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# A Comprehensive Review on Several Techniques for Solving 2D and 3D Partial Differential Equations

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**ABSTRACT**

Many Physical phenomenon in Sciences and engineering are modeled in the form of Partial differential equations. Partial differential equations arise in formulation of many mathematical problems may be of one dimensional, two dimensional, three dimensional etc. In this paper, various types of Partial differential equations and methods available in literature for solving two dimensional and three dimensional Partial differential equations are discussed.

**Keywords:** Two dimensional Partial differntial equation; Three dimensional partial differential equation.

**2010 AMS Subject Classification:** 44A10, 35E15, 47J30.

## 1. Introduction

In mathematics, a partial differential equation (PDE) is an equation which establishes relations between the various partial derivatives of a multivariable function. Partial differential equations arise in many physical phenomenon occurred in physical sciences, chemical sciences, engineering etc in the form of wave equations, diffusion equations, Telegraph equations, Burgers equations etc. There are large number of mathematical methods, which can be used to solve the partial differentials equations. Some methods are used to find the numerical solution, some methods are used to find the exact solution and some methods are applied to find both numerical as well as analytical solutions.

As per the dimensions of the material or physical problems, Partial differential equations may be of one dimensional, two dimensional or three dimensional. Some methods are only applicable for one dimensional problems, while other can be applicable to solve two and three dimensional problems.

Many types of one dimensional partial differential equations and the methods to solve one dimensional partial differential equations have been discussed in the Literature. Our main focus in this paper is to present various types of two and three dimensional partial differential equations and several techniques to solve these two and three dimensional partial differential equations.

Partial differential equations have many applications in sciences and engineering. For examples partial differential equations are used to modeled the problems of waves, heat, general relativity, diffusion, electrostatics, fluid dynamics, electrodynamics, elasticity and quantum mechanics.

## 2. Types of Partial Differntial Equations

Partial Differential Equation can be classified into three types:

2.1 Linear Partial Differential Equation

2.2 Non Linear Partial Differential Equation

2.3 Coupled Partial Differential Equations

### *2.1 Linear Partial Differential Equation:*

A partial differential equation is said to be linear if the degree of unknown function i.e. dependent variable is one and the dependent variable or its derivatives should not multiplied in the equation, should be linear.

e.g. (i) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

(ii)$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial u}{\partial t}$

(iii)$\frac{\partial u}{\partial t}+\frac{\partial^2 u}{\partial x\partial y}+2\frac{\partial^2 u}{\partial y^2}=0$

***2.2 Non Linear Partial Differential Equation :***

The Partial Differential Equation which is not linear is said to be non linear.

e.g. (i) $u\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

(ii)$\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}=\frac{\partial u}{\partial t}$

(iii)$\left(\frac{\partial u}{\partial x}\right)^2+2\,u\frac{\partial^2 u}{\partial x^2}=0$

***2.3 Coupled Partial Differential Equations:***

When there are two or more dependent variables corresponding to two or more independent variables, such a system of differential equations is known as Simultaneous system of Partial Differential equations.

e.g.: (i) $\frac{\partial \mathrm{u}}{\partial t}+u\frac{\partial \mathrm{u}}{\partial x}+v\frac{\partial \mathrm{u}}{\partial y}=\frac{1}{R}\left(\frac{\partial^2 \mathrm{u}}{\partial \mathrm{x}^2}+\frac{\partial^2 \mathrm{u}}{\partial \mathrm{y}^2}\right)$

$$\frac{\partial \mathrm{v}}{\partial t}+v\frac{\partial \mathrm{v}}{\partial x}+u\frac{\partial \mathrm{v}}{\partial y}=\frac{1}{R}\left(\frac{\partial^2 \mathrm{v}}{\partial \mathrm{x}^2}+\frac{\partial^2 \mathrm{v}}{\partial \mathrm{y}^2}\right)$$

## 3. Some examples of Linear and Non Linear 2D & 3D Partial differential equations

***3.1 Schrodinger Wave Equation:***

The Schrödinger equation is a partial differential equation that describes the wave function of a quantum mechanical system.it was first formulated by Erwin Schrödinger in 1925. Schrödinger equation is an important result of the quantum mechanics. Schrödinger equation may be of one dimensional, two dimensional or three dimensional. As our main focus in this paper is to discuss the methods for two dimensional and three dimensional problems, so we are introducing only two dimensional and three dimensional partial differential equations here,

*2-Dimensional Linear Schrodinger Wave Equation*

$$-i\frac{\partial u(x,y,t)}{\partial t}=\nabla^2 u(x,y,t)+v(x,y)u(x,y,t)$$

*3-Dimensional Linear Schrodinger Wave Equation*

$$-i\frac{\partial u(x,y,z,t)}{\partial t}=\nabla^2 u(x,y,z,t)+v(x,y,z)u(x,y,z,t)$$

***3.2 Heat Equation:***

The Heat equation is an important Partial differential equations, which was first introduced by Joseph Fourier in 1822. The heat equations describes the flow of heat in a specific region or diffusion of heat through a given region. In heat equations , 'u' represents the temperature at a specific point. Heat equations may be of one dimensional, two dimensional and three dimensional according to the dimension of the regions through which heat flows. Two dimensional and three dimensional heat equations are given below.

*2-Dimensional Linear Diffusion Equation*

$$\frac{\partial u(x,y,t)}{\partial t}=c^2\nabla^2 u(x,y,t)$$

*3-Dimensional Linear Diffusion Equation*

$$\frac{\partial u(x,y,z,t)}{\partial t}=c^2\nabla^2 u(x,y,z,t)$$

***3.3 Wave Equation:***

The one dimensional wave equation was first discovered by d'Alembert IN 1746 and then later on the model of three dimensional wave equations was first introduced by Euler in 1756.The wave equation has many applications in sciences and engineering. It is used to discuss the wave function in case of water wave, sound wave, light wave etc in acoustics, fluid dynamics and electromagnetics. The models of two dimensional and three dimensional wave equations are given below.

*2-Dimensional wave equation*

$$\frac{\partial^2 u(x,y,t)}{\partial t^2} = c^2\nabla^2 u(x,y,t)$$

*3-Dimensional wave equation*

$$\frac{\partial^2 u(x,y,z,t)}{\partial t^2} = c^2\nabla^2 u(x,y,z,t$$

### *3.4 Telegraphic Equations:*

A partial differential equation usually written in the form

$$\frac{\partial^2 u(x,t)}{\partial x^2} = a\frac{\partial^2 u(x,t)}{\partial t^2} + b\frac{\partial u(x,t)}{\partial t} + cu(x,t)$$

where *t* is a time coordinate, *x* is a space coordinate, and *a, b and c* are positive constants.

The one dimensional telegraph equation represents the flow of electricity in a cable. This model checks how the cable transmits the voltage using the telegraph equation. The telegraph equation models can be modelled in two dimensional and three dimensional also as per the dimension of the material through which voltage flows. Here we represent the two dimensional and three dimensional telegraph equations.

*2-Dimensional Telegraph equation*

$$a\frac{\partial^2 u(x,y,t)}{\partial t^2} + b\frac{\partial u(x,y,t)}{\partial t} + cu(x,y,t) = \frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2}$$

*3-Dimensional Telegraph equation*

$$a\frac{\partial^2 u(x,y,z,t)}{\partial t^2} + b\frac{\partial u(x,y,z,t)}{\partial t} + cu(x,y,z,t) = \frac{\partial^2 u(x,y,z,t)}{\partial x^2} + \frac{\partial^2 u(x,y,z,t)}{\partial y^2} + \frac{\partial^2 u(x,y,z,t)}{\partial z^2}$$

### *3.5 Burgers Equations:*

Burgers' equation is a nonlinear Partial Differential equation. It is also known as Bateman–Burgers equation as it was first modeled by Harry Bateman in 1915 and then later on it was restudied by Johannes Martinus Burgers in 1948. It is usually arises in fluid mechanics traffic control, gas dynamics etc. The two dimensional and three dimensional coupled system of Burger's equations are given here.

*2-Dimensional Burgers Equation*

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{R}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + u\frac{\partial v}{\partial y} = \frac{1}{R}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

*3-Dimensional Burgers equation*

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{R}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$
$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \frac{1}{R}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$
$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \frac{1}{R}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

## 4 Several Techniques for solving the partial differential equations

### *4.1: Finite Difference method*

Finite Difference method is a numerical method used to solve the differential equations by approximation of derivatives by taking finite differences. By this method, the time interval and domain are divided into finite number of steps and then the values are solved by solving the algebraic equations using nearby values or initial values.By this method, we can solve ordinary as well as partial differential equations. First the linear or non linear differential equations are converted into linear algebraic equations and then using explicit or implicit techniques, these equations are solved. In literature, Generalized Finite difference method has been used to solve the Two dimensional Burgers' equations [1]. A fourth-order compact finite difference scheme of the two-dimensional convection–diffusion equation has been used to solve groundwater pollution problems [2]. The generalized finite difference method with the Euler method has been utilized to solve time dependent boundary values viscoelastic problems [3]. A method based upon the two dimensionalHopf-Cole transformation known as Fourth order compact Finite difference scheme has been used to solve the system of two dimensional Burgers equations [4]. The Stokes interface problems have been solved by using generalized finite difference method [5]. The two-dimensional long wave equations have been solved using a form of Finite difference scheme [6]. Finite difference method has been used to solve hyperbolic partial differential equations [7]. A novel meshless method based on Finite difference technique for solving partial differential equations has been presented [8]. The finite difference approximation method has been used to solve parabolic boundary value problems [9].

### 4.2: Finite Element Method

The Finite element method is an important numerical method for solving the two dimensional and three dimensional partial differential equations arising in physical sciences, chemical sciences, engineering and technology. By this method, the large region is divided into smaller and simple parts, which are known as finite elements. Then these finite elements are computed by using several approximate techniques. Finite element method has been used to solve various ordinary differential equations and partial differential equations. The three dimensional vorticity differential equations have been solved by Finite element method [10]. The finite element method for solving two dimensional non linear water wave equations has been described [11]. The mixed finite element technique has been used to solve two-dimensional Sobolev differential equations [12]. The Galerkin finite element method has been used to solve two dimensional wave equations [13].The fully discrete stabilized finite element technique has been presented to solve two-dimensional Fluid flow problems [14]. The Backward Euler with Crank Nicolson Galerkin method has been utilized to solve two dimensional advection dispersion equations [15]. The two-dimensional Boussinesq equations has been solved using Finite element method [16]. In [17], the formulation of three-dimensional elements has been presented following the standard step-by-step method. The three dimensional space problem has been solved by using Finite element method [18]. The three dimensional space three phase flow problems has been solved using Finite element method [19]. The Galerkin Finite element method has been presented to solve three dimensional diffusion equations [20]. The discontinuous Finite element method has been presented to solve the three dimensional elastic wave equations [21].

### 4.3: Adomian decomposition method

ADM was introduced by GrorgeAdomian in 1980. Adomian decomposition method is a semi analytical method for solving differential equations. A large number of linear and non linear differential equations have been solved by Adomian decomposition method. By Adomian decomposition method, The unknown function or dependent variable of any equation is decomposed into a sums of infinite number of components defined by the decomposition series. In Literature, Adomian decomposition method has been presented to solve the two-dimensional wave equations [22]. Adomian decomposition method has been applied to solve two dimensional parabolic partial differential equations [23]. The two dimensional Lane Emden equations have been solved by Adomian Decomposition method [24]. Adomian decomposition method has been applied to solve the homogeneous partial differential equations in three dimension [25]. The Adomian decomposition method has been used to solve the system of linear or non linear partial differential equations [26].

### 4.4: The Variational Iteration Method

The variational iteration technique was introduced in 1997 by Ji-Huan He. The variational iteration method has been used to solve both linear and non linear partial differential equations arising in various physical phenomena in sciences and engineering. The variational iteration method is a semi analytical method, which is very convenient and effective for solving nonlinear partial differential equations. In literature, The variational iteration method has been presented to solve linear non homogeneous partial differential equations [27]. The parabolic partial differential equations have been solved using variational iteration method [28]. The differential equations with variable coefficients have been solved by Variational iteration method [29]. The variational iteration method has been presented to solve the non linear parabolic and hyperbolic partial differential equations [30]. The Cauchy's problems of Nonlinear partial differential equations have been solved using Modified variational iteration method [31]. The variational iteration method has been discussed to solve the system of linear or non linear partial differential equations [32]. Solutions of some systems of partial differential equations have been computed by variational iteration method [33].

### 4.5: Homotopy Perturbation Method

Homotopy Perturbation method is a well known method to solve the partial differential equations. It was first introduced by Ji-Huan He. Homotopy perturbation method consists from the combination of Perturbation method and Homotopy method. It is very easy to use method and solve the linear or non linear partial differential equations efficiently. In literature, New Homotopy Perturbation method has been presented to solve two-dimensional partial differential equiations [34]. It has been successfully used to solve the system of partial differential equations [35]. Homotopy Perturbation method has been solved to solve the delay differential equations [36]. Homotopy Perturbation method has been applied to solve the multidimensional partial differential equations [37]. The non linear Schrodinger equations have been solved by using Homotopy Perturbation method [38]. The exact solutions of partial differential equations have been obtained using modified homotopy perturbation method [39]. The Homotopy perturbation method combined with Elzaki transform has been used to solve linear and nonlinear Schrodinger's equations [40]. Natural Homotopy perturbation method has been applied to solve the two dimensional system of partial differential equations like Burger's equations [41].

### 4.6: Laplace Adomian Decomposition Method

The Laplace transform Adomian decomposition technique was proposed by K khuri. It is very important combination for solving the linear or nonlinear partial differential equations. In literature, Laplace Adomian decomposition method has been used to solve linear and nonlinear pdes [42].

This combination has been presented to solve the system of fractional partial differential equations [43]. The three dimensional Laplace adomian decomposition method has been used to solve coupled system of partial differential equations [44].The Radhakrishnan–Kundu–Lakshmanan equation has been solved by Laplace adomian combination [45]. The fifth order KdV equation has been solved using modified Laplace adomian method [46]. Laplace decomposition method has been used for solving some non linear models of partial differential equations [47]. Double Laplace decomposition method has been presented to solve Hirota, Schrödinger and modified KdV equations [48]. Laplace decomposition method has been applied to solve the non-linear fractional Sine-Gordon equations [49]. Benjamin-Bona-Mahony equation has been solved analytically using Laplace Adomian decomposition method [50]. Laplace adomian decomposition technique has been presented to find the exact solution of multi-dimensional system of fractional Navier-Stokes equations [51].

### *4.7: Laplace Homotopy Perturbation method*

Laplace Homotopy perturbation method is a comination of the Laplace transform and Homotopy perturbation method. In this method ,first the given differential equation is transformed to an algebraic expression and then that algebraic expression is solved by Homotop perturbation method. Linear or nonlinear ODEs and PDEs are studied successfully by using Laplace Homotopy Perturbation method. In literature, Laplace Homotopy Perturbation method has been presented to solve fractional Burgers equations [52]. The Rosenau-Hyman equation has been solved successfully by using Laplace Homotopy Perturbation method [53]. This method has been applied to solve some nonlinear models of partial differential equations [54].

### *4.8: The Variational iteration Decomposition Method*

The variational iteration decomposition method is technique based on the combination of two most powerful mathematical methods for solving a large class of differential Equations, namely variational iteration method and Adomian decomposition method. In literature, Variational iteration decomposition method has been used to solve the higher dimensional initial values problems in 2009 [55]. In 2016, Variational iteration decomposition method has been presented to solve the three dimensional coupled Burgers equations [56]. This technique has been utilized to solve some models of the fractional order diffusion equations [57].

### *4.9: The variational iteration Homotopy Perturbation method:*

The variational iteration homotopy perturbation method is combination of two well-known methods, namely variational iteration method and homotopy perturbation method. In this method, the partial differential equations is simplified using variational iteration technique and then Homotopy Perturbation method is used to solve the simplified equation iteratively. In literature, Variational iteration Homotopy Perturbation method has been used to solve two dimensional unsteady flow and heat transfer problems [58]. This technique has been presented to solve some model of nonlinear heat equations [59]. In 2007, Non linear evolution equations have been solved by using Variational iteration Homotopy Perturbation method [60]. This method has been presented to solve system of Burgers equations arising in fluid dynamics [61].

## Conclusion

After reviewing the literature, it is observed that a number of numerical methods, analytical methods and semi analytical methods have been developed for solving Two dimensional and three dimensional linear and nonlinear partial differential equations. But because of the complexity of calculation or the size of calculation, there is a need of an efficient method which can solve the two dimensional and three dimensional partial differential equations more efficiently and accurately. New Laplace Variational Iterative Method is not used for solving some specific linear and non-linear two dimensional (2D) and three dimensional (3D) partial differential equations. Therefore we used New Laplace variational iterative method for solving such equations. The proposed method is capable of greatly reducing the size of calculation by maintaining high accuracy of analytical solution.

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