WWW.IJRPR.COM

International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Observations on the Hyperbola $x^2=20y^2-4$

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Abstract:

The aim of this paper is to determine the non-homogeneous quadratic equations with two unknowns $x^2=20y^2-4$. A few interesting properties among the solution are given. Employing the linear combination among the solution of the given equation, integer solutions for other choices of hyperbola and parabola are determined.

Keywords: Binary quadratic, non-homogeneous quadratic, Pell equation, negative

Pell equation, hyperbola

2010 Mathematics Subject Classification: 11D09

Introduction:

A binary quadratic equation of the form $y^2=Dx^2+1$, where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1-2]. For an extensive review of various problems, one may refer [3-20]. In this communication, yet another interesting hyperbola given by $x^2=20y^2-4$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are obtained. Further, employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbola, parabola.

METHOD OF ANALYSIS:

The negative Pell equation representing hyperbola under consideration is

$$x^2 = 20y^2 - 4 \tag{1}$$

whose initial solution is

$$x_0 = 4, \quad y_0 = 1$$

To obtain the other solutions of (1), consider the Pell equation

$$x^2 = 20y^2 + 1$$

whose general solution is given by

$$\tilde{x}_n = \frac{1}{2} f_n, \quad \tilde{y}_n = \frac{1}{2\sqrt{20}} g_n$$

where

$$f_n = (9+2\sqrt{20})^{n+1} + (9-2\sqrt{20})^{n+1}$$
$$g_n = (9+2\sqrt{20})^{n+1} - (9-2\sqrt{20})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) & $(\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by

$$20x_{n+1} = 40 f_n + 10\sqrt{20} g_n$$
$$20y_{n+1} = 10 f_n + 2\sqrt{20} g_n$$

Replacing n by n+1, n+2 in turn in the above two equations, we have

$$20x_{n+1} = 40f_n + 10\sqrt{20}g_n \tag{2}$$

$$20x_{n+2} = 760f_n + 170\sqrt{20}g_n \tag{3}$$

$$20x_{n+3} = 13640f_n + 3050\sqrt{20}g_n \tag{4}$$

$$20y_{n+1} = 10f_n + 2\sqrt{20}g_n \tag{5}$$

$$20y_{n+2} = 170f_n + 38\sqrt{20}g_n \tag{6}$$

$$20y_{n+3} = 3050f_n + 682\sqrt{20}g_n \tag{7}$$

Eliminating f_n & g_n among (2)-(4) and (5)-(7), the recurrence relation for x &y are given by

$$x_{n+1} - 18x_{n+2} + x_{n+3} = 0$$
$$y_{n+1} - 18y_{n+2} + y_{n+3} = 0$$

A few numerical examples are given in the following Table: 1

Table: 1 Numerical examples

N	x_{n+1}	y_{n+1}
-1	$x_0 = 4$	$y_0 = 1$
0	$x_1 = 76$	$y_1 = 17$
1	$x_2 = 1364$	$y_2 = 305$
2	$x_3 = 24476$	$y_3 = 5473$
3	$x_4 = 439204$	$y_4 = 98209$

From the above table x values are even and y values are odd.

1. Relations between solutions

- $x_{n+3} = 18x_{n+2} - x_{n+1}$
- $40y_{n+1} = x_{n+2} - 9x_{n+1}$
- $40y_{n+2} = 9x_{n+2} - x_{n+1}$
- $80y_{n+3} = 161x_{n+2} - 9x_{n+1}$
- $18y_{n+2} = y_{n+3} + y_{n+1}$
- $720y_{n+2} = 9x_{n+3} - 19446x_{n+1}$
- $25920y_{n+1} = 36x_{n+3} - 5796x_{n+1}$
- $1440y_{n+3} = -2x_{n+1} + 322x_{n+3}$
- $10y_{n+2} = 90y_{n+1} + 20x_{n+1}$
- $2x_{n+3} = 1440y_{n+1} + 322x_{n+1}$

- $10y_{n+3} = 1610y_{n+1} + 360x_{n+1}$
- $x_{n+2} = 40y_{n+1} + 9x_{n+1}$
- $18x_{n+2} = 2x_{n+1} + 80y_{n+2}$
- $2x_{n+3} = 1440y_{n+2} + 18x_{n+1}$
- $90y_{n+1} = 10y_{n+2} - 20x_{n+1}$
- $90y_{n+3} = 1610y_{n+2} + 20x_{n+1}$
- $1610y_{n+1} = 360x_{n+1} + 10y_{n+3}$
- $322x_{n+2} = 18x_{n+1} + 80y_{n+3}$
- $1610y_{n+2} = -20x_{n+1} + 90y_{n+3}$
- $3220x_{n+3} = 14400y_{n+3} + 20x_{n+1}$

2. Each of the following expressions represents a cubical integer

- $\frac{1}{4}\left[x_{3n+4} - 17x_{3n+3} + 3x_{n+2} - 51x_{n+1}\right]$
- $\frac{1}{72}\left[x_{3n+5} - 305x_{3n+3} + 3x_{n+3} - 915x_{n+1}\right]$
- $10y_{3n+3} - 2x_{3n+3} + 30y_{n+1} - 6x_{n+1}$
- $\frac{1}{9}\left[10y_{3n+4} - 38x_{3n+3} + 30y_{n+2} - 114\right]$
- $\frac{1}{161}\left[10y_{3n+5} - 682x_{3n+3} + 30y_{n+3} - 2046\right]$

3. Each of the following expression represents a bi-quadratic integer

- $\frac{1}{4}\left[x_{4n+5} - 17_{4n+4} + 4x_{2n+3} - 68x_{2n+2} + 24\right]$
- $\frac{1}{72}\left[x_{4n+6} - 305x_{4n+4} + 4x_{2n+4} - 1220x_{2n+2} + 432\right]$
- $10y_{4n+4} - 2x_{4n+4} + 40y_{2n+2} - 2x_{2n+2} + 6$
- $\frac{1}{9}\left[10y_{4n+5} - 38x_{4n+4} + 40y_{2n+3} - 152x_{2n+2} + 54\right]$
- $\frac{1}{161}\left[10y_{4n+6} - 682x_{4n+4} + 40y_{2n+4} - 2728x_{2n+2} + 966\right]$

4. Each of the following expressions represents a quintic integer

- $\frac{1}{4}\left[x_{5n+6}-17x_{5n+5}+5x_{3n+4}-85x_{3n+3}+15x_{n+2}-255x_{n+1}-5x_{n+2}+85x_{n+1}\right]$
- $\frac{1}{72}\left[x_{5n+7}-305x_{n+5}+5x_{3n+5}-1525x_{3n+3}+15x_{n+3}-4575x_{n+1}-5x_{n+3}+1525x_{n+1}\right]$
- $10y_{5n+5}-2x_{5n+5}+50y_{3n+3}-10x_{3n+3}+150y_{n+1}-6x_{n+1}-50y_{n+1}-10x_{n+1}$
- $\frac{1}{9}\left[10y_{5n+6}-38x_{5n+5}+50y_{3n+4}-114x_{3n+3}+150y_{n+2}-570-50y_{n+2}+190x_{n+1}\right]$
- $\frac{1}{161}\left[10y_{5n+7}-682x_{5n+5}+50y_{3n+5}-3410x_{3n+3}+150y_{n+3}-10230-50y_{n+3}+3410x_{n+1}\right]$

REMARKABLE OBSERVATIONS:

1. Employing linear combinations among the solutions of (1),one may generate integer solutions for other choices of hyperbola which are presented in the Table: 2 below

Table: 2 Hyperbola

S.No	Hyperbola	(P,Q)
1	$5P^2-4Q^2=320$	$(x_{n+2}-17x_{n+1},19x_{n+1}-x_{n+2})$
2	$5P^2-4Q^2=103680$	$(x_{n+3}-305x_{n+1},341x_{n+1}-x_{n+3})$
3	$20P^2-Q^2=80$	$(10y_{n+1}-2x_{n+1},10x_{n+1}-40y_{n+1})$
4	$20P^2-Q^2=6480$	$(10y_{n+2}-38x_{n+1},170x_{n+1}-40y_{n+2})$
5	$20P^2-Q^2=2073680$	$(10y_{n+3}-682x_{n+1},3050x_{n+1}-40y_{n+3})$

2. Employing linear combinations among the solutions of (1),one may generate integer solutions for other choices of parabola which are presented in the Table: 3

Table: 3 Parabola

S. No	Parabola	(R,Q)
1	$5R-Q^2=80$	$(x_{2n+3}-17x_{2n+2}+8,19x_{n+1}-x_{n+2})$
2	$90R-Q^2=25920$	$(x_{2n+4}-305x_{2n+2}+144,341x_{n+1}-x_{n+3})$
3	$20R-Q^2=80$	$(10y_{2n+2}-2x_{2n+2}+2,10x_{n+1}-40y_{n+1})$
4	$180R-Q^2=6480$	$(10y_{2n+3}-38x_{2n+2}+18,170x_{n+1}-40y_{n+2})$
5	$3220R-Q^2=518420$	$(10y_{2n+4}-682x_{2n+2}+322,3050x_{n+1}-40y_{n+3})$

CONCLUSION:

In this paper, we have presented infinitely many integer solutions for the hyperbola represented by the negative Pell equation $x^2=20y^2-4$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of positive Pell equations and determine their integer solutions along with suitable properties.

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