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Second Law Analysis of Passive Control MHD Flow in the Presence of Arrhenius Chemical Reaction with Heat Generation/Absorption

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A B S T R A C T

The motivation for this research paper is to analyze second law analysis of passive control MHD flow in the presence of Arrhenius Chemical reactions with Heat Generation Absorption. In this paper, a mathematical model for second law analysis of passive control MHD flow in the presence of Arrhenius Chemical reactions with Heat Generation Absorption was formulated. The momentum, energy, magnetic, and species equations were non-dimensionalized to arrive at dimensionless equations. The dimensionless equations were solved analytically with the use of asymptotic expansions defined about activation energy parameter ϵ . With graphical representation, the effect of various important physical parameters on entropy generation, velocity, energy, concentration and chemical species for reactivity parameter, convective heat transfer, heat generation, thermal buoyancy, soret number, Eckert number, mass buoyance, thermal buoyance, Hartman number, velocity slip factor, Frank-Kamenestkii and Prandtl number were investigated. A table is also given that provides the results of different parameters on Entropy, Velocity, Temperature and Concentration. The total entropy generation is reduced to the entropy generation due to heat transfer for tiny thermal Grashof numbers because there is virtually no or very little convection and zero entropy creation due to fluid friction. At larger Grashof numbers, convectional heat transfer starts to have a major impact on flow velocity and, as a result, entropy formation due to viscous effects. Additionally, the deformed isotherms increase the temperature gradient, which in turn causes a heat-induced entropy formation.

Keywords: Heat transfer, Mass transfer, Entropy generation, Fluid friction, MHD flow, irreversibility ratio, Arrhenius heat

MSC Subject Classification: 76A05, 76A20 ,76B75, 76D55, 76M45 76Q05,76R10, 76V05, 76W05

Introduction

The boundary layer along material handling conveyors, the cooling of an infinite metallic plate in a cooling bath, glass blowing, continuous casting, and fiber spinning are just a few manufacturing processes in industry that involve the flow caused by a stretching surface. The study of flow and, heat and mass transfer in the boundary layer induced by a surface moving with a uniform or non-uniform velocity in a quiescent ambient fluid is important in these processes.Numerous academics are interested in the study of boundary-layer behavior on continuously moving solid surfaces. Numerous processes, including the aerodynamic extrusion of plastic sheets and the boundary layer along a liquid film in condensation processes, Chamka [1], make use of the analysis of magneto-hydrodynamic (MHD) flows of electrically conducting fluid. According to Sajid & Hayat [2], the Prandtl number and radiation parameter have differing effects on the fluid's temperature on an exponentially stretched sheet.The temperature of the fluid fell as the radiation parameter and Prandtl number grew, according to research by Aliakbar et al. [3] on the effect of thermal radiation on the MHD flow of a Maxwellian fluid. Furthermore, it was discovered that the temperature of the fluid increased as the Eckert number increased.Siddheshwar&Mahabaleswar's inquiry [4] examined the effects of radiation and heat sources on the MHD flow of viscoelastic fluid over a stretching sheet. Makinde and Sibanda took into account MHD mixed-convective flow and heat transfer past a vertical plate in a porous medium with constant wall suction in their work [5].

Nomenclature

y	flow axis	**Dimensionless group**	
u, v	Velocity component along x and y-axis	$\boldsymbol{Bi}$	Convective heat transfer
T	Temperature field	$\boldsymbol{\theta}$	dimensionless temperature
C	Species concentration field	$\boldsymbol{\phi}$	dimensionless concentration
g	gravitational acceleration	$\boldsymbol{Grc}$	Mass Grashof number
B_0	Magnetic field of uniform strength	$\boldsymbol{Grt}$	Thermal Grashof number
T_w	surface temperature	N	buoyancy ratio
T_∞	ambient temperature	λ	Chemical reaction parameter

C_w	surface concentration	$\boldsymbol{\alpha}$	heat generation parameter
C_∞	ambient concentration	M	Magnetic parameter
β_t	Volumetric coefficient of thermal expansion	$\boldsymbol{Nu}$	Nusselt number
β_c	Volumetric coefficient of mass expansion	$\boldsymbol{Sh}$	Sherwood number
k	thermal conductivity	H	Hartmann number
c_p	specific heat capacity at constant pressure	Sr	Soret number
D	Molecular diffusivity	Ec	Eckert Number
D_T	Thermophoretic Diffusion Coefficient	λ	Reactivity parameter
U_∞	ambient velocity	β	Heat generation parameter
Q	Heat source/sink parameter	D_B	Brownian Diffusion Coefficient,
L_1	Slip velocity	h	plate heat transfer coefficient
k_r	Binary chemical reaction parameter	$\boldsymbol{Pr}$	Prandtl number
R_G	Universal gas constant	$\boldsymbol{Sc}$	Schmidt number
Ea	Activation Energy	**Subscript**	
Greek Symbol		∞	ambient condition
ρ	fluid density	$\boldsymbol{w}$	wall condition
σ	Electrical conductivity		
μ	Fluid viscosity		$0 < \epsilon \ll 1$

In their study [6], Chamkha& Aly focused on the MHD free convection flow of a nanofluid via a vertical permeable plate in the presence of a heat source or sink, while Aziz & Khan [7] examined the flow of nanofluid over a vertical plate that had been convectively heated in a natural convective boundary layer.In their examination of MHD free convective boundary layer flow across a flat vertical plate under Newtonian heating boundary conditions, Uddin et al. [8] investigated a nanofluid. Samad & Mansur-study Rahman's [9] examined how thermal radiation and unsteady MHD flow interacted as they passed by a vertical porous plate.The plate was immersed in a porous substance. Makinde& Sibanda [5] concentrated on MHD mixed convective flow and heat transfer past a vertical plate dipped in a porous medium with constant wall suction, while Md. Anwar Hossain & Munir [10] provided analysis of a 2-D mixed convection flow of viscous incompressible temperature dependent viscous fluid past a vertical plate.Fang [11] investigated how changes in fluid properties affect the boundary layers of a stretching surface, while Mahmoud [12] showed how changes in viscosity affect the flow of a hydromagnetic boundary layer along a continuously rotating vertical plate sensitive to radiation.M. Anwar Hossain et al study's [13] found that radiation affects the free convection flow of a fluid with varying viscosity on a porous vertical plate.Using a non-linear stretching sheet, Poornima & Reddy [14] generated sustained free convective boundary layer flow of a radiating nanofluid in the presence of a transverse magnetic field.Kandasamy et al. [15] looked at how thermal stratification caused by solar radiation, Brownian motion, and thermophoresis affected the MHD boundary layer flow of nanofluid. For every heat absorption situation and a limited heat generation condition, Chamkha [16] found answers. The effects of a magnetic field are consistent with those previously reported in the literature when there is no heat creation or absorption.Makinde, [17] investigates the steady-state solutions for the substantially exothermic decomposition of a combustible substance uniformly distributed between symmetrically heated parallel plates under bimolecular, Arrhenius, and sensitized reaction rates, ignoring the material consumption. Using perturbation technique and a particular kind of Hermite-Padé approximants, analytical solutions are created for the governing nonlinear boundary-value problem. Makinde et al.[18] explored the rate of entropy production in a laminar flow across a saturated porous media channel. To calculate the entropy generation number and the irreversibility ratio for the big Darcy number (Da) and group parameter ($B_r\Omega^{-1}$), the velocity and temperature profiles are acquired. The outcome demonstrates that heat transfer irreversibility outweighs fluid friction irreversibility (i.e., $0 \le \varphi < 1$) and that viscous dissipation has no impact on the rate of entropy development at the channel centreline.

Governing equations

In light of many physical issues, such as fluid experiencing exothermic or endothermic chemical reactions, it is crucial to study the effects of heat generation or absorption in moving fluids. In many chemical engineering processes, a foreign mass and the working fluid move as a result of stretching a surface, which causes chemical reactions to occur. The chemical reaction's sequence is determined by a number of variables. One of the most basic chemical reactions is a first-order reaction, in which the rate of reaction is inversely proportional to the species concentration. This steady-state examination of the convection problem gave us the idea that adding in the magnetic field and the Arrhenius reaction would be intriguing and practical for applications.

The surface is kept at a constant temperature, the flow is considered to be laminar, two-dimensional, and stable, and the concentration of the species is supposed to be indefinitely long. Furthermore, it is believed that the applied transverse magnetic Reynolds number is low enough to ignore the induced magnetic field. Apart from the density in the buoyancy components of the momentum equation, which is approximated using the Boussinesq approximation, there is no applied electric field, and all of the Hall effect, viscous dissipation, and Joule heating are omitted.

The steady equations that explain the physical condition are given as follows under these assumptions:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta_t(T - T_\infty) + g\beta_c(C - C_\infty) - \frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$\rho c_p v \frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) + \mu \left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

$$\rho v \frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + k_r^2 (T - T_\infty)^r \exp\left(-\frac{E_a}{R_G T}\right)(C - C_\infty) \quad (4)$$

where y is the horizontal or transverse coordinate, it is the axial velocity, $v$is the transverse velocity, T is the fluid temperature, C is the concentration, T_∞ is the ambient temperature C_∞ is the ambient concentration, and $\rho, g, B_T, \beta c\ v\ \sigma, Bc, Q, D$ and r are the density, gravitational acceleration, coefficient of thermal expansion, coefficient of concentration expansion, kinematics viscosity, fluid electrical conductivity, magnetic induction, heat generation/coefficient and the chemical reaction parameter and real number respectively.

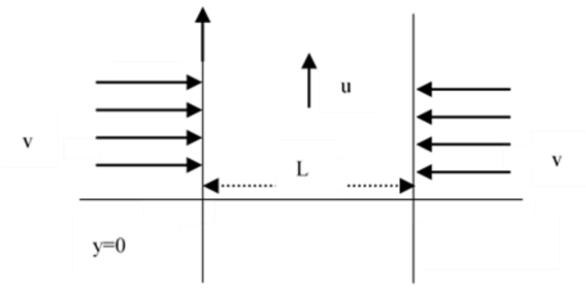

Figure 1: The channel of flow.

With the physical boundary conditions

$$\begin{aligned} &u(0) = u_w, v(0) = -v_0, T(0) = T_w, C(0) = C_w \\ &u \to \infty, T \to \infty, C \to \infty, \qquad \text{as } y \to \infty \end{aligned} \quad (5)$$

where $u_w, v_0 > 0, T_w$ and C_w are surface velocity, suction velocity, surface temperature and concentration respectively.

2.2Non – Dimensionalisation

From the continuity equation (1)

$$\frac{\partial v}{\partial y} = 0, \qquad v(0) = -v_0$$

Integrating we have the solution

$$v(y) = -v_0 \quad (6)$$

Using the solution (6), the momentum, energy, magnetic and species equations (2-4) can be non – dimensionalised using the following non – dimensional variables.

$$y' = y\,\frac{v_w}{v}, u' = \frac{u}{u_w}, v' = \frac{v}{v_w}, T - T_\infty = \frac{R_g T_\infty^2}{E_a}\theta, C - C_\infty = (C_w - C_\infty)\phi \quad (7)$$

After dropping primes (‘), we have

$$\frac{d^2 u}{dy^2} + v_0 \frac{du}{dy} + Grt\theta + Grc\phi - Hu = 0 \quad (8)$$

$$\frac{d^2\theta}{dy^2} + \mathrm{Pr} v_0 \frac{d\theta}{dy} + \mathrm{Pr}\,\beta\theta + \epsilon\delta\left(\frac{\partial u}{\partial y}\right)^2 = 0 \quad (9)$$

$$\frac{d^2\phi}{dy^2} + Sc\ v_0 \frac{d\phi}{dy} + \epsilon\lambda\theta^r e^{\frac{\theta}{1+\epsilon\theta}}\phi = 0 \quad (10)$$

The dimensionless boundary conditions are

$$\begin{aligned} &u(0) = 1, \theta(0) = 1, \phi(0) = 1 \\ &u \to 0, \theta \to 0, \phi \to 0, as\ y \to \infty \end{aligned} \quad (11)$$

Where

$$\begin{aligned} &\frac{\mu c_p}{k} = Pr, \frac{\mu}{D} = Sc, g\beta_t \frac{R_g T_\infty^2 v}{v_w^2 u_w E_a} = Grt, \frac{g\beta_c (C_w - C_\infty) v}{v_w^2 u_w} Grc, \\ &\frac{\sigma B_0^2 v}{v_w^2 \rho} = H, \frac{Q v^2}{v_w k} = \mathrm{Pr}\,\beta, \frac{E_a^2}{R_g^2 T_\infty^3}\frac{\mu}{k} u_w^2 = \delta, \frac{k_r^2}{D}\frac{v^2}{v_w^2}\frac{\epsilon^{r-1} T_\infty^r}{(C_w - C_\infty)} e^{-\frac{1}{\epsilon}} = \lambda \end{aligned} \quad (12)$$

We assume exponential approximation similar to the one in Ayeni et. al [19], the polynomial approximation of the exponential term

$$\exp\left(\frac{\theta}{1+\epsilon\theta}\right) \approx 1 + \theta + \left(\frac{1}{2} - \epsilon\right)\theta^2$$

Then the equation (10) becomes

$$\frac{d^2\phi}{dy^2} + Sc\, v_0 \frac{d\phi}{dy} + \epsilon\lambda\theta^r \left(1 + \theta + \left(\frac{1}{2} - \epsilon\right)\theta^2\right)\phi = 0 \tag{13}$$

which has a quadratic temperature field.

The equivalent of the boundary condition becomes

$$u(0) = L_0 \left.\frac{\partial u(y)}{\partial y}\right|_{y=0}, \left.\frac{\partial \theta(y)}{\partial y}\right|_{y=0} = Bi(\theta(y) - 1)|_{y=0}, \frac{\partial \phi(y)}{\partial y} + Sr \left.\frac{\partial \theta(y)}{\partial y}\right|_{y=0} = 0, \quad (14)$$
$$u \to 0, \theta \to 0, \phi \to 0, \text{ as } y \to \infty$$

Entropy generation rate

In convective heat and mass transfer and MHD flows, irreversibility arises due to viscous, heat and mass transfer effects. The entropy generation rate is expressed as the sum of contributions due to viscus, thermal and diffusive effects, and thus it depends functionally on the local values of temperature, velocity and concentration in the domain of interest.

According to Bejan. [20], the characteristics entropy transfer rate is given by

$$\boldsymbol{\Gamma_0 = k\left(\frac{\Delta T}{LT_0}\right)} \tag{15}$$

Where $\boldsymbol{k, L, T_0}$ and $\boldsymbol{\Delta T}$ are respectively, the thermal conductivity, the characteristics length of the enclosure, a reference temperature and a reference temperature difference.

Magherbi Mourad et al [21], give two-dimensional entropy generation rate as

$$\boldsymbol{\Gamma = \frac{\mu}{T_0}\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] + \frac{k}{T_0^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{RD}{C_0}\left[\left(\frac{\partial C}{\partial x}\right)^2 + \left(\frac{\partial C}{\partial y}\right)^2\right] + \frac{RD}{T_0}\left[\left(\frac{\partial T}{\partial x}\right)\left(\frac{\partial C}{\partial x}\right) + \left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial C}{\partial y}\right)\right]} \tag{16}$$

Where $\boldsymbol{C_0}$and $\boldsymbol{T_0}$are respectively the reference concentration and temperature, which are in our case, the bulk concentration and the bulk temperature.

Also, we note that the flow is in y – direction only, hence all derivatives with respect to x vanishes.

Thus, the dimensionless form of local entropy generation rate can be obtained in using the system of the dimensionless variables defined in (7).

$$\boldsymbol{\Gamma_{n,\theta} = \left(\frac{\partial \theta}{\partial y}\right)^2, \Gamma_{n,u} = \delta_1\left(\frac{\partial u}{\partial y}\right)^2, \Gamma_{n,\phi} = \delta_2\left(\frac{\partial \phi}{\partial y}\right)^2, \Gamma_{n,\tau} = \delta_3\left(\frac{\partial \theta}{\partial y}\right)\left(\frac{\partial \phi}{\partial y}\right)} \tag{17}$$

Dimensionless terms denoted $\boldsymbol{\delta_i, (1 \le i \le 3)}$, and called irreversibility distribution ratios, are given by:

$$\boldsymbol{\delta_1 = \frac{u_w^2 T_0 \mu}{k}\left(\frac{E_a}{R_g T_\infty^2}\right)^2, \delta_2 = \frac{RDT_0^2}{kC_0}\left(\frac{E_a \Delta C}{R_g T_\infty^2}\right)^2, \delta_3 = \frac{T_0 RD}{k}\frac{E_a \Delta C}{R_g T_\infty^2}} \tag{18}$$

The dimensionless total entropy generation is the integral over the system volume of the dimensionless local entropy generation. In our case here total entropy is given by summation of all local entropy stated as

$$\boldsymbol{E_G = \underbrace{\left(\frac{\partial \theta}{\partial y}\right)^2}_{\text{Thermal irreversibility}} + \underbrace{\delta_1\left(\frac{\partial u}{\partial y}\right)^2}_{\text{Viscous irreversibility}} + \underbrace{\delta_2\left(\frac{\partial \phi}{\partial y}\right)^2 + \delta_3\left(\frac{\partial \theta}{\partial y}\right)\left(\frac{\partial \phi}{\partial y}\right)}_{\text{Diffusive irreversibility}}} \tag{19}$$

The set of the dimensionless equations (8, 9 and 13), show that the problem is governed by the thermal and mass Grashof number Hartmann number, $\boldsymbol{\beta, \delta, \lambda}$.

4. Method of solution

Now, we consider the dimensionless equations (8), (9) and (13) with the boundary conditions (14). Using the asymptotic expansions defined about activation energy parameter ϵ, as

$$\left.\begin{array}{l} u \cong u_0 + \epsilon u_1 \\ \theta \cong \theta_0 + \epsilon\theta_1 \\ \phi \cong \phi_0 + \epsilon\phi_1 \end{array}\right\} \tag{20}$$

The solutions approximated by (20) are obtained using method of undetermined coefficient and the result for zero order ϵ^0 are:

$$\left.\begin{array}{r} \frac{d^2}{dy^2}u_0(y) + v_0\frac{d}{dy}u_0(y) + Grc\,\phi_0(y) + Grt\,\theta_0(y) - Ha\,u_0(y) = 0, \\ \frac{d^2}{dy^2}\theta_0(y) + Prv_0\frac{d}{dy}\theta_0(y) + Pr\boldsymbol{\beta}\theta_0(y) = 0, \\ \frac{d^2}{dy^2}\phi_0(y) + Scv_0\frac{d}{dy}\phi_0(y) = 0. \end{array}\right\} \tag{21}$$

$$u_0 = L_0 \frac{\partial u_0}{\partial y}, \frac{\partial \theta_0}{\partial y} = Bi(\theta_0 - 1), \frac{\partial \phi_0}{\partial y} + Sr\ \frac{\partial \theta_0}{\partial y} = 0, u_0 \to A, \theta_0 \to 0, \phi_0 \to 0, \text{as } y \to \infty \tag{22}$$

and for first order ϵ^1

$$\left.\begin{aligned} &\frac{d^2}{dy^2}u_1(y) + v_0\frac{d}{dy}u_1(y) + Grc\phi_1(y) + Grt\theta_1(y) - Hau_1(y) = 0, \\ &Prv_0\frac{d}{dy}\theta_1(y) + Pr\boldsymbol{\beta}\theta_1(y) + \frac{d^2}{dy^2}\theta_1(y) + Ec\left(\frac{d}{dy}u_0(y)\right)^2 = 0 \\ &\frac{d^2}{dy^2}\phi_1(y) + Scv_0\frac{d}{dy}\phi_1(y) + \lambda\theta_0(y)^r\left(1 + \theta_0(y) + \frac{1}{2}\theta_0(y)^2\right)\phi_0(y) = 0. \end{aligned}\right\} \tag{23}$$

$$u_1 = L_0 \frac{\partial u_1}{\partial y}, \frac{\partial \theta_1}{\partial y} = Bi\theta_1, \frac{\partial \phi_1}{\partial y} + Sr\frac{\partial \theta_1}{\partial y} = 0, u_1 \to 0, \theta_1 \to 0, \phi_1 \to 0, \quad \text{as } y \to \infty \tag{24}$$

The solutions approximated by equations (21) – (24) are obtained using method of undetermined coefficient and the result for zero order ϵ^0 are:

$$\left.\begin{aligned} &\theta_0(y) = a_1e^{-my} \\ &\phi_0(y) = a_2e^{-v_0Scy}, \\ &u_0(y) = a_3e^{-my} + a_4e^{-v_0Scy} + a_5e^{-ny} \end{aligned}\right\} \tag{25}$$

While the first order solutions are:

$\phi_0(y) < 0$ destructive chemical reaction

$$\left.\begin{aligned} &\theta_1(y) = a_6e^{-my} + a_7e^{-2my} + a_8e^{-(Scv_0+m)y} + a_9e^{-(n+m)y} \\ &+(a_{10} + a_{12})e^{-2Scv_0y} + a_{11}e^{-(Scv_0+n)y} \\ &\phi_1(y) = a_{14}e^{-(v_0Sc+m(r+2))y} + a_{15}e^{-(v_0Sc+m(r+1))y} + a_{16}e^{-(v_0Sc+mr)y} \\ &+a_{17}e^{-v_0Scy} \\ &u_1(y) = a_{18}e^{-ny} + a_{19}e^{-my} + a_{20}e^{-2my} + a_{21}e^{-(Scv_0+m)y} + a_{22}e^{-(n+m)y} \\ &+a_{23}e^{-2v_0Scy} + a_{24}e^{-(Scv_0+n)y} + a_{26}e^{-(Scv_0+m(r+2))y} \\ &+a_{27}e^{-(Scv_0+m(r+1))y} + a_{28}e^{-(Scv_0+mr)y} + a_{28}e^{-(Scv_0+mr)y} \end{aligned}\right\} \tag{26}$$

Solution for order of $\epsilon^i, i \geq 2$ are negligible.

Using equations (25) and (26) in equation (20), we have the solutions for velocity, temperature and species concentrations, respectively as

$$\begin{aligned} u(y) \cong\ & a_3e^{-my} + a_4e^{-v_0Scy} + a_5e^{-ny} + \epsilon(a_{18}e^{-ny} + a_{19}e^{-my} \\ &+a_{20}e^{-2my} + a_{24}e^{-(Scv_0+n)y} + a_{21}e^{-(Scv_0+m)y} + a_{22}e^{-(n+m)y} \\ &+a_{23}e^{-2v_0Scy} + a_{26}e^{-(Scv_0+m(r+2))y} + a_{27}e^{-(Scv_0+m(r+1))y} \\ &+a_{28}e^{-(Scv_0+mr)y} + a_{28}e^{-(Scv_0+mr)y} \end{aligned} \tag{27}$$

$$\begin{aligned} \phi(y) \cong\ & a_2e^{-v_0Scy} + \epsilon\big(a_{14}e^{-(v_0Sc+m(r+2))y} + a_{15}e^{-(v_0Sc+m(r+1))y} \\ &+a_{16}e^{-(v_0Sc+mr)y} + a_{17}e^{-v_0Scy}\big) \end{aligned} \tag{28}$$

$$\begin{aligned} \theta(y) \cong\ & a_1e^{-my} + \epsilon\big(a_6e^{-my} + a_7e^{-2my} + a_8e^{-(Scv_0+m)y} + a_9e^{-(n+m)y} \\ &+(a_{10} + a_{12})e^{-2Scv_0y} + a_{11}e^{-(Scv_0+n)y}\big) \end{aligned} \tag{29}$$

All parameters are as define in the appendix.

Now using equations (27) – (29), the local entropy generation are

$$\begin{aligned} \Gamma_{n,\theta} =\ & \big(a_1me^{-my} + \epsilon\big(a_6me^{-my} + 2a_7me^{-2my} + a_8(Scv_0 + m)e^{-y(Scv_0+m)} \\ &+a_9(n + m)e^{-y(n+m)} + \big(2(a_{10} + a_{12})\big)Scv_0e^{-2Scv_0y} \\ &+a_{11}(Scv_0 + n)e^{-y(Scv_0+n)}))^2 \end{aligned} \tag{26}$$

$$\begin{aligned} \Gamma_{n,u} =\ & \delta_1(a_3me^{-my} + a_4Scv_0e^{-Scv_0y} + a_5ne^{-ny} + \epsilon(a_{18}me^{-my} + a_{19}me^{-my} \\ &+2a_{20}me^{-2my} + a_{21}(Scv_0 + m)e^{-y(Scv_0+m)} + a_{22}(n + m)e^{-y(n+m)} \\ &+2a_{23}Scv_0e^{-2Scv_0y} + a_{24}(Scv_0 + n)e^{-y(Scv_0+n)} + a_{29}Scv_0e^{-Scv_{0y}} \\ &+a_{26}\big(Scv_0 + m(r + 2)\big)e^{-y(Scv_0+m(r+2))} + a_{28}(Scv_0 + mr)e^{-(Scv_0+mr)y} \\ &+a_{27}\big(Scv_0 + m(r + 1)\big)e^{-y(Scv_0+m(r+1))}\big)\big)^2 \end{aligned} \tag{27}$$

$$\begin{aligned} \Gamma_{n,\phi} =\ & \delta_2\big(a_2Scv_0e^{-Scv_0y} + \epsilon\big(a_{14}\big(Scv_0 + m(r + 2)\big)e^{-y(Scv_0+m(r+2))} \\ &+a_{15}\big(Scv_0 + m(r + 1)\big)e^{-y(Scv_0+m(r+1))} + a_{16}(Scv_0 + mr)e^{-(Scv_0+mr)y} \\ &+a_{17}Scv_0e^{-Scv_0y}\big)\big)^2 \end{aligned} \tag{28}$$

$$\Gamma_{n,\tau} = \delta_3\big(a_1 me^{-my} + \epsilon\big(a_6 me^{-my} + 2a_7 me^{-2my} + a_8(Scv_0+m)e^{-y(Scv_0+m)} +a_9(n+m)e^{-y(n+m)} + \big(2(a_{10}+a_{12})\big)Scv_0 e^{-2Scv_0 y} +a_{11}(Scv_0+n)e^{-y(Scv_0+n)}\big)(a_2 Scv_0 e^{-Scv_0 y} + a_{17} Scv_0 e^{-Scv_0 y} +\epsilon\big(a_{14}\big(Scv_0+m(r+2)\big)e^{-y(Scv_0+m(r+2))} +a_{16}(Scv_0+mr)e^{-(Scv_0+mr)y} + a_{15}\big(Scv_0+m(r+1)\big)e^{-y(Scv_0+m(r+1))}\big)\big) \tag{29}$$

Using equations (26) - (29) in equation (19) we obtain an explicit total entropy generation is obtained as

$$E_G = \big(a_1 me^{-my} + \epsilon\big(a_6 me^{-my} + 2a_7 me^{-2my} + a_8(Scv_0+m)e^{-y(Scv_0+m)} +a_9(n+m)e^{-y(n+m)} + \big(2(a_{10}+a_{12})\big)Scv_0 e^{-2Scv_0 y} +a_{11}(Scv_0+n)e^{-y(Scv_0+n)}\big)\big)^2 + \delta_1(a_3 me^{-my} + a_4 Scv_0 e^{-Scv_0 y} +a_5 ne^{-ny} + \epsilon(a_{18} me^{-my} + a_{19} me^{-my} + 2a_{20} me^{-2my} +a_{21}(Scv_0+m)e^{-y(Scv_0+m)} + a_{22}(n+m)e^{-y(n+m)} + 2a_{23} Scv_0 e^{-2Scv_0 y} +a_{24}(Scv_0+n)e^{-y(Scv_0+n)} + a_{29} Scv_0 e^{-Scv_{0y}} +a_{26}\big(Scv_0+m(r+2)\big)e^{-y(Scv_0+m(r+2))} + a_{28}(Scv_0+mr)e^{-(Scv_0+mr)y} +a_{27}\big(Scv_0+m(r+1)\big)e^{-y(Scv_0+m(r+1))}\big)\big)^2 + \delta_2(a_2 Scv_0 e^{-Scv_0 y} +\epsilon\big(a_{14}\big(Scv_0+m(r+2)\big)e^{-y(Scv_0+m(r+2))} + a_{16}(Scv_0+mr)e^{-(Scv_0+mr)y} +a_{15}\big(Scv_0+m(r+1)\big)e^{-y(Scv_0+m(r+1))} + a_{17} Scv_0 e^{-Scv_0 y})\big)^2 +\delta_3\big(a_1 me^{-my} + \epsilon\big(a_6 me^{-my} + 2a_7 me^{-2my} + a_8(Scv_0+m)e^{-y(Scv_0+m)} +a_9(n+m)e^{-y(n+m)} + \big(2(a_{10}+a_{12})\big)Scv_0 e^{-2Scv_0 y} +a_{11}(Scv_0+n)e^{-y(Scv_0+n)}\big)(a_2 Scv_0 e^{-Scv_0 y} + a_{17} Scv_0 e^{-Scv_0 y} +\epsilon\big(a_{14}\big(Scv_0+m(r+2)\big)e^{-y(Scv_0+m(r+2))} +a_{16}(Scv_0+mr)e^{-(Scv_0+mr)y} + a_{15}\big(Scv_0+m(r+1)\big)e^{-y(Scv_0+m(r+1))}\big)\big) \tag{30}$$

where all parameters are as defined above!

Equation (30) is the expression for total entropy generation in passive control MHD flow in the presence of Arrhenius chemical reaction with heat generation/absorption

It is quite essential to calculate the significant input of each source of entropy production in a system, in view of this, the Bejan number describes the proportion of the entropy production by heat transfer to the total proportion as represented in eqn (47) or (48),

$$Be = \frac{\Gamma_{n,\theta}}{E_G} = \frac{\Gamma_{n,\theta}}{\Gamma_{n,\theta} + \Gamma_{n,u} + \Gamma_{n,d}} \tag{31}$$

It is important to note that the entropy generation due to diffusion ($\Gamma_{n,d} = \Gamma_{n,\phi} + \Gamma_{n,\tau}$) is the sum of a pure term ($\Gamma_{n,\phi}$) which involves concentration gradient only and a crossed term ($\Gamma_{n,\tau}$) with both thermal and concentration gradients. Therefore, a coupling effect between thermal gradient and concentration gradient can be shown in the expression of the entropy generation, whereas this coupling effect was neglected in the energy and specie conservation equations (Soret and Dufour effects) and also in the mass diffusion flux equation (first Fick's law).

Results and discussions

The local entropy generation rate is a function of concentration temperature and velocity gradients in the y directions in the entire calculation domain. The analytical simulations presented in this work has been conducted in order to study the effects of the thermal Grashof number, heat generation/absorption, chemical reaction parameter and the Hartmann number on entropy generation in steady state conditions.

Figures 2 and 3 shows the effect of Lorentz force and heat generation on the entropy generation respectively. From the figures we observe that the Lorentz force in term of Hartman and heat generation decline the bulk entropy generation of the flow field with maximum entropy generation at the surface. Increasing the Hartman number by 2.5%, 37.5% and 1.8% consecutively result into 43.2%, 85.6% and 59.1% decrease in entropy generation respectively as shown in Figure 2 while increasing the heat generation by 3.33%, 6.5 % and 6.1% consecutively as shown in Figure 3 produces 78.5%, 68.6% and 76.1% decrease in total entropy generation respectively. It was further discovered that Eckert number enhances entropy generation. From Figure 4, doubling the value of Eckert number from 0.05 to 0.10 increases the entropy generation from 17.92 to 68.47 corresponding 282.14% and from $Ec = 0.2$ to $Ec = 0.5$ yielded 516.72% increment. The impact of convective heat transfer on Entropy generation was displayed in Figure 5. It was discovered that increase in convective heat transfer result in increase in entropy generation. The velocity slip factor was seen to increase the entropy generation as seen in Figure 6. From this figure we could see that increasing the velocity slip factor from 0 to 0.5, 1.0 and 2.0 produces entropies 197.09, 321.28, 476.80, 881.07 at the surface respectively. The chemical reaction parameter effect was displayed in Figure 7, where increase in generative chemical reaction was seen to increase the entropy generation and conversely destructive chemical reaction lowers the entropy generation.

From Figures 8-12, we displayed the impact of Frank Kamnetski parameter (ϵ), mass (Grc) and thermal (Grt) buoyancy and Soret number respectively on entropy generation. From the figures, it was discovered that an increase in each parameter resulted in increase in entropy generation of the flow field. The impact of Prandtl number is displayed in Figures 12. As expected, as much heat is generated, the bulk available heat in the flow system decrease resulting into lowering of entropy generation in the flow field.

For small thermal Grashof number, there is practically little or no convection and the entropy generation due to fluid friction is zero, consequently the total entropy generation is reduced to the entropy generation due to heat transfer. At higher Grashof number heat transfer due to convection begins to play a significant role increasing the flow velocity and in turn the entropy generation due to the viscous effects. Also, the isotherms are deformed increasing the temperature gradient and consequently the entropy generation due to heat transfer..

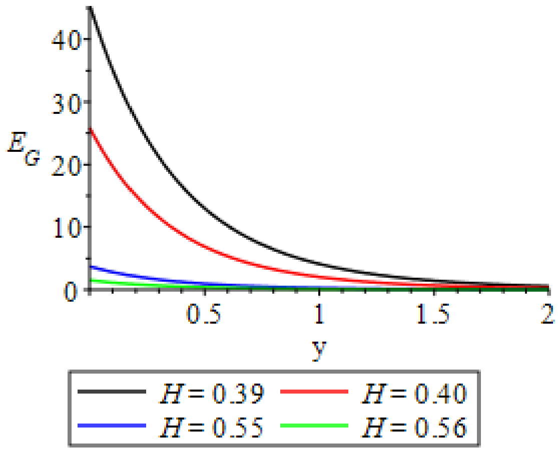

Figure 2: entropy Generation distributionfor various values of Hartman numbers

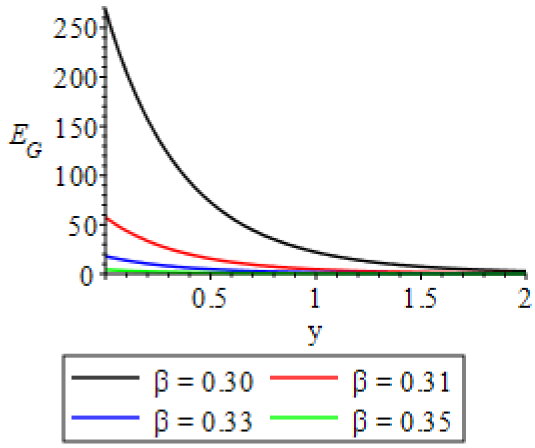

Figure 3: Entropy generation distribution for various values of heat **g**eneration/absorption

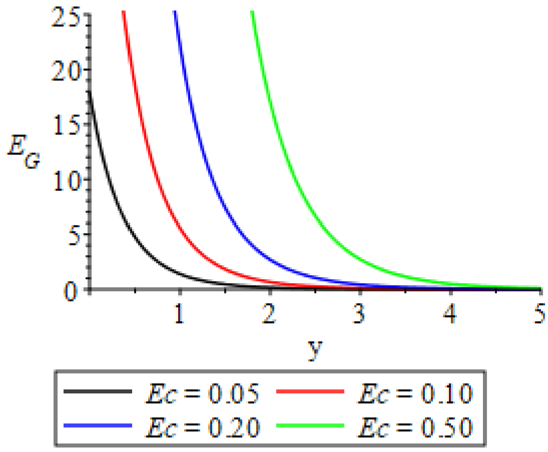

Figure 4: entropy Generation distribution for various values of Eckert numbers.

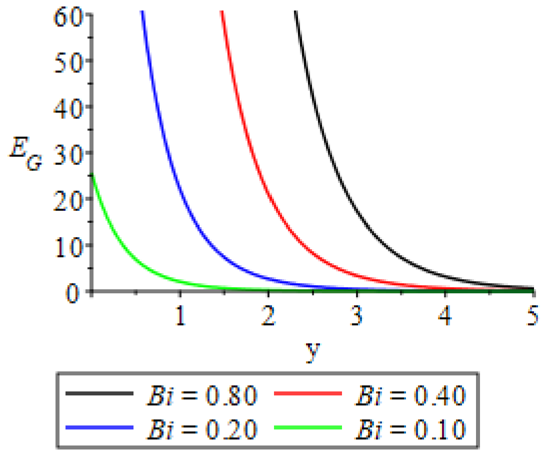

Figure 5: Entropy generation distribution for various values of Convective heat transfer

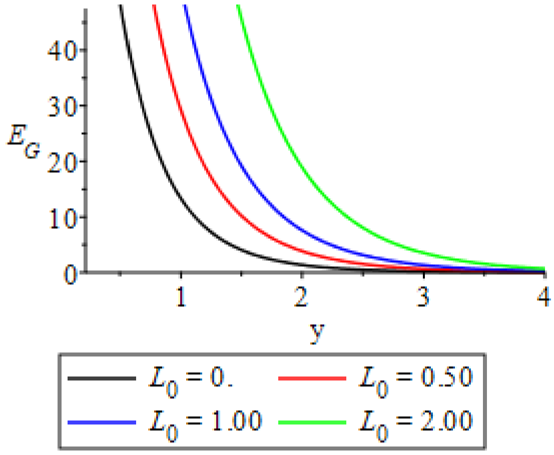

Figure 6: entropy Generation distribution for various values of velocity slip factor

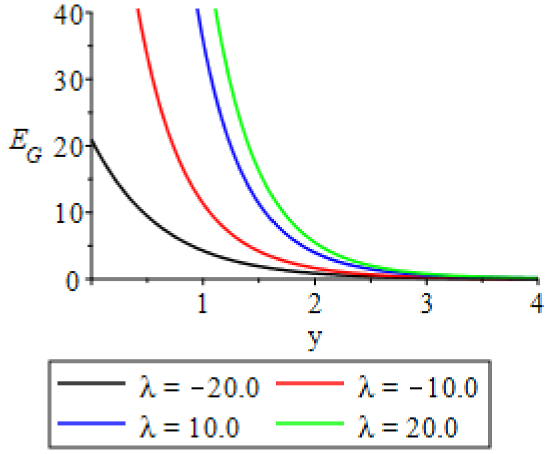

Figure7: **E**ntropy generation distribution for various values of reactivity parameter

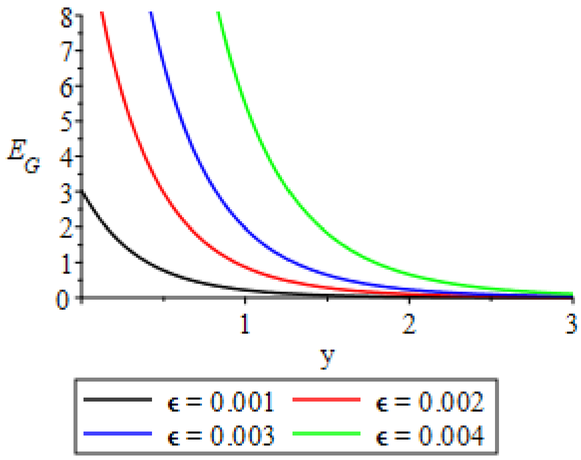

Figure 8: entropy Generation distributionfor various values of **of**Frank Kamnetskiparameter

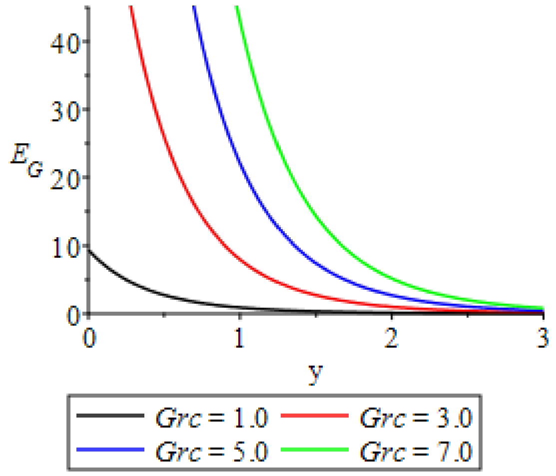

Figure 9: Entropy Generation distribution for various values of mass buoyancy.

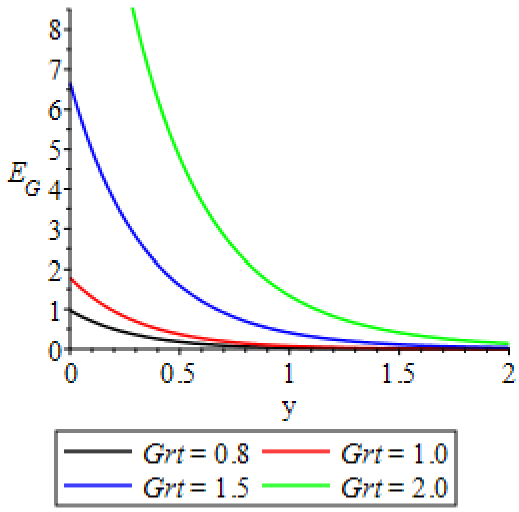

Figure 10: entropy Generation distributionfor various values of of thermal buoyancy

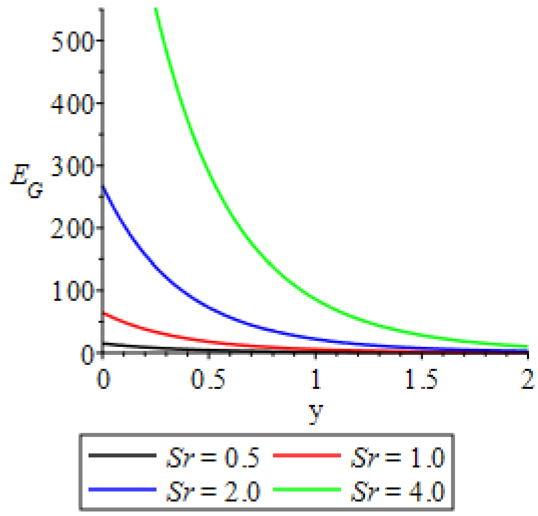

Figure 11: Entropy Generation distribution for various values of Soret number.

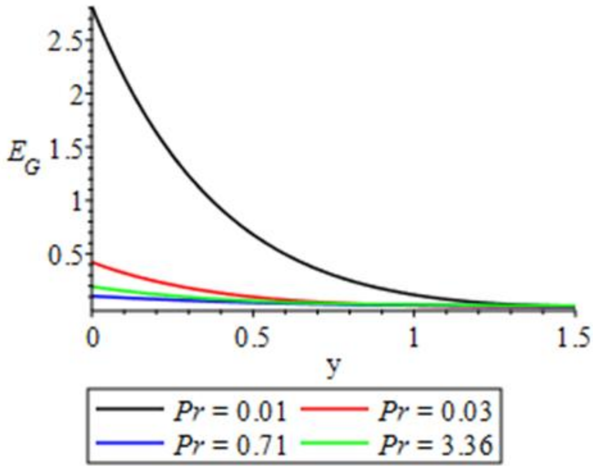

Figure12: Entropy Generation distribution for various values of Prandtl number

Table 2: Correlation between the entropy generated and Velocity, temperature and Concentration at Flow Surface

	$u(0)$	$\theta(0)$	$\phi(0)$
H	0.96	-	0.89
β	0.98	-0.89	-0.87
Ec	0.98	0.97	-0.98
λ	0.99	-	0.98
Grc	0.97	-	-0.59
Grt	0.99	-0.97	-1.00
Bi	0.98	0.92	-0.93
L_0	0.99	-	-
ϵ	0.98	-	-0.94
Sr	0.98	-	-0.99
Pr	0.89	0.82	0.82

Table 3: Effect of governing parameters on surface values of Entropy, Velocity, Temperature and Concentration.

	$E_G(0)$	$u(0)$	$\theta(0)$	$\phi(0)$
H =0.39	45.60	13.91	0.20	1.09
H =0.40	25.89	4.23	0.20	1.14
H =0.55	3.73	1.04	0.20	-1.78
H =0.56	1.52	0.40	0.20	-1.04
β =0.30	267.82	13.91	0.10	-0.79
β =0.31	57.47	6.38	0.12	-0.66
β =0.32	18.01	3.51	0.13	-0.39
β =0.35	4.31	1.65	0.15	-0.35
Ec =0.05	17.92	3.50	0.20	-0.37
Ec =0.10	68.48	6.97	0.24	-0.39
Ec =0.20	267.82	13.91	0.28	-0.44
Ec =0.50	1651.64	34.73	0.40	-0.58
λ =-20.0	20.94	13.85	0.20	-0.49
λ =-10.0	108.69	17.91	0.20	-0.40
λ =10.0	489.73	23.99	0.20	-0.31
λ =20.0	783.04	34.12	0.20	-0.22
Grc =1.0	9.38	2.80	0.20	-0.35
Grc =3.0	94.09	8.36	0.20	-0.35
Grc =5.0	267.82	13.91	0.20	-0.37
Grc =7.0	529.48	19.42	0.20	-0.36
Grt =0.8	0.97	0.45	-0.36	-0.36
Grt =1.0	1.79	0.78	-2.05	-0.64
Grt =1.5	6.66	1.89	-4.87	-1.57
Grt =2.0	18.78	3.44	-8.82	-5.35
Bi =0.8	10528.3	87.50	0.50	-0.92
Bi =0.4	2069.15	38.76	0.34	-0.60
Bi =0.2	267.82	13.91	0.20	-0.35
Bi =0.1	25.65	4.28	0.11	-0.19
L_0 =0.0	197.09	10.98	0.20	-0.35
L_0 =0.5	321.28	15.87	0.20	-0.35
L_0 =1.0	476.80	20.78	0.20	-0.35
L_0 =2.0	881.07	30.62	0.20	-0.35
ϵ =0.01	3.03	1.37	0.20	-0.34
ϵ =0.02	11.27	2.76	0.20	-0.36
ϵ =0.03	24.80	4.15	0.20	-0.37
ϵ =0.05	67.75	6.94	0.20	-0.39
Sr =0.5	15.06	3.49	0.20	-0.09

Sr =1.0	64.52	6.97	0.20	-0.17
Sr =2.0	267.82	13.91	0.20	-0.35
Sr =4.0	1079.53	27.50	0.20	-0.76
Pr =0.01	2.81	0.40	0.46	-0.24
Pr =0.03	0.42	0.17	0.33	-0.29
Pr =0.71	0.11	-0.11	0.07	-0.39
Pr =2.36	0.19	-0.11	0.03	-0.41

6.Concluding remark

From the solutions obtained, we deduced the following: That

- Lorentz force in term of Hartman and heat generation decline the bulk entropy generation of the flow field
- maximum entropy generation occur at the surface of the flow.
- increasing the heat generation by decrease total entropy generation
- Eckert number enhances entropy generation.
- increase in convective heat transfer result in increase in entropy generation.
- increase in generative chemical reaction increase the entropy generation and conversely destructive chemical reaction lowers the entropy generation.
- Frank Kamnetski parameter (ϵ), mass (Grc) and thermal (Grt) buoyancy and Soret number bring about enhancement in entropy generation of the flow field.

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Conflict of Interest: According to the authors, there is no conflict of interest.

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Appendix

$$m = \frac{1}{2}\left(Prv_0 + \sqrt{Pr^2v_0^2 - 4Pr\beta}\right), n = \frac{1}{2}\left(v_0 + \sqrt{v_0^2 + 4H}\right)$$

$$b_2 = \frac{\lambda}{m(r+1)(Scv_0 - mr - m)}, b_3 = \frac{\lambda}{2(r+2)m(Scv_0 - mr - 2m)}$$

$$b_4 = \frac{\lambda}{rm(Scv_0 - mr)}, b_4 = \frac{1}{Scv_0}, a_0 = 1 - a_1 - a_2$$

$$a_2 = \frac{Grc}{(-Sc^2 + Sc)v_0^2 + H}, a_1 = \frac{Grt}{-m^2 + mv_0 + H}$$

$$a_3 = -\frac{b_2 + b_3 + b_4}{b_5}, a_4 = \frac{\delta a_0^2 n^2}{-2Prnv_0 + Pr\beta + 4}$$

$$a_5 = (-m^2 + mv_0 + H)(-m^2r^2 + mrv_0 + H)\big((-Sc^2 + Sc)v_0^2 + H\big)Grc(-(r+2)^2m^2 + v_0(r+2)m + H)\frac{b_2}{a_{11}}$$

$$a_6 = (-(r+1)^2m^2 + v_0(r+1)m + H)(-m^2 + mv_0 + H)(-m^2r^2 + mrv_0 + H)\big((-Sc^2 + Sc)v_0^2 + H\big)b_3\frac{Grc}{a_{11}}$$

$$a_7 = \frac{b_4Grc}{a_{11}}(-(r+2)^2m^2 + v_0(r+2)m + H)(-(r+1)^2m^2 + v_0(r+1)m + H)(-m^2 + mv_0 + H)\big((-Sc^2 + Sc)v_0^2 + H\big)$$

$$a_8 = \frac{a_3}{a_{11}}(-(r+2)^2m^2 + v_0(r+2)m + H)(-(r+1)^2m^2 + v_0(r+1)m + H)(-m^2r^2 + mrv_0 + H)(-m^2 + mv_0 + H)b_5Grc$$

$$a_9 = \frac{Grta_4}{a_{11}}(-(r+1)^2m^2 + v_0(r+1)m + H)\big((-Sc^2 + Sc)v_0^2 + H\big)(-m^2r^2 + m\,rv_0 + H)\big(-(r+2)^2m^2 + v_{0(r+2)m} + H\big)$$

$$a_{10} = \frac{m^2 - mv_0 - H}{H}, a_{12} = -a_5 - a_6 - a_7 - a_8 - a_9(1 + a_{10})$$

$$a_{11} = (n + (-r-2)m)(n-m)(mr + n - v_0)(n + m(r+1) - v_0)(-Scv_0 + n)(n - v_0 + m)(n + (r+2)m - v_0)(-mr + n)(n + (-r-1)m)v_0(Sc - 1)$$

References

[1]. Chamkha A.J. (2003). MHD flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction,Int. Comm. *Heat Mass Transfer* 30, pp 413 – 422.

[2]. Sajid, M., & Hayat, T. (2008). Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. *International Communications in Heat and Mass Transfer*, *35*(3), 347–356.

[3]. Aliakbar, V., Alizadeh-Pahlavan, A., &Sadeghy, K. (2009). The influence of thermal radiation on MHD flow of Maxwellian fluids above stretching sheets. *Commun. Nonlinear Science Numerical Simulation*, *14*, 779–794.

[4]. Siddheshwar, P. G., &Mahabaleswar, U. S. (2005). Effects of radiation and heat source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet. *International Journal of Non-Linear Mechanics*, *40*(6), 807–820.

[5]. Makinde, O. D., & Sibanda, P. (2008). Magnetohydrodynamic Mixed-Convective Flow and Heat and Mass Transfer Past a Vertical Plate in a Porous Medium With Constant Wall Suction. *Journal of Heat Transfer*, *130*(11).

[6]. Chamkha, A. J., & Aly, A. M. (2010). Mhd Free Convection Flow Of A Nanofluid Past A Vertical Plate In The Presence Of Heat Generation Or Absorption Effects. *Chemical Engineering Communications*, *198*(3), 425–441.

[7]. Aziz, A., & Khan, W. A. (2012). Natural convective boundary layer flow of a nanofluid past a convectively heated vertical plate. *International Journal of Thermal Sciences*, *52*, 83–90.

[8]. Uddin, M. J., Khan, W. A., & Ismail, A. I. (2012). MHD Free Convective Boundary Layer Flow of a Nanofluid past a Flat Vertical Plate with Newtonian Heating Boundary Condition. *PLoS ONE*, *7*(11), 49499.https://doi.org/10.1371/journal.pone.0049499

[9]. Samad, M. A., & Mansur-Rahman, M. (1970). Thermal Radiation Interaction with Unsteady MHD Flow Past a Vertical Porous Plate Immersed in a Porous Medium. *Journal of Naval Architecture and Marine Engineering*, *3*(1), 7–14.

[10]. Hossain, Md.Anwar, & Munir, M. S. (2000). Mixed convection flow from a vertical flat plate with temperature dependent viscosity. *International Journal of Thermal Sciences*, *39*(2), 173–183.

[11]. Fang, T. (2004). Influences of fluid property variation on the boundary layers of a stretching surface. *Acta Mechanica*, *171*(1–2).

[12]. Mahmoud, M. A. A. (2007). Variable viscosity Effects on hydromagnetic boundary layer flow along a continuously moving vertical plate in the presence of radiation. *Applied Mathematical Sciences*, *1(17)*, 799–814.

[13]. Hossain, M.Anwar, Khanafer, K., &Vafai, K. (2001). The effect of radiation on free convection flow of fluid with variable viscosity from a porous vertical plate. *International Journal of Thermal Sciences*, *40*(2), 115–124.

[14]. Poornima, T., & N.B., R. (2013). Radiation effects on MHD free convective boundary layer flow of nanofluids over a nonlinear stretching sheet. *Advances in Applied Science Research*, *4(2)*, 190–202.

[15]. Kandasamy, R., Muhaimin, I., & Mohamad, R. (2013). Thermophoresis and Brownian motion effects on MHD boundary-layer flow of a nanofluid in the presence of thermal stratification due to solar radiation. *International Journal of Mechanical Sciences*, *70*, 146–154.

[16]. Chamkha A.J. (2003)**:** MHD flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction,Int. Comm. Heat Mass Transfer 30, pp 413 – 422.

[17]. O.D. Makinde, (2004)**:** Exothermic explosions in a slab: A case study of series summation technique. *Int. Comm. Heat and Mass Transfer* 31,8, 1227-1231.

[18]. O.D. Makinde and E. Osalusi (2005)**:** Second Law Analysis of Laminar Flow in a Channel Filled With Saturated Porous Media. *Entropy* ISSN 1099-4300**,** *7[2], 148-160*

[19]. Ayeni R.O., Okedoye A.M., Balogun F.O. and Ayodele T.O. (2004)**:** Higher order MHD flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction, *J. Nigeria Association of Mathematical Physics vol. 8 pp 163 – 166*.

[20]. Bejan, A., **1**996**.** Entropy generation minimization, CRC Press, New York,

[21]. Magherbi Mourad, Abbassi Hassen, HidouriNejib&Ben Brahim Ammar (2006): Second Law Analysis in Convective Heat and Mass Transfer. *Entropy 8[1], 1-17*