



Scheduling a Stochastic Flow Shop using Modified Metaheuristics

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ABSTRACT –

The present work deals with scheduling of jobs in a stochastic flow shop environment with exponentially distributed processing time. The objective of the study is to determine an ordering of the jobs on the machines or the sequence that minimizes the makespan. Minimizing makespan enables firms to reduce jobs in the queue and hence reduce the in-process inventory. Since scheduling problems belong to the category of NP-hard problems, the metaheuristic method of genetic algorithm is adopted to solve the problem. A modified version of genetic algorithm is proposed by combining genetic algorithm with variable neighborhood search to solve the stochastic flow shop scheduling problem. A new neighborhood structure for variable neighborhood search is developed by combining swap and reversion. It is further hybridized by changing the sequence having maximum makespan with a better sequence before the algorithm enters variable neighborhood search. The best set of parameters for the proposed metaheuristics is determined using Taguchi's robust design. The proposed metaheuristics are applied on problem sizes of 20 job \times 5 machine, 20 job \times 10 machine, 20 job \times 20 machine and 50 job \times 50 machine. The makespan values obtained from both the metaheuristics for all the problem sizes indicate the superior performance of the modified genetic algorithm. The relative performance improvement (RPI) values are determined and it shows an improvement of 9.13%, 6.87%, 7.99% and 1.90% for the 20 job \times 5 machine, 20 job \times 10 machine, 20 job \times 20 machine and 50 job \times 50 machine respectively. The positive RPI values indicate the superior performance of the modified metaheuristics.

Keywords: Scheduling, stochastic flow shop, genetic algorithm, variable neighborhood search, makespan

1. INTRODUCTION

Scheduling is the allocation of limited resources such as men, money, materials and labour to perform a set of tasks with the view of optimizing one or more objectives. Scheduling occurs in both the service sector and in the manufacturing sector. Scheduling activities in service sectors include allocating staffs in hospitals, scheduling of aircraft and crew, drivers and duty staff in buses and railways etc. In manufacturing sector, scheduling enables the allocation of machinery and plant resources, arrange production process, purchase raw materials and to select human resources. Shop floor scheduling problem deals with the determination of a job schedule which minimizes an objective or a set of objectives. In manufacturing sector, a schedule is to be developed in such a way that it reduces the idle time thereby increasing the efficiency of manufacturing activities and better control of operations. The works that discuss sequencing and scheduling include Lawler et.al (1993), Pinedo et.al (1995), Blazewicz et.al (1996), Chung-Yee Lee (1997), Anderson et.al (1997), Allahverdi et.al (2004), Behnamian et.al (2009), Ying (2014), Rahman et.al (2015), Peng et.al (2018), Framinan et.al (2019), Venkateshan et.al (2019).

Shop floors are subjected to random events which may disturb their working process like machine break down, operator unavailability, stock shortages etc. In the deterministic model, all these random events are neglected to reduce the complexity of the problem. In the stochastic context, the objective is to determine a solution that optimizes the criterion considering one or more of these random events that occur in an industrial environment. The flow shop scheduling problem have attracted a lot of attention however, progress with the stochastic version of the flow shop problem have been very limited. The present work deals with the scheduling of jobs in a stochastic flow shop with the objective of minimizing makespan.

Scheduling problems are NP hard problems and the randomness in the shop floor activities further increases the complexity of the problem. Hence, metaheuristics are adopted for solving such scheduling problems. Some of the works on stochastic flow shop scheduling include Pinedo (1982), Gourgand et.al (2003), Kalczyński et.al (2006), Laha et.al (2007), Baker (2010), Framinan et.al (2015), Tyagi et.al (2017). The review of the literature unveils that the works that discuss the hybridization of metaheuristic in stochastic flow shop environment is scarce. Besides, no other works is reported with the hybridization of genetic algorithm with variable neighborhood search algorithm for minimizing makespan in the stochastic flow shop.

In the present work, the evolutionary method of genetic algorithm (GA) is hybridized with the variable neighborhood search (VNS) algorithm to minimise the makespan of the stochastic flow shop scheduling problem. Unlike other algorithms and optimization techniques, genetic algorithm promises convergence but not optimality. GA alone is not good at identifying the optimal solution but works very well in finding the region where the optimal solution lies hence we hybridize GA with local search algorithms like VNS to get near optimal solutions. The effectiveness of Hybrid GA when compared to regular GA has already been proved in various works.

The objectives of the present study are as follows.

- Development of a hybrid metaheuristic based on GA for solving the single objective stochastic flow shop scheduling problem.

- Determination of the best set of parameters of the proposed metaheuristics.
- Experimentation of the metaheuristic using exponentially distributed processing time with mean value equal to 40.
- Comparison of the proposed metaheuristic using RPI analysis.

The rest of the paper is organized as follows. The next section provides the problem statements and the assumptions of the study. Section 3 discuss a comprehensive literature review of many existing heuristics and metaheuristics for solving flow shop and stochastic flow shop problems with single and multiple objectives. Section 4 and 5 describes the methodology adopted and the experimentation details respectively. Section 6 provides the results and discussion. Section 7 presents the conclusion.

2. LITERATURE REVIEW

Scheduling covers a wide range of problems and include all industrial systems. Most of the early works in scheduling are concerned with the analysis of single machine. Later Johnson's algorithm (1954) is introduced to solve the two-machine flow shop scheduling problem. Now more complex systems like m machine regular flow shop, job shop, open shop, flexible flow shop etc. are being studied intensively. The complexity of scheduling problem renders exact solution methods that are impractical for instances of more than a few jobs and m machines. The important works on regular flow shop include Sethi (1984), Rajendran et.al (1990), Cheng et.al (2007), Gao et. al (2008), Framinan et.al (2015), Gao et.al (2018), Framinan et.al (2019) etc.

In a stochastic flow shop, the processing times are not deterministic in nature i.e. the system can be subjected to random events such as machine breakdowns, failures, unavailability of raw materials etc. The most common stochastic version of the flow shop problem assumes that the processing times are allowed to be random variables. There are very few works that considers stochastic flow shop scheduling. Some of them include, Niu et.al (1985) worked on Johnson's two machine flow shop with the objective of minimising makespan using random processing times. Their study proved that makespan becomes stochastically smaller when two adjacent jobs in a given job sequence were interchanged. Allahverdi (1996) studied two machine flow shop problem subjected to random breakdown and introduced five rules namely Talwars rule, Jhonsons rule, Diagonal rule, Shortest processing time rule(SPT) and Longest processing time rule(LPT) to find out the makespan. It is found that different rules perform well in various stochastic conditions. Ronconi et.al (2015) dealt with stochastic single-machine problem to find out the job sequence and the due dates which minimize the expected total earliness and tardiness costs. Chandramouli et.al (2017) combined the probabilistic processing time with weight of the jobs and transportation times in stochastic scheduling of two machines to find out an optimal sequence that minimises the expected makespan. The above mentioned works deal with single machine or two-machine stochastic flow shop scheduling problem.

Dutta et.al (1973) considered the exponentially distributed processing times to schedule jobs in a stochastic environment such that it minimises the expected makespan. Pinedo (1982) considered the minimization of makespan in a stochastic flow shop assuming that the processing times of a job are independent and identically distributed random variable. Weber (1982) identified the scenario in stochastic flow shop, i.e. in order to reduce makespan, a number of identical machines should be run in parallel for the processing of similar jobs and it was found that reduction in flow time is possible when the processing times were distributed exponentially. Foley et.al (1984) focused on scheduling and minimising overall makespan of stochastic flow shop with m machines assuming that there are no intermediate storages.

Pinedo et.al (1995) worked on minimizing the makespan in flow shops subjected to breakdowns. Allahverdi (1999) focussed on stochastically minimizing total flow time in flow shops with no waiting space. Gourgand et.al (2000) studied the various scenarios of stochastic flow shop scheduling problem with randomly distributed processing times and stochastic systems which considers machine breakdowns. Gourgand et.al (2003) proposed a recursive algorithm based on a Markov chain to compute the expected makespan and a discrete event simulation model to evaluate the expected makespan. Draper et.al (2007) studied leading stochastic optimization methods such as simulated annealing, genetic algorithms and tabu search and suggested the solution using Bayesian decision theory.

Laha et.al (2007) uses simulated annealing in permutation flow shop scheduling problem to minimize makespan. Mora (2008) discussed various models for scheduling in stochastic flow shops. Baker (2010) suggested three constructive heuristics procedures for stochastic flow shop scheduling with modest computational requirements. Framinan et.al (2015) proposed a procedure to find out the percentage error in the computation of expected makespan. Framinan et.al (2018) developed a simheuristic algorithm to set up starting times in the stochastic parallel flow shop problem by providing a deadline on starting with user defined probability to reduce makespan in deterministic as well as in stochastic version.

3. PROBLEM STATEMENT

- The work deals with the scheduling of jobs in a production shop floor having stochastic flow shop configuration.
- The study aims to determine a sequence that minimizes the makespan.

$$Makespan = \sum_{j=1}^n C_j \quad \text{----- (1)}$$

where n and C_j denote the number of jobs and completion time of job j respectively.

The assumptions for the present study are as follows.

- The processing times of operations of jobs are assumed as exponentially distributed with mean value equal to 40.
- Each machine can process only one job at a time.
- No pre-emption is allowed.
- The machines are continuously available. i.e. no breakdown of machines.

4. METHODOLOGY

The metaheuristic method of genetic algorithm is adopted to solve the single objective stochastic flow shop scheduling problem. In the present study, two hybrid versions of genetic algorithm are proposed to solve the stochastic flow shop scheduling problem.

Genetic algorithm (John Holland, 1960) is an adaptive heuristic search algorithm which is formulated from Charles Darwin's theory of natural evolution. GA simulates the process of natural selection which promotes the survival of the fittest among individuals of consecutive generations for solving a problem. The population required is generated randomly where each individual in the population represents a solution in the search space of the given problem. The selection of fittest population is performed using Roulette wheel selection method. A fitness number is provided to each individual and the individuals with better fitness number are selected for crossover using the selection probability. Crossover is performed on the selected individuals from the population with a specific crossover probability. The selected offspring's obtained from crossover is subjected to mutation based on mutation probability. The algorithm is repeated until the specified termination criteria.

4.1 Hybrid genetic algorithm

In order to improve the performance of genetic algorithm, the genetic algorithm is hybridized with variable neighborhood search algorithm. The offsprings obtained from GA are subjected to VNS. VNS (Mladenovic and Hansen, 1997) is a local search algorithm which uses various neighborhood operators. In the present study, operators namely swap, reversion and a new operator which is a combination of swap and reversion is adopted and is shown in Figures 1, 2 and 3 respectively.

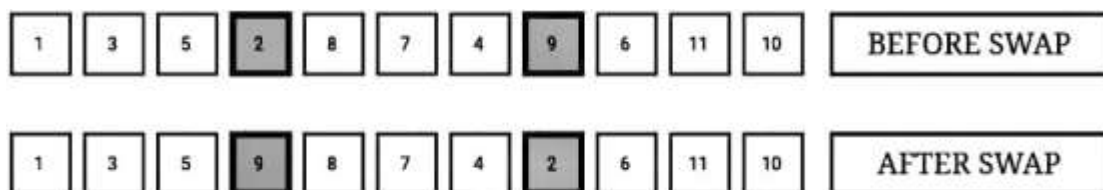


Fig 1. Swap

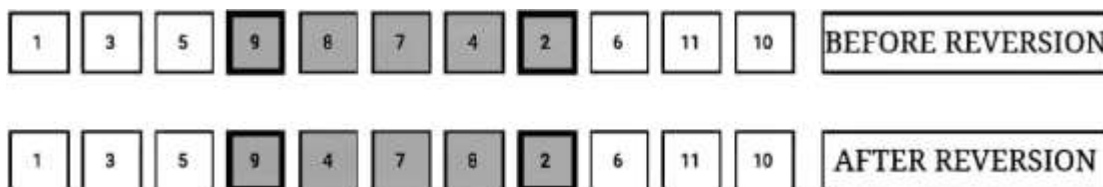


Fig 2. Reversion

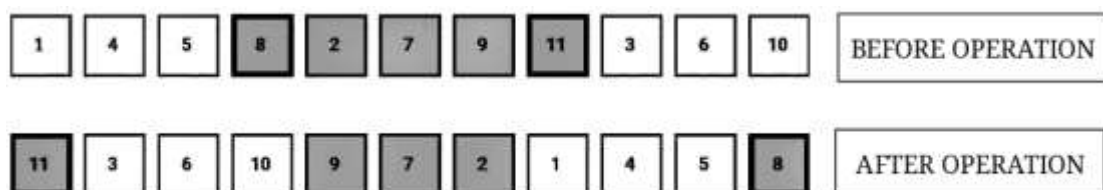


Fig 3. Combination of swap and reversion

The hybridized versions of genetic algorithm are discussed in this section. In the first hybrid algorithm (HGA1), as the sequence with best makespan enters the VNS algorithm it activates the first operator which in our case is swap. If the makespan value obtained by swapping 2 elements at two randomly selected points is lesser than the makespan value obtained after GA then the operator works again until this condition fails. When the condition fails the next operator is activated, here we have used reversion. In the reversion operator the elements between two randomly chosen points are reversed. The test condition follows the same operation in all the cases. Once the operator fails the next operator is activated. The third neighborhood structure is developed by combining swap and reversion. The reversion operation happens as such and the other two ends are swapped in each others place. When the test condition fails, the algorithm ends and the convergence curve is plotted.

In the second version of hybrid genetic algorithm (HGA2), the offsprings obtained from genetic algorithm is improved by replacing the sequence having maximum makespan with randomly generated sequence with a lower makespan value. This is followed by a variable neighborhood search similar to the first version. The flow chart depicting the second hybrid version is shown in Figure 4.

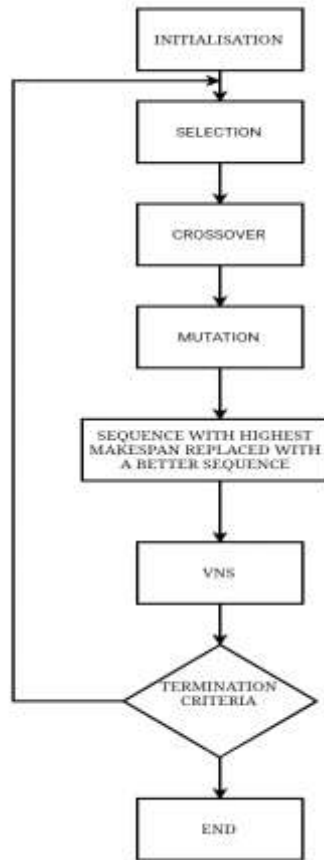


Fig 4. Hybrid genetic algorithm 2

4.2 Parameter Configuration

The parameter configuration for the proposed algorithms is performed using Taguchi’s robust design. The parameters of the algorithms include crossover type (single point and two point), crossover probability (0.7, 0.8, 0.9), mutation probability (0.1, 0.2, 0.3) and number of generations (100, 150, 200). Considering the number of parameters and their values L18 orthogonal array is selected to perform the parameter configuration.

Table 1. Parameter and the different levels

Parameters	Cross over type	Cross over probability	Mutation probability	Number of generations
Values	Single point cross over (A1)	0.7 (B1)	0.1 (C1)	100 (D1)
	Two point cross over (A2)	0.8 (B2)	0.2 (C2)	150 (D2)
		0.9 (B3)	0.3 (C3)	200 (D3)

The best parameters are selected by calculating the S/N (signal to noise) ratio using Equation 2.

$$S/N = -10 \times \log (\Sigma (Y^2) / n) \dots\dots\dots (2)$$

The parameter value with lower S/N ratio is selected as the best value. The near optimal set of parameters of each algorithm for each problem size is depicted in Figures 5,6,7 and 8 respectively. The values are also provided in Tables 2 and 3 respectively.

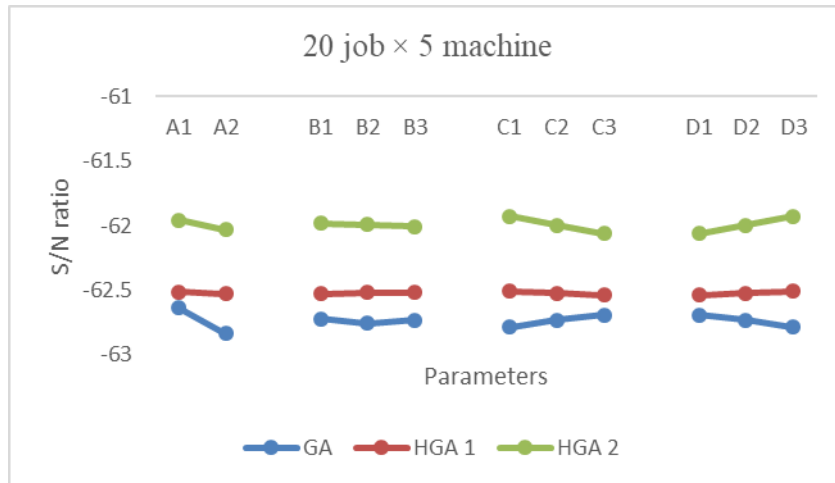


Fig. 5 Best set of parameters for the three algorithms of problem size 20 job × 5 machine

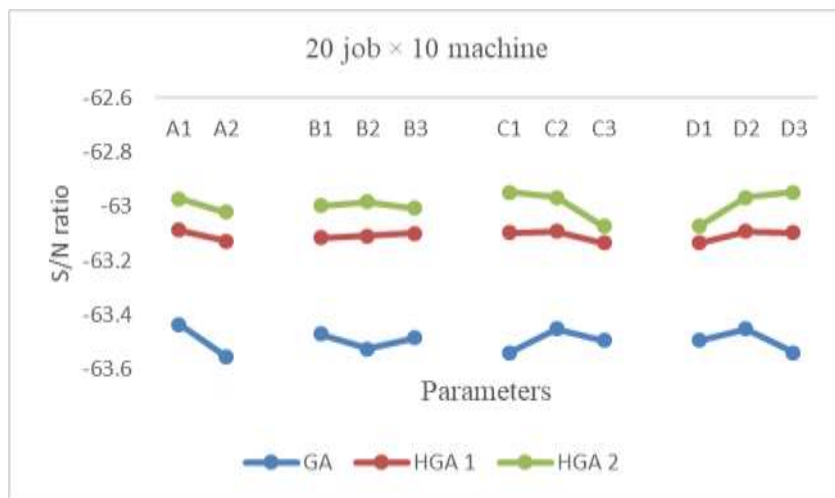


Fig. 6 Best set of parameters for the three algorithms of problem size 20 job × 10 machine



Fig. 7 Best set of parameters for the three algorithms of problem size 20 job × 20 machine

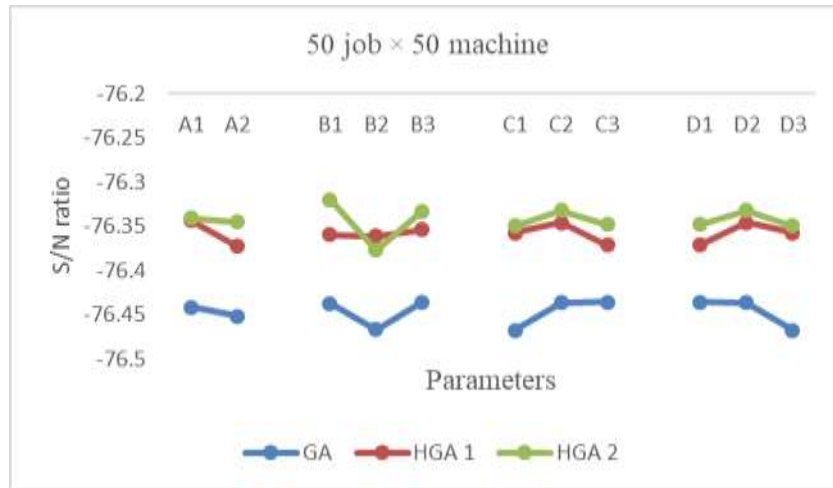


Fig. 8 Best set of parameters for the three algorithms of problem size 50 job x 5 machine

Table 2 Best set of parameters for the three algorithms for the problem size 20 job x 5 machine and 20 job x 10 machine

Parameters	20 job x 5 machine			20 job x 10 machine		
	GA	HGA 1	HGA 2	GA	HGA 1	HGA 2
Cross over type	Two point crossover	Two point crossover	Two point crossover	Two point crossover	Single point crossover	Two point crossover
Cross over probability	0.8	0.7	0.9	0.8	0.7	0.9
Mutation probability	0.1	0.3	0.3	0.1	0.3	0.3
Number of generations	200	100	100	200	100	100

Table 3 Best set of parameters for the three algorithms for the problem size 20 job x 10 machine and 50 job x 50 machine

Parameters	20 job x 20 machine			50 job x 50 machine		
	GA	HGA 1	HGA 2	GA	HGA 1	HGA 2
Cross over type	Two point crossover	Two point crossover	Two point crossover	Two point crossover	Two point crossover	Two point crossover
Cross over probability	0.7	0.8	0.8	0.8	0.8	0.8
Mutation probability	0.1	0.3	0.3	0.1	0.3	0.1
Number of generations	200	100	100	200	100	200

4. EXPERIMENTATION DETAILS

The processing time for the study is assumed to be exponentially distributed with mean value equal to 40 (Pinedo, 1995). In the present study, we have considered four problem instances 20 job x 5 machine, 20 job x 10 machine, 20 job x 20 machine and 50 job x 50 machine. The genetic algorithm and the hybrid versions with best set of parameters are applied to solve the stochastic flow shop scheduling problem. Each algorithm is iterated 20 times and the mean makespan value is determined for each problem size.

5. Performance Analysis of the algorithms

The objective is to determine the sequence of jobs with minimum makespan for a stochastic flow shop. Two version of hybrid algorithm are developed and the makespan values are determined. The results obtained from hybrid versions are compared with the simple genetic algorithm for all the problem instances and the convergence curves are plotted. The performance of the algorithms is compared using relative percentage improvement (RPI). RPI is determined from the relation provided below.

$$RPI = \frac{(GA_{makespan} - Hybrid_GA_{makespan})}{GA_{makespan}} \times 100 \dots \dots \dots (3)$$

The mean makespan values obtained from each algorithm for all the problem instances of the stochastic flow shop is provided in Table 8.

Table 8. Makespan values obtained from GA and the modified GA's

Makespan with best parameter values												
Sl.no	20 job × 5 machine			20 job × 10 machine			20 job × 20 machine			50 job × 50 machine		
	GA	HGA1	HGA 2	GA	HGA1	HGA 2	GA	HGA1	HGA 2	GA	HGA1	HGA 2
1	1390	1330	1296	1533	1433	1424	2449	2345	2304	6879	6742	6736
2	1460	1330	1255	1522	1433	1402	2606	2352	2208	6842	6704	6719
3	1361	1327	1279	1528	1445	1383	2299	2297	2249	6830	6685	6689
4	1385	1330	1277	1492	1432	1400	2429	2243	2266	6817	6683	6674
5	1361	1349	1277	1541	1477	1411	2481	2258	2315	6804	6682	6657
6	1397	1330	1280	1578	1458	1427	2423	2296	2275	6795	6612	6645
7	1476	1329	1272	1559	1459	1439	2446	2286	2231	6735	6611	6639
8	1385	1344	1289	1511	1442	1330	2510	2331	2286	6697	6610	6637
9	1355	1339	1276	1519	1448	1422	2469	2381	2264	6643	6599	6582
10	1427	1330	1252	1557	1464	1445	2439	2317	2264	6639	6581	6547
11	1355	1337	1252	1492	1428	1442	2491	2336	2332	6627	6573	6508
12	1422	1330	1272	1555	1442	1394	2575	2274	2322	6611	6531	6501
13	1489	1351	1262	1486	1449	1440	2470	2327	2302	6608	6528	6460
14	1386	1329	1297	1529	1370	1470	2428	2254	2236	6576	6527	6456
15	1415	1355	1260	1519	1421	1414	2497	2320	2285	6559	6504	6420
16	1355	1343	1250	1554	1455	1464	2421	2352	2273	6548	6502	6388
17	1386	1330	1246	1471	1440	1423	2443	2296	2170	6548	6454	6382
18	1475	1334	1279	1514	1448	1400	2557	2288	2282	6534	6412	6359
19	1418	1350	1314	1541	1458	1433	2458	2328	2270	6457	6396	6335
20	1361	1345	1313	1511	1463	1454	2523	2360	2329	6446	6394	6324
Mean	1403	1337	1274.9	1526	1443	1420.9	2471	2312	2273.2	6660	6567	6532.9

From Table 8, it is observed that for all the problem instances the hybrid versions of genetic algorithm performs superior to the simple genetic algorithm. Further, the comparison of the modified versions reveals the better performance of the second modified version of GA (HGA 2). The convergence of makespan values for each algorithm for the problem sizes of 20 job × 5 machine, 20 job × 10 machine and 20 job × 20 machine is plotted and is shown in Figures 5, 6, 7 and 8 respectively.

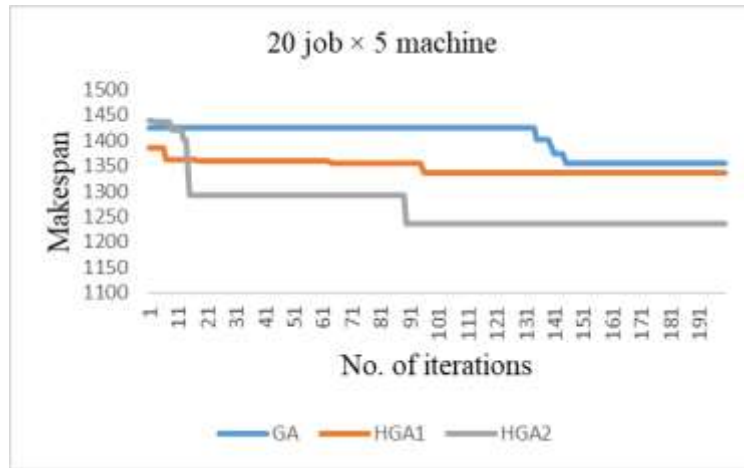


Fig. 5 Convergence plot for 20 job x 5 machine stochastic flow shop

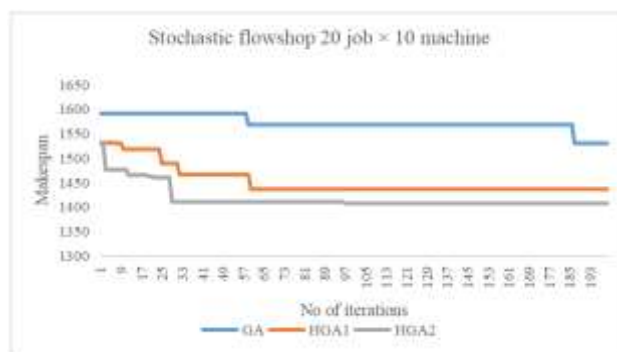


Fig. 6 Convergence plot for 20 job x 10 machine stochastic flow shop

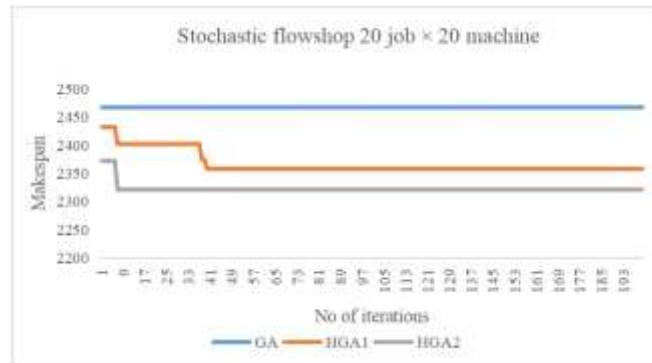


Fig. 7 Convergence plot for 20 job x 20 machine stochastic flow shop

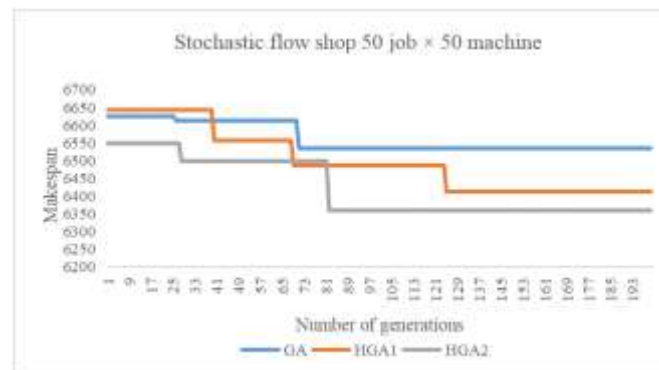


Fig. 8 Convergence plot for 50 job x 50 machine stochastic flow shop

The algorithms are repeated 20 times and the makespan values are determined for each trial. Graphs are plotted with the number of trials and makespan values for genetic algorithm and its hybrid versions. The graphs for 20 job \times 5 machine, 20 job \times 10 machine and 20 job \times 20 machine are shown in figures 9, 10, 11 and 12 respectively.

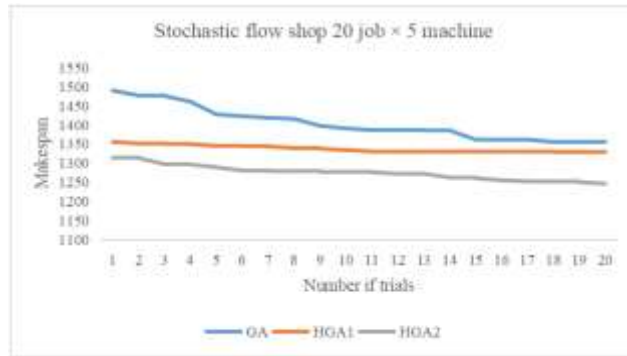


Fig. 9 No. of trials and makespan for 20 job \times 5 machine stochastic flow shop

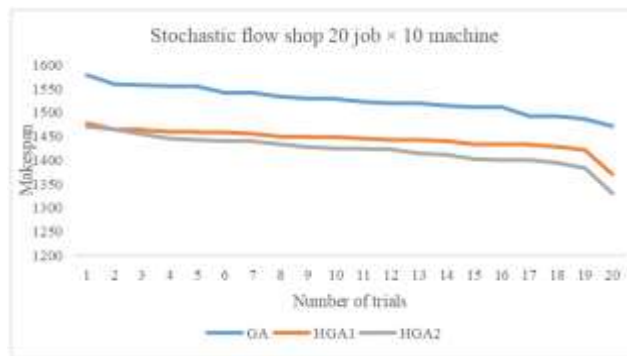


Fig. 10 No. of trials and makespan for 20 job \times 10 machine stochastic flow shop

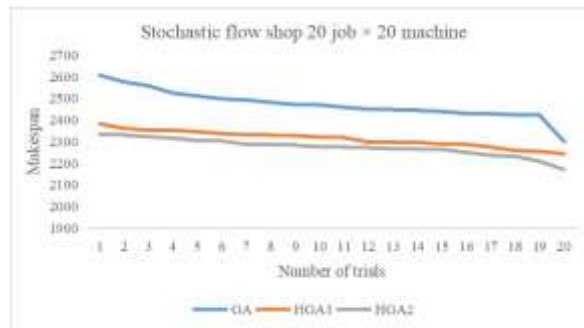


Fig. 11 No. of trials and makespan for 20 job \times 20 machine stochastic flow shop

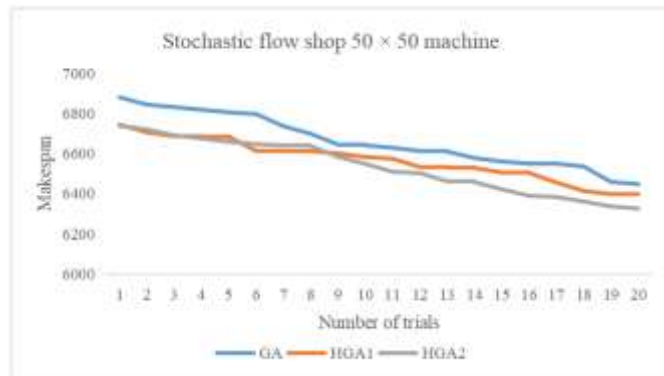


Fig. 12 No. of trials and makespan for 50 job \times 50 machine stochastic flow shop

The graphs indicate the superior performance of proposed hybrid versions of genetic algorithm.

5.2 Comparison of GA and hybrid versions using RPI values

The RPI values are determined from the mean makespan values of each problem size obtained from GA and the hybrid versions. The mean makespan and the RPI values determined for each problem size of stochastic flow shop are provided in Table 9.

Table 9. Mean makespan and RPI values for stochastic flow shop

Job × Machine	Mean makespan			RPI (%)		
	GA	HGA1	HGA2	HGA 1 with GA	HGA 2 with GA	HGA1 with HGA2
20×5	1403	1337.1	1275	4.69	9.13	4.65
20×10	1526	1443.3	1421	5.39	6.87	1.55
20×20	2471	2312.1	2273	6.42	7.99	1.68
50×50	6660	6566.5	6533	1.4	1.9	0.51

The highest RPI value observed in the stochastic configuration is 9.13 % when comparing genetic algorithm with HGA2 for the 20 job × 5 machine problem and the least value is 1.4 % when comparing genetic algorithm with HGA1 for 50 job × 50 machine problem. Comparing the Genetic algorithm with HGA1, an improvement of 1.4 - 6.42 % can be observed whereas comparison of genetic algorithm with HGA2 provides an improvement of 1.92 - 9.12 %. Further, comparing HGA1 and HGA2 a percentage improvement of 0.51 - 4.65 % can be observed in the makespan values.

6. CONCLUSION

We consider the problem of scheduling jobs in a stochastic flow shop environment. The objective of our work is to obtain a sequence of jobs which provides an optimum value for makespan. Since scheduling problems belong to the category of NP hard problems, metaheuristic method of genetic algorithm is adopted to solve the problem. We have developed two hybrid versions of genetic algorithm by combining genetic algorithm with VNS. The work deals with scheduling of jobs in stochastic flow shop. Since stochastic flow shop problems does not have a benchmark problem we have considered the processing time exponentially distributed with mean value equal to 40. We have conducted our study in problem sizes 20 job × 5 machine, 20 job × 10 machine, 20 job × 20 machine and 50 job × 50 machine.

The performance of the modified versions is compared with genetic algorithm and the analysis of the results reveals the superior performance of the hybrid versions of the genetic algorithm. When the hybrid versions are compared, the HGA2 perform better than HGA1. The positive RPI values also indicate the same. HGA1 has an improvement of 4.69%, 5.39%, 6.42% and 1.40% than simple genetic algorithm. For the same flow shop configuration, HGA2 provides an improvement of 9.13%, 6.87%, 7.99% and 1.90% in the makespan values of 20 job × 5 machine, 20 job × 10 machine, 20 job × 20 machine and 50 job × 50 machine respectively.

The study can be further extended by considering objective other than makespan. The present work focuses on only one objective. However, the real-life situations involve the optimisation of more than one objective. Other types of crossover and mutation operators can also be experimented. Methods other than VNS can be used for the hybridization of metaheuristic.

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