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On Non-Homogeneous Ternary Cubic Diophantine Equation $w^2+5z^2-2wx-10zx=6x^3-6x^2$

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ABSTRACT:

The non-homogeneous ternary cubic diophantine equation $w^2+5z^2-2wx-10zx=6x^3-6x^2$ is analyzed for its patterns of non-zero distinct integral solutions.

Keywords: Ternary cubic, Non- Homogeneous cubic, Integral solutions

INTRODUCTION:

The Diophantine equation offers an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-16] for cubic equations with three unknowns. This communication concerns with yet another interesting equation $w^2+5z^2-2wx-10zx=6x^3-6x^2$ representing non-homogeneous cubic with three unknowns for determining its infinitely many non-zero integral points.

METHOD OF ANALYSIS:

The given non-homogeneous ternary cubic diophantine equation is

$$w^2+5z^2-2wx-10zx=6x^3-6x^2 \tag{1}$$

To start with, it is seen that (1) is satisfied by the integer triples given below:

$$(x,z,w)=(5\alpha^2,5\alpha^2+5\alpha^3,5\alpha^2+25\alpha^3),(6\alpha^2,6\alpha^2+12\alpha^3,6\alpha^2+24\alpha^3),$$
$$(9\alpha^2,9\alpha^2+9\alpha^3,9\alpha^2+63\alpha^3)$$

However, we have other sets of integer solutions to (1).We illustrate below the process of obtaining different sets of integer solutions to (1):

Set 1:

On completing the squares, (1) is written as

$$P^2+5Q^2=6x^3 \tag{2}$$

where

$$P=w-x,Q=z-x \tag{3}$$

After some algebra, it is observed that (2) is satisfied by

$$P=36m(m^2+5n^2),Q=36n(m^2+5n^2) \tag{4}$$

and

$$x=6(m^2+5n^2) \tag{5}$$

From (4) and (3), we have

$$w=(36m+6)(m^2+5n^2),z=(36n+6)(m^2+5n^2) \tag{6}$$

Thus, (5) and (6) represent the integer solutions to (1).

Set 2:

Write 6 as

$$6=(1+i\sqrt{5})(1-i\sqrt{5}) \quad (7)$$

Let

$$x=a^2+5b^2 \quad (8)$$

Substituting (7) and (8) in (2) and employing the method of factorization one has

$$P+i\sqrt{5}Q=(1+i\sqrt{5})(a+i\sqrt{5}b)^3 \quad (9)$$

On equating the real and imaginary parts, we have

$$P=(a^3-15ab^2)-5(3a^2b-5b^3),\ Q=(a^3-15ab^2)+(3a^2b-5b^3) \quad (10)$$

Using (10) in (3), note that

$$\begin{aligned} z &= a^3-15ab^2+3a^2b-5b^3+a^2+5b^2, \\ w &= a^3-15ab^2-15a^2b+25b^3+a^2+5b^2, \end{aligned} \quad (11)$$

Thus, (8) and (11) represent the integer solutions to (1).

Note 1:

The integer 6 on the R.H.S. of (2) is also represented by

$$6=\frac{(7+i\sqrt{5})(7-i\sqrt{5})}{9},$$
$$6=\frac{(17+i\sqrt{5})(17-i\sqrt{5})}{49}$$

The repetition of the above process leads to a different set of solutions to (1).

Set 3:

Write (2) as

$$P^2+5Q^2=6x^3*1 \quad (12)$$

Consider 1 as

$$1=\frac{(2+i\sqrt{5})(2-i\sqrt{5})}{9} \quad (13)$$

Using (8), (7) and (13) in (12) and employing the method of factorization, one has

$$P+i\sqrt{5}Q=\frac{(-1+i\sqrt{5})(a+i\sqrt{5}b)^3}{3}$$

Following the procedure as in Set 2, the integer solutions to (1) are given by

$$z=a^3-15ab^2-3a^2b+5b^3+a^2+5b^2,$$
$$w=-a^3+15ab^2-15a^2b+25b^3+a^2+5b^2,$$

along with (8)

Note 2:

One may also take 1 on the R.H.S. of (12) as

$$1=\frac{(5r^2-s^2+i\,2rs\sqrt{5})(5r^2-s^2-i\,2rs\sqrt{5})}{(2r^2+s^2)^2},$$

$$1=\frac{(2+i3\sqrt{5})(2-i3\sqrt{5})}{7^2}$$

The repetition of the above process leads to different sets of solutions to (1).

CONCLUSION:

In this paper, we have made an attempt to obtain all integer solutions to (1). To conclude, one may search for integer solutions to other choices of homogeneous or non-homogeneous ternary cubic Diophantine equations.

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