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On the Non-homogeneous Ternary Bi-quadratic Equation

$$2xz(x-z)=y^4$$

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ABSTRACT:

This paper focuses on finding non-zero distinct integer solutions to the non-homogeneous ternary bi-quadratic diophantine equation given by $2xz(x-z)=y^4$. Different sets of integer solutions are presented.

Key words: non-homogeneous bi-quadratic, ternary bi-quadratic ,integer solutions

Notations:

$$t_{m,n}=n(1+\frac{(n-1)(m-2)}{2}),P_n^3=\frac{n(n+1)(n+2)}{6},P_n^5=\frac{n^2(n+1)}{2}$$

$$CP_n^3=\frac{(n^3+n)}{2},CP_n^{12}=2n^3-n,CP_n^8=\frac{(4n^3-n)}{3},CP_n^4=\frac{(2n^3+n)}{3}$$

Introduction:

The Diophantine equations are rich in variety and offer an unlimited field for research [1-4]. In particular refer [5-28] for a few problems on Biquadratic equation with 3 unknowns. This paper concerns with yet another interesting Biquadratic Diophantine equation with three variables given by $2xz(x-z)=y^4$ for determining its infinitely many non-zero distinct integral solutions.

Method of analysis:

The non-homogeneous ternary bi-quadratic equation under consideration is

$$2xz(x-z)=y^4 \tag{1}$$

Introduction of the linear transformations

$$x=u+v, z=u-v, y=2v, u\neq v\neq 0 \tag{2}$$

in (1) leads to

$$u^2=v^2(1+4v)$$

which is satisfied by

$$v=n(n+1), u=n(n+1)(2n+1)$$

In view of (2), the corresponding integer solutions to (1) are given by

$$x=2n(n+1)^2, z=2n^2(n+1), y=2n(n+1) \tag{3}$$

A few interesting relations among the solutions are exhibited below:

(i). $x+z=24P_n^3-12t_{3,n}$

(ii). $y+z=6P_n^3+2P_n^5$

(iii). $x-y=4P_n^5$

(iv). $x+z-3CP_n^8-3n$ is a square multiple of 6

(v). $x-y-CP_n^{12}-2t_{8,n}$ is a square integer

(Vi).Each of the following is a square multiple of 2:

$$\frac{yx}{z}, \frac{zx}{y}, \frac{zy}{x}$$

(Vii). $x-y-CP_n^{12}-t_{6,n}\equiv 0 \pmod 2$

(Viii). $z+y-3CP_n^4-t_{10,n}\equiv 0 \pmod 4$

(IX). $2xyz$ is a bi-quadratic integer

(X). $y + z - 4CP_n^3$ is a square integer

It is worth mentioning that, apart from (2), one may employ other choices of linear transformations leading to different sets of solutions . For simplicity and brevity , we present below a few sets of solutions to (1) :

Set 1:

$$x = (k+4)(k^2-16), z = (k-4)(k^2-16), y = 2(k^2-16) \quad k \neq \pm 4$$

Set 2:

$$x = (2k+6)(2k^2+4k), z = (2k-2)(2k^2+4k), y = 2(2k^2+4k) \quad , k \neq 1$$

Set 3:

$$x = (2k+6)(3k^2+9k), z = 2k(3k^2+9k), y = 2(3k^2+9k)$$

Set 4:

$$x = (2k+10)(5k^2+25k), z = 2k(5k^2+25k) \ , y = 2(5k^2+25k)$$

Set 5:

$$x = (2k^2+8k)(4k+16), z = 4k(2k^2+8k), y = 2(2k^2+8k)$$

Set 6:

$$x = (k+8)(2k^2-128), z = (k-8)(2k^2-128), y = 2(2k^2-128) \quad k \neq \pm 8$$

Conclusion:

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous bi-quadratic diophantine equation with three unknowns given by $2xz(x-z) = y^4$ .One may search for other sets of integer solutions to the considered equation as well as other choices of the fourth degree diophantine equations with multi-variables

References:

[1] L.J. Mordell, Diophantine Equations, Academic press, New York, 1969.

[2] R.D. Carmichael, The Theory of numbers and Diophantine Analysis, Dover publications, New York, 1959.

[3] L.E. Dickson, History of theory of Numbers, Diophantine Analysis, Vol.2, Dover publications, New York, 2005.

[4] S.G. Telang, Number Theory, Tata Mc Graw Hill publishing company, New Delhi, 1996.

[5] M.A.Gopalan, and G. Janaki, Integral solutions of ternary quartic equation $x^2 - y^2 + xy = z^4$, Impact J. Sci. Tech, 2(2), Pp 71-76, 2008.

[6] M.A.Gopalan, and V. Pandichelvi, On ternary biquadratic diophantine equation $x^2 + ky^3 = z^4$, Pacific- Asian Journal of Mathematics, Volume 2, No.1-2, Pp 57-62, 2008.

[7] M.A.Gopalan, A.Vijayasankar and Manju Somanath, Integral solutions of $x^2 - y^2 = z^4$, Impact J. Sci. Tech., 2(4), Pp 149-157, 2008.

[8] M.A.Gopalan, and G.Janaki, Observation on $2(x^2 - y^2) + 4xy = z^4$, Acta Ciencia Indica, Volume XXXVM, No.2, Pp 445-448, 2009.

[9] M.A.Gopalan, and R. Anbuselvi, Integral solutions of binary quartic equation $x^3 + y^3 = (x - y)^4$, Reflections des ERA-JMS, Volume 4, Issue 3, Pp 271-280, 2009.

[10] M.A.Gopalan, Manjusomanath and N. Vanitha, Integral solutions of $x^2 + xy + y^2 = (k^2 + 3)^n z^4$, Pure and Applied Mathematical Sciences, Volume LXIX, No.(1-2), Pp 149-152, 2009.

[11] M.A.Gopalan, and G.Sangeetha, Integral solutions of ternary biquadratic equation $(x^2 - y^2) + 2xy = z^4$, Antartica J.Math., 7(1), Pp 95-101, 2010.

[12] M.A.Gopalan and A.Vijayashankar, Integral solutions of ternary biquadratic equation $x^2 + 3y^2 = z^4$, Impact.J.Sci.Tech., Volume 4, No.3, Pp 47-51, 2010.

[13] M.A.Gopalan and G. Janaki, Observations on $3(x^2 - y^2) + 9xy = z^4$, Antartica J.Math., 7(2), Pp 239-245, 2010.

[14] M.A. Gopalan, S. Vidhyalakshmi, S. Devibala, Ternary bi-quadratic Diophantine equation $2^{4n+3}(x^3 - y^3) = z^4$, Impact J. Sci. Tech, Vol.4(3), 57-60, 2010.

[15] M.A. Gopalan, G. Sangeetha, Integral solutions of ternary non-homogeneous bi-quadratic equation $x^4 + x^2 + y^2 - y = z^2 + z$, Acta Ciencia Indica, Vol. XXXVIIM, No.4, 799-803, 2011.

[16] M.A. Gopalan, S. Vidhyalakshmi, G. Sumathi, Integral solutions of ternary bi-quadratic non-homogeneous equation $(\alpha + 1)(x^2 + y^2) + (2\alpha + 1)xy = z^4$, JARCE, Vol.6(2), 97-98, July-December 2012.

[17] M.A. Gopalan, G. Sumathi, S. Vidhyalakshmi, Integral solutions of ternary non-homogeneous bi-quadratic equation $(2k + 1)(x^2 + y^2 + xy) = z^4$, Indian Journal of Engineering, Vol.1(1), 37-39, 2012.

[18] Manju Somanath, G.Sangeetha, and M.A.Gopalan, Integral solutions of a biquadratic equation $xy + (k^2 + 1)z^2 = 5w^4$, PAJM, Volume 1, Pp 185-190, 2012.

[19] M.A. Gopalan, G. Sumathi, S. Vidhyalakshmi, On the ternary bi-quadratic non-homogeneous equation $x^2 + ny^3 = z^4$, Cayley J.Math, Vol.2(2), 169-174, 2013.

[20] M.A.Gopalan , V.Geetha , (2013), Integral solutions of ternary biquadratic equation $x^2 + 13y^2 = z^4$, IJLRST, Vol 2, issue2, 59-61

[21] M.A.Gopalan ,S. Vidhyalakshmi ,A. Kavitha , (2013), Integral points on the biquadratic equation $(x + y + z)^3 = z^2(3xy - x^2 - y^2)$, IJMSEA,Vol 7, No.1, 81-84

[22] A. Vijayasankar, M.A. Gopalan, V. Kiruthika, On the bi-quadratic Diophantine equation with three unknowns $7(x^2 - y^2) + x + y = 8z^4$, International Journal of Advanced Scientific and Technical Research, Issue 8, Volume 1, 52-57, January-February 2018.

[23] Shreemathi Adiga,N.Anusheela,M.A.Gopalan,Non-Homogeneous Bi-Quadratic EquationWith Three Unknowns $x + 3xy + y = z$,Vol.7,Issue.8,Version -3,pp.26-29,2018

[24] S.Vidhyalakshmi, M.A. Gopalan, S. Aarthy Thangam and Ozer, O., On ternary biquadratic diophantine equation $11(x^2 - y^2) + 3(x + y) = 10z^4$, NNTDM, Volume 25, No.3, Pp 65-71, 2019.

[25] A.Vijayasankar, Sharadha Kumar, M.A.Gopalan, “A Search For Integer Solutions To

Ternary Bi-Quadratic Equation $(a+1)(x^2+y^2)-(2a+1)xy=\left[p^2+(4a+3)q^2\right]z^4$", EPRA(IJMR), 5(12), Pp: 26-32, December 2019.

[26] A.Vijayasankar, Sharadha Kumar, M.A.Gopalan, "On Non-Homogeneous Ternary Bi-Quadratic Equation $x^2+7xy+y^2=z^4$", Compliance Engineering Journal, 11(3), Pp:111-114, 2020.

[27] S.Vidhyalakshmi, M.A.Gopalan, On The Non-Homogeneous Ternary Bi-quadratic Equation $xz(x+z)=2y^4$,IJRPR,Vol,3 ,Issue7,pp.3465-3469,2022

[28]] S.Vidhyalakshmi, M.A.Gopalan, On The Non-Homogeneous Ternary Bi-quadratic Equation $8xz(x+z)=15y^4$,IRJMETS,Vol,4 ,Issue7,pp.3623-3625,2022