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On Finding General Form of Integral Solutions to the Quinary Homogeneous Bi-Quadratic Equation

$$(x+y)(x^3+y^3)=\alpha(z^2-w^2)P^2$$

S. Vidhyalakshmi[1], M. A. Gopalan[2]

[1]Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.

[2]Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.

ABSTRACT:

The purpose of this paper is to obtain a general form of non-zero distinct integral solutions of quinary bi-quadratic homogeneous diophantine equation $(x+y)(x^3+y^3)=\alpha(z^2-w^2)P^2$ where α is a given non-zero integer.

Keywords: Quinary bi-quadratic equation, Homogeneous bi-quadratic, Integer solutions

Introduction:

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, bi-quadratic diophantine equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity[1-5]. In this context, one may refer [6-21] for various problems on the bi-quadratic diophantine equations with five variables ,where ,in each of the problems , different sets of non-zero integer solutions are obtained. However, often we come across homogeneous bi-quadratic equations and as such one may require its integral solution in its most general form. It is towards this end, this paper concerns with the problem of determining a general form of non-trivial integral solutions of the homogeneous bi-quadratic equation with five unknowns given by $(x+y)(x^3+y^3)=\alpha(z^2-w^2)P^2$ where α is a given non-zero square-free integer.

Method of Analysis:

The quinary homogeneous bi-quadratic diophantine equation to be solved for its distinct non-zero integral solution is

$$(x+y)(x^3+y^3)=\alpha(z^2-w^2)P^2 \tag{1}$$

Considering the change of variables in (1) given by

$$x=u+v,\ y=u-v,\ z=u^2+1,\ w=u^2-1 \tag{2}$$

we have

$$u^2+3v^2=\alpha P^2 \tag{3}$$

Let u_0, v_0, p_0 be any given integer solution to (3) so that

$$u_0^2+3v_0^2=\alpha P_0^2 \tag{4}$$

Now, consider

$$u=Au_0+3Bv_0,\ v=Bu_0-Av_0 \tag{5}$$

where A, B are non-zero integers to be determined such that (5) satisfies (3).

Substituting (5) in (3), we have

$$\text{L.H.S. of (3)}=(u_0^2+3v_0^2)(A^2+3B^2) \tag{6}$$

In view of (4), it is seen that

$$\text{L.H.S. of (3)}=\alpha p_0^2(A^2+3B^2) \tag{7}$$

On comparing the R.H.S. of (3) and (7), note that we have to choose A and B

so that A^2+3B^2 is a perfect square. Choosing

$$B=2rs, A=3r^2-s^2 \tag{8}$$

it is seen that

$$A^2+3B^2=(3r^2+s^2)^2$$

From (8), (5) and (3), one obtains

$$u=(3r^2-s^2)u_0+6rsv_0, v=2rsu_0-(3r^2-s^2)v_0 \tag{9}$$

$$p=(3r^2+s^2)p_0 \tag{10}$$

In view of (2), we get

$$\left.\begin{aligned} &x=(3r^2-s^2)(u_0-v_0)+6rsv_0+2rsu_0,\\ &y=(3r^2-s^2)(u_0+v_0)+6rsv_0-2rsu_0,\\ &z=((3r^2-s^2)u_0+6rsv_0)^2+1,\\ &w=((3r^2-s^2)u_0+6rsv_0)^2-1 \end{aligned}\right) \tag{11}$$

Thus, (10) and (11) represent the general form of integral solutions to (1).

In otherwords, various integer solutions to the given equation (1) are obtained by taking different values of initial particular solutions (u_0, v_0, p_0) to (3) for given value of α.

A few examples are presented below:

Example 1:

$$\left.\begin{aligned} &\alpha=7, u_0=2, v_0=1, p_0=1\\ &x=3r^2-s^2+10rs,\\ &y=9r^2-3s^2+2rs,\\ &z=(6r^2-2s^2+6rs)^2+1,\\ &w=(6r^2-2s^2+6rs)^2-1,\\ &p=3r^2+s^2 \end{aligned}\right)$$

Example 2:

$$\left.\begin{aligned} &\alpha=7, u_0=5, v_0=1, p_0=2\\ &x=12r^2-4s^2+16rs,\\ &y=18r^2-6s^2-4rs,\\ &z=(15r^2-5s^2+6rs)^2+1,\\ &w=(15r^2-5s^2+6rs)^2-1,\\ &p=6r^2+2s^2 \end{aligned}\right)$$

Example 3:

$$\left.\begin{aligned} &\alpha=13, u_0=1, v_0=-2, p_0=1\\ &x=9r^2-3s^2-10rs,\\ &y=-3r^2+s^2-14rs,\\ &z=(3r^2-s^2-12rs)^2+1,\\ &w=(3r^2-s^2-12rs)^2-1,\\ &p=3r^2+s^2 \end{aligned}\right)$$

It is worth to observe that there are other choices of general form of integer solutions to (1).

An example is illustrated below:

Let

$$u_1=h-2u_0, v_1=h-2v_0, p_1=2p_0 \tag{12}$$

be the second solution to (1), where h is a non-zero constant to be determined. Substituting (12) in (3) and simplifying, one gets

$$h = u_0 + 3v_0$$

and thus

$$u_1 = -u_0 + 3v_0, v_1 = u_0 + v_0$$

which is written in the matrix form as

$$(u_1, v_1)^t = M(u_0, v_0)^t$$

where $M = \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix}$ and t is the transpose.

Repeating the above process, the general solution (u_n, v_n, p_n) to (3) is given by

$$\left.\begin{aligned} u_n &= 2^{n-2}[(1+3(-1)^n)u_0 + 3(1-(-1)^n)v_0], \\ v_n &= 2^{n-2}[(1-(-1)^n)u_0 + (3+(-1)^n)v_0] \end{aligned}\right\} \quad (13)$$

$$p_n = 2^n p_0 \quad (14)$$

Substituting (13) in (2), we have

$$\left.\begin{aligned} x_n &= 2^{n-2}[(2+2(-1)^n)u_0 + (6-2(-1)^n)v_0], \\ y_n &= 2^{n-2}[4(-1)^n u_0 - 4(-1)^n)v_0], \\ z_n &= \{2^{n-2}[(1+3(-1)^n)u_0 + 3(1-(-1)^n)v_0]\}^2 + 1, \\ w_n &= \{2^{n-2}[(1+3(-1)^n)u_0 + 3(1-(-1)^n)v_0]\}^2 - 1 \end{aligned}\right\} \quad n = 1,2,3,... \quad (15)$$

Thus, (14) and (15) represent the general form of integer solutions to (1).

Conclusion:

In this paper, we have made an attempt to find a general form of non-zero distinct integer solutions to the homogeneous bi-quadratic equation with five unknowns given by $(x+y)(x^3+y^3) = \alpha(z^2-w^2)P^2$. To conclude, one may search for other choices of general form of integer solutions to the bi-quadratic equation with five unknowns in title.

References:

[1]. Dickson L.E., History of Theory of Numbers, Vol.11, Chelsea publishing company, New York (1952).

[2]. Mordell L.J., Diophantine equations, Academic press, London (1969).

[3]. Carmichael R.D., The theory of numbers and Diophantine Analysis, Dover publications, New York (1959).

[4]. Telang S.G., Number theory, Tata Mc Graw Hill publishing company, New Delhi (1996).

[5]. Nigel, D. Smart, The Algorithmic Resolutions of Diophantine Equations, Cambridge University press, London (1999).

[6]. Gopalan. M. A, Vidhyalakshmi. S, Kavitha. A, Premalatha. E, On the Homogeneous Bi-quadratic equation with Five Unknowns $x^4 - y^4 = 5(z^2 - w^2)R^2$,International Archieve of Applied Science and Technology ,Vol.4,Issue.3,Sep.2013,Pg..37-44.

[7]. Gopalan. M. A, Vidhyalakshmi. S, Premalatha. E., On the Homogeneous Bi-Quadratic Equation with Five Unknowns $x^4 - y^4 = 8(z+w)p^3$, International journal of Scientific and Research Publications.,Vol.4,Issue.1,Jan-2014 Pg.1-5.

[8]. Vidhyalakshmi S., Lakshmi K., Gopalan M.A., Observation on the bi-quadratic equation with five unknowns $2(x^3+y^3) = (k^2+3s^2)(z^2-w^2)P^2$, International Journal of Innovative Research and Review, 2(2), 12-19, June 2014.

[9]. Vidhyalakshmi S., Lakshmi K., Gopalan M.A., Integral solution of the bi-quadratic equation with five unknowns $(x+y)^2 + xy + (z+w)^2 - zw = (5a^2+3b^2)\,p^4$, IJIRT, Volume-1, Issue-10, 2014, 47-52.

[10]. Gopalan. M. A, Vidhyalakshmi. S, Maheswari. D., Observations on the Biquadratic Equation with Five unknowns $(x-y)(x^3+y^3) = 39(w^2-z^2)P^2$,International Journal Of Multidisciplinary Research and Development.Vol.2,Issue.5.May 2015,pg.83-86.

[11]. Geetha.V, Gopalan.M.A,On the Non-homogeneous Biquadratic Diophantine equation with Five Unknowns $x^3 - y^3 = z^3 - w^3 + 12t^4$,International Journal of current Research Vol.7,Issue.8,August 2015, Pg.19519-19522.

[12]. Gopalan M.A., Kavitha A., Kiruthika R., Observations on the bi-quadratic equation with five unknowns $2(x-y)(x^3+y^3) = (1+3k^2)(X^2-Y^2)w^2$, IRJET, Volume-02, Issue-03, June 2015, 1562-1573.

[13]. Gopalan M.A., Thiruniraiselvi N., Menaka P., On the homogeneous bi-quadratic equation with five unknowns $x^4 - y^4 = 40(z+w)p^3$

, International Journal of Current Research, Vol-7(07), July 2015, 18435-18438.

[14]. Gopalan M.A., Thiruniraiselvi N., Sridevi R., Observations on the bi-quadratic equation with five unknowns $2\left(x^3+y^3\right)=19\left(z^2-w^2\right)p^2$, Transactions on Mathematics, Vol.2(1), January 2016, 01-18.

[15]. Gopalan M.A., Vidhyalakshmi S., Sumathi G., "Special higher degree diophantine problems with solutions", LAP LAMBERT Academic Publishing, Omni Scriptum, Germany, 2017.

[16]. Janaki.G,Saranya.C,Integral Solutions of the Homogeneous Biquadratic Diophantine equation $3(x^4-y^4)-2xy(x^2-y^2)=972(z+w)p^3$,Vol.5,Issue VIII, August 2017, Pg.1123-1127.

[17]. Anbuselvi R., Ahila N., On the Homogeneous Bi-quadratic Equation with Five Unknowns $x^4-y^4=37\left(z^2-w^2\right)T^2$, IJSR, Volume 6, Issue.9, September 2017, 201-205.

[18]. Umamaheswari R., Kavitha A., Integer solution of the Homogeneous Bi-Quadratic Diophantine Equation with Five Unknowns $(x-y)\left(x^3-y^3\right)=\left(z^2-w^2\right)p^2$, Journal of Mathematics and Informatics, Volume 11, 2017, 55-61.

[19]. Anbuselvi R., Ahila N., Observation on the Bi-quadratic Equation with Five Unknowns $(x-y)\left(x^3+y^3\right)=\left(1+12k^2\right)\left(X^2-Y^2\right)w^2$, International Journal of Research in Advent Technology, Volume 6, No.9, September 2018, 5204-5212.

[20]. Sharadha Kumar, Gopalan M.A., The Homogeneous Bi-quadratic Equations with Five Unknowns $x^4-y^4+2\left(x^2-y^2\right)\left(w^2+p^2\right)=4\left(x^3+y^3\right)z$, IJRASET, Volume 6, Issue X, October 2018, 700-703.

[21]. Vijayasankar.A,Gopalan.M.A,and Krithika.V,On the Biquadratic Diophantine Equation with Five Unknowns $2(x^4-y^4)=13(z^2-w^2)p^2$,International Journal of Mathematical Archive Vol.9,Issue.4,2018, Pg.26-31.