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On Finding Integer Solutions To Non-homogeneous Ternary Cubic Equation $x^2 - y^2 + x + y = 2z^3$

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ABSTRACT:

This paper concerns with the problem of obtaining non-zero distinct integer solutions to the non-homogeneous ternary cubic equation $x^2 - y^2 + x + y = 2z^3$. Different sets of integer solutions are illustrated.

Keywords: Non-Homogeneous Cubic ,Ternary Cubic , Integer Solutions

I.INTRODUCTION:

The Diophantine equations are rich in variety and offer an unlimited field for research [1-4]. In particular refer [5-24] for a few problems on ternary cubic equation with 3 unknowns. This paper concerns with yet another interesting ternary cubic diophantine equation with three variables given by $x^2 - y^2 + x + y = 2z^3$ for determining its infinitely many non-zero distinct integral solutions

II. METHOD OF ANALYSIS:

The non-homogeneous ternary cubic equation under consideration is

$$x^2 - y^2 + x + y = 2z^3 \tag{1}$$

Different methods of solving (1) are illustrated below:

Method 1:

Introduction of the linear transformations

$$x = u + v, y = u - v, u \neq v \neq 0 \tag{2}$$

in (1) leads to

$$u(2v+1) = z^3 \tag{3}$$

We solve (3) through different ways and using (2) , obtain different sets of solutions to (1).

Way 1:

The choice

$$u = 1 \tag{4}$$

in (3) gives

$$v = \frac{z^3 - 1}{2}$$

For integer solution , take

$$z = 2k + 1 \tag{5}$$

and thus,

$$v = 4k^3 + 6k^2 + 3k \tag{6}$$

Substituting (4) and (6) in (2) ,we have

$$x = 4k^3 + 6k^2 + 3k + 1, y = 1 - (4k^3 + 6k^2 + 3k) \tag{7}$$

Thus,(5) and (7) represent the integer solutions to (1).

Way 2:

The choice

$$u = z \tag{8}$$

in (3) gives

$$v = \frac{z^2 - 1}{2}$$

For integer solution , take

$$z = 2k + 1 \tag{9}$$

and thus,

$$v = 2k^2 + 2k \tag{10}$$

Substituting (8) and (10) in (2) ,we have

$$x = 2k^2 + 4k + 1, y = 1 - 2k^2 \tag{11}$$

Thus,(9) and (11) represent the integer solutions to (1).

Way 3:

The choice

$$u = z^2 \tag{12}$$

in (3) gives

$$v = \frac{z-1}{2}$$

For integer solution , take

$$z = 2k + 1 \tag{13}$$

and thus,

$$v = k, u = 4k^2 + 4k + 1 \tag{14}$$

Substituting (14) in (2) ,we have

$$x = 4k^2 + 5k + 1, y = 4k^2 + 3k + 1 \tag{15}$$

Thus,(13) and (15) represent the integer solutions to (1).

Method 2:

Employing factorization ,(1) is written as

$$(x + y)(x - y + 1) = 2z^3 \tag{16}$$

Express (16) as the system of double equations as in Table 1 below:

Table 1-System of double equations

System	I	II
$x + y$	z^3	z^2
$x - y$	1	$2z - 1$

Solving each of the above system of equations ,the corresponding values of x, y, z satisfying (1) are exhibited below:

Solutions from system I :

$$x = 4k^3 + 6k^2 + 3k + 1, y = 4k^3 + 6k^2 + 3k, z = 2k + 1$$

Solutions from system II :

$$x = 2k^2 + 4k + 1, y = 2k^2, z = 2k + 1$$

Method 3:

Treating (1) as the quadratic in x and solving for x ,one obtains

$$x=\frac{-1\pm\sqrt{(2y-1)^2+8z^3}}{2} \tag{17}$$

To remove the square-root on the R.H.S. of (17) , take

$$\alpha^2=(2y-1)^2+8z^3$$

and express it as the system of double equations as in Table 2 below:

Table 2 –System of double equations

System	I	II	III	IV	V
$\alpha+2y-1$	$2z^3$	$4z^3$	$2z^2$	$4z^2$	$8z^2$
$\alpha-2y+1$	4	2	$4z$	$2z$	z

Solving each of the above system of equations and using (17) , the corresponding integer solutions to (1) are obtained. For simplicity and brevity, the integer solutions are presented below:

Solutions from system I :

$$x=-4k^3-6k^2-3k-2,\ y=4k^3+6k^2+3k,\ z=2k+1$$

Solutions from system II :

$$x=-k^3-1,\ y=k^3,\ z=k$$

Solutions from system III :

$$x=2k^2+4k+1,\ y=2k^2,\ z=2k+1,$$
$$x=-2k^2-4k-2,\ y=2k^2,\ z=2k+1$$

Solutions from system IV :

$$x=-4k^2-5k-2,\ y=4k^2+3k+1,\ z=2k+1$$

Solutions from system V :

$$x=32k^2-33k+8,\ y=32k^2-31k+8,\ z=-4k+2,$$
$$x=-32k^2+33k-9,\ y=32k^2-31k+8,\ z=-4k+2$$

III.CONCLUSION:

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous cubic diophantine equation with three unknowns given by $x^2 - y^2) + x + y = 2z^3$ .One may search for other sets of integer solutions to the considered equation as well as other choices of the third degree diophantine equations with multi-variables

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