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GENERAL FORM OF INTEGRAL SOLUTIONS TO THE TERNARY NON-HOMOGENEOUS CUBIC EQUATION $\mathbf{y^2 + Dx^2 = \alpha z^3}$

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Abstract:

The purpose of this paper is to obtain a general form of non-zero distinct integral solutions of ternary non-homogeneous cubic diophantine equation $y^2 + Dx^2 = \alpha z^3$ where α is a given non-zero integer.

Keywords: Ternary cubic equation, Non-homogeneous cubic, Integer solutions

Introduction:

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, cubic diophantine equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity [1-4]. In this context, one may refer [5-24] for various problems on the cubic diophantine equations with three variables, where, in each of the problems, different sets of non-zero integer solutions are obtained. However, often we come across homogeneous and non-homogeneous cubic equations and as such one may require its integral solution in its most general form. It is towards this end, this paper concerns with the problem of determining a general form of non-trivial integral solutions of the non-homogeneous cubic equation with three unknowns given by $y^2 + Dx^2 = \alpha z^3$ where α is a given non-zero integer and D is a given non-zero square-free integer.

Method of Analysis:

The ternary non-homogeneous cubic diophantine equation to be solved for its distinct non-zero integral solution is

$$y^2 + D x^2 = \alpha z^3 \tag{1}$$

Let x_0, y_0, z_0 be any given non-zero integer solution to (1) so that

.
$$y_0^{\,2} + D x_0^{\,2} = \alpha z_0^{\,3} \tag{2}$$

Now, consider

$$y = A y_0 \pm D B x_0 , x = B y_0 \mp A x_0 \tag{3}$$

where A, B are non-zero integers to be determined such that (3) satisfies (1).
Substituting (3) in (1),we have

$$\text{L.H.S. of } (1) = (y_0^{\,2} + D x_0^{\,2})(A^2 + D B^2) \tag{4}$$

In view of (2), it is seen that

$$\text{L.H.S. of } (1) = \alpha z_0^{\,3} (A^2 + D B^2) \tag{5}$$

On comparing the R.H.S. of (1) and (5), note that we have to choose A and B so that $A^2 + DB^2$ is a perfect cubical integer. Choosing

$$A = m(m^2 + D n^2), B = n(m^2 + D n^2) \tag{6}$$

it is seen that

$$A^2 + D B^2 = (m^2 + D n^2)^3$$

and thus, one obtains

$$z = z_0 (m^2 + D n^2) \tag{7}$$

From (6) and (3), we have

$$x = (m^2 + Dn^2)(ny_0 \mp mx_0), y = (m^2 + Dn^2)(my_0 \pm Dnx_0) \tag{8}$$

Thus, (7) and (8) represent the general form of integral solutions to (1).

Note :

It is worth to mention that

$A^2 + D B^2$ is a perfect cubical integer when

$$A = m(m^2 - 3D n^2), B = n(3m^2 - D n^2)$$

In this case, the general form of integer solution to (1) is given by

$$y = m(m^2 - 3Dn^2)\, y_0 \pm Dn(3m^2 - Dn^2)\, x_0,$$
$$x = n(3m^2 - Dn^2)\, y_0 \mp m(m^2 - 3Dn^2)\, x_0,$$
$$z = (m^2 + Dn^2)\, z_0$$

A few examples are presented below:

Example: 1

$$D = 10, \alpha = 11, y_0 = x_0 = z_0 = 1$$
$$x = (m^2 + 10n^2)(n \mp m),$$
$$y = (m^2 + 10n^2)(m \pm 10n),$$
$$z = (m^2 + 10n^2)$$

Example: 2

$$D = 10, \alpha = 11, y_0 = x_0 = z_0 = 1$$
$$x = n(3m^2 - 10n^2) \mp m(m^2 - 30n^2),$$
$$y = m(m^2 - 30n^2) \pm 10n(3m^2 - 10n^2),$$
$$z = (m^2 + 10n^2)$$

Example: 3

$$D = 19, \alpha = 35, y_0 = 4, x_0 = z_0 = 1$$
$$x = (m^2 + 19n^2)(4n \mp m),$$
$$y = (m^2 + 19n^2)(4m \pm 19n),$$
$$z = (m^2 + 19n^2)$$

Example: 4

$$D = 19, \alpha = 35, y_0 = 4, x_0 = z_0 = 1$$
$$x = 4n(3m^2 - 19n^2) \mp m(m^2 - 57n^2),$$
$$y = 4m(m^2 - 57n^2) \pm 19n(3m^2 - 19n^2),$$
$$z = (m^2 + 19n^2)$$

It is worth to mention that, various integer solutions to the given equation (1) are obtained by taking different values of initial particular solutions (x_0, y_0, z_0) to (1) for given values of D, α.

Example: 5

$$D = 10, \alpha = 11, y_0 = 11^2 * 29, x_0 = 11^2 * 7, z_0 = 11^2$$
$$x = 11^2(m^2 + 10n^2)(29n \mp 7m),$$
$$y = 11^2(m^2 + 10n^2)(29m \pm 70n),$$
$$z = 11^2(m^2 + 10n^2)$$

Example: 6

$$D=10, \alpha=11, y_0=11^2*29, x_0=11^2*7, z_0=11^2$$
$$x=11^2*29n(3m^2-10n^2)\mp 11^2*7m(m^2-30n^2),$$
$$y=11^2*29m(m^2-30n^2)\pm 11^2*70n(3m^2-10n^2),$$
$$z=11^2(m^2+10n^2)$$

Conclusion:

In this paper, we have made an attempt to find a general form of non-zero distinct integer solutions to the non-homogeneous cubic equation with three unknowns given by $y^2+Dx^2=\alpha z^3$. To conclude, one may search for other choices of general form of integer solutions to the cubic equation with three unknowns in title.

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