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On the Non-homogeneous Ternary Bi-quadratic Equation

$$4xz(x+z)=5y^4$$

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ABSTRACT:

This paper focuses on finding non-zero distinct integer solutions to the non-homogeneous ternary bi-quadratic diophantine equation given by $4xz(x+z)=5y^4$. Different sets of integer solutions are presented.

Key words: non-homogeneous bi-quadratic, ternary bi-quadratic ,integer solutions

Notations:

$$CP_n^{10}=\frac{(5n^3-2n)}{3},CP_n^{12}=2n^3-n,CP_n^5=\frac{(5n^3+n)}{6},$$

$$CP_n^{25}=\frac{(25n^3-19n)}{6},CP_n^6=n^3,CP_n^{30}=5n^3-4n)$$

Introduction:

The Diophantine equations are rich in variety and offer an unlimited field for research [1-4]. In particular refer [5-30] for a few problems on Biquadratic equation with 3 unknowns. This paper concerns with yet another interesting Biquadratic Diophantine equation with three variables given by $4xz(x+z)=5y^4$ for determining its infinitely many non-zero distinct integral solutions.

Method of analysis:

The non-homogeneous ternary bi-quadratic equation under consideration is

$$4xz(x+z)=5y^4 \quad (1)$$

Introduction of the linear transformations

$$x=u+v, z=u-v, y=2u, u\neq v\neq 0 \quad (2)$$

in (1) leads to

$$v^2=u^2(1-10u)$$

which is satisfied by

$$u=k(2-10k), v=k(2-10k)(10k-1)$$

In view of (2), the corresponding integer solutions to (1) are given by

$$x=20k^2(1-5k), z=4k(1-5k)^2, y=4k(1-5k) \quad (3)$$

A few interesting relations among the solutions are exhibited below:

(i). $x+60\,CP_k^{10}\equiv 0(\bmod 20)$

(ii). $x+20\,CP_k^{12}+60\,CP_k^{6}=0$

(iii). $x+120\,CP_k^{5}\equiv 0(\bmod 40)$

(iv). $x+y+24\,CP_k^{25}\equiv 0(\bmod 72)$

(v). $x+20\,CP_k^{30}\equiv 0(\bmod 60)$

(vi). xyz is a square multiple of 20

(vii). xyz is 20 times a bi-quadratic integer

(viii). $5zy^3=4x(y-x)^2$

(ix). $5\,y^3=4x(y-x)$

(x). $5xy^3=4z(y-z)^2$

(xi). $5(x+z)y^2=4x(y-x)$

(xii). $5(x+z)y^2=4z(y-z)$

(xiii). $5(y-x)y^3=4x(y-x)^2$

(xiv). $5(y-x)y^3=4z^2(y-z)$

It is worth mentioning that, apart from (2), one may employ other choices of linear transformations leading to different sets of solutions . For simplicity and brevity , we present below a few sets of solutions to (1) :

Set 1:

$$x = -4k\,(1+5k)^2\,,\, z = 20k^2\,(1+5k),\, y = -\,4k(1+5k)$$

Set 2:

$$x = 5k\,(-5k^2+4k)\,,\, z = (4-5k)\,(-5k^2+4k),\, y = 2\,(-5k^2+4\,k)$$

Set 3:

$$x = (5k+4)\,(-5k^2-4k)\,,\, z = 5k\,(5k^2+4k),\, y = -\,2\,(5k^2+4k)$$

Set 4:

$$x = 20k\,(-10k^2+4k)\,,\, z = (8-20k)\,(-10k^2+4k)\ ,\, y = 4\,(-10k^2+4k)$$

Set 5:

$$x = (-10k^2-4k)\,(20k+8),\, z = 20k(10k^2+4k)\,,\, y = -\,4\,(10\,k^2+4k)$$

Conclusion:

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous bi-quadratic diophantine equation with three unknowns given by $4xz\,(x+z) = 5y^4$ .One may search for other sets of integer solutions to the considered equation as well as other choices of the fourth degree diophantine equations with multi-variables.

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