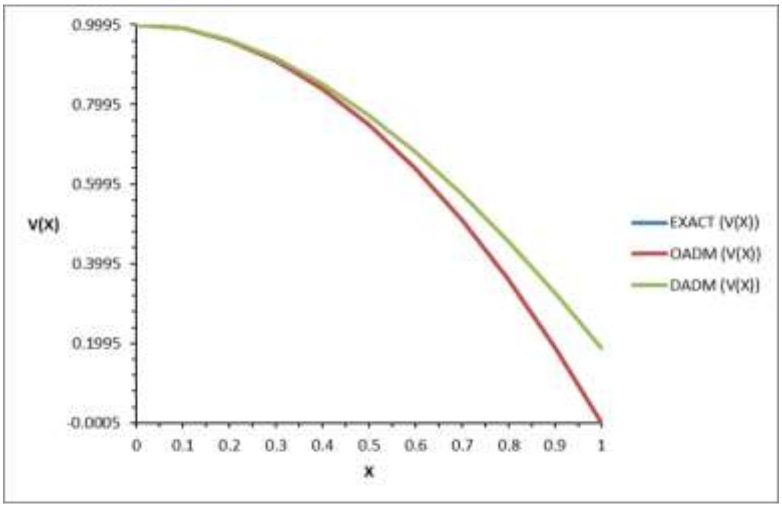

Figure 6: Graphs of the exact solution$v(x)$, the approximate solution using OADM and DADM

5.0 Conclusion

In this work, a semi-analytical method based on the ADM and the inverse of the differential operator in the problem under consideration is introduced, then it is used to solve nonlinear integro-differential and systems of nonlinear IDEs. To support the analysis, two nonlinear systems of IDEs and one nonlinear IDE of Volterra form are solved. The obtained results reveal that this method is simpler and shorter in its computational procedures and time than the other methods. Therefore, this method is more suitable and convenient for solving nonlinear problems.

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