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Normal Spaces on Bipolar Intuitionistic Fuzzy Sets

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ABSTRACT

In this paper the concepts of bipolar intuitionistic fuzzy normal space, bipolar intuitionistic fuzzy generalized alpha normal space, bipolar intuitionistic fuzzy generalized alpha almost normal space, bipolar intuitionistic fuzzy mildly normal space, bipolar intuitionistic fuzzy generalized alpha strongly normal space are put forward. Further few theorems bridging te above mentioned spaces are studied

Keywords: Bipolar intuitionistic fuzzy normal space, bipolar intuitionistic fuzzy generalised alpha normal space,bipolar intuitionistic fuzzy generalised alpha almost normal space, bipolar intuitionistic fuzzy generalised alpha mildly normal space, bipolar intuitionistic fuzzy strongly normal space

1. Introduction

The concept of fuzzy set was introduced by Zadeh[16] in the year 1965 and later Atanassov[3] generalised this idea to a new class of intuitionistic fuzzy sets using the notions of fuzzy sets. C.L. Chang[4] described the new concept of Fuzzy Topological Spaces and Dogan Coker[5] gave an introduction to intuitionistic fuzzy topological spaces. Bipolar valued fuzzy sets, which was introduced by Lee[10] in 2000,which is an extension of fuzzy sets whose membership degree range is enlarged from the interval [0,1] to [-1,1]. D. Ezhilmaran & K. Sankar [6] in the year 2015,discussed on the morphism of bipolar intuitionistic fuzzy graphs and developed its related properties. The property of almost normality was introduced by the authors Singal and Arya[14] .The notion of mildly normal space was introduced by Singal and Singal[15] independently. Also,$\pi\beta$-normality was discussed by Sharma.M.C and Kumar.H[13] in the year 2010.Almost normality was discussed by Nidhi Sharma[11],in the year 2014,almost γ – normal spaces was studied by Hamant Kumar and M.C.Sharma[8].In the year 2016,Jothimani[9] introduced and studied new types of $\pi g\beta$ normal spaces in intuitionistic fuzzy topological spaces. Furthermore ,in the year 2019,[1]introduced and discussed the notions of b- regularity and normality in intuitionistic fuzzy topological spaces. Later in the year 2018,[2]introduced and studied the new classes of β *and* β^*normal spaces in intuitonistic fuzzy topological spaces.

2. Preliminaries

Definition 2.1[7]:

An ITS,(X,τ) will be called normal if for each IFCSs U and V such that U⋂V=0∼ there eists IFOSs U_1 and V_1 such that U⊆ U_1 and V⊆ V_1 and U_1⋂V_1=0∼.

Definition 2.2[14]:

An intuitionistic fuzzy (IF) topological space (X,τ) is called mildly normal if for any two IF disjoint regularly closed subsets A and B of X,there exist two IF open disjoint subsets U and V of X such that A⊆U and B⊆V.i.e.,any two IF disjoint regularly closed subsets are separated.

Definition 2.3[15]:

An intuitionistic fuzzy (IF) topological space (X,τ) is called almost normal if for any two IF disjoint closed subsets A and B of X,one of which is IF regularly closed,there exist two IF disjoint open subsets U and V of X such that A⊆U and B⊆V.

Definition 2.4[12]:

An intuitionistic fuzzy (IF) topological space (X,τ) is called strongly normal if and only if for every pair L,M of disjoint intuitionistic fuzzy closed sets of T,there exists intuitionistic fuzzy open sets U and V such that L⊆U,M⊆V and U∩V=0∼.

3. Bipolar intuitionistic fuzzy gα normal space

Definition 3.1:

A space (X, τ) is said to be a BIF- normal space if for any pair of disjoint, BIF-closed sets A and B there exists disjoint, BIF-open sets U and V such that A ⊆ U and B ⊆ V

Definition 3.2. A space (X, τ) is said to be a BIFgα-normal space if for any pair of disjoint BIF-closed sets A and B there exists disjoint, BIFgα-open sets U and V such that A ⊆ U and B ⊆ V .

Theorem 3.3. Every BIF-normal space is BIF-gα normal space.

Proof. Let (X,τ) be a BIF-normal space.Then for any two disjoint,BIF-closed sets A and B in X,there exists disjoint,BIF-closed sets U and V such that A ⊆ U and B ⊆ V .Every BIF-open set is BIF-gα open set .Therefore U and V are BIF-gα open sets.Therefore for any pair of disjoint,BIF-closed sets A and B ,there exists disjoint, BIFgα-open sets U and V such that A ⊆ U and B ⊆ V .Hence (X,τ) is a BIFgα-normal space.

The converse of the above theorem need not be true as shown in the example below.

Example 3.4. Let X={a,b} and let τ={0∼,1∼,A,B} be a BIFT on X where

$$A=\left\{x,\frac{a}{<0,,-1,1,0>},\frac{b}{<0,-1,1,0>}\right\},\ B=\left\{x,\frac{u}{<1,0,0,-1>},\frac{v}{<1,0,0,-1>}\right\}.$$

For the pair of disjoint,BIF-closed sets 0∼ and Ac there exists a pair of disjoint,BIFgα-open sets 0∼ and B such that 0∼ ⊆ 0∼ and Ac ⊆ B.Therefore (X,τ) is BIFgα-normal space.But for the pair of BIF-closed sets,0∼ and Bc there does not exist disjoint BIFgα-open sets,satisfying the condition of BIF -normal space.Therefore BIFgα-normal space need not be BIF- normal space.

Theorem 3.5. If (X,τ) is a BIFgα-normal space,Tgα space,then (X,τ) is a BIF- normal space.

Proof. Let (X,τ) be a BIFgα-normal space.Then for each pair of disjoint,BIF-closed sets A and B in X there exists disjoint,BIFgα-open sets U and V such that A ⊆ U and B ⊆ V .Since (X,τ) is a Tgα-space,every BIFgα-open set is BIF-open set.Therefore U and V are BIF-open sets in X.Therefore (X,τ) is a BIF-normal space.

Theorem 3.6. Let (X, τ) be a bipolar intuitionistic fuzzy topological space,then following are equivalent

(i) X is an BIFgα- normal space.

(ii) For every pair of bipolar intuitionistic fuzzy open sets U and V whose union is 1∼ , there exist BIFgα-closed sets A and B such that A ⊆ U,B ⊆ V and A∪B = 1∼.

(iii) For every bipolar intuitionistic fuzzy closed set H and every bipolar intuition- istic fuzzy open set K containing H, there exists an BIFgα-open set U such that H ⊆ U ⊆ BIFgαcl(U) ⊆ K.

(iv) For every pair of BIFgα-closed sets H and K of X there exists an BIFgα-open set U of X such that H ⊆ U and BIFgαcl(U)∩ K = 0∼.

(v) For every pair of disjoint,BIFgα- closed sets H and K of X there exists BIFgα- open sets U and V of X such that H ⊆ U,K ⊆V and BIFgαcl(U)∩BIFgαcl(V)= 0∼.

Proof.

(i)⇒(ii) Let U and V be two BIFgα- normal space X such that U ∪ V = 1∼. Then U c, V c are BIF-closed sets.Since X is an BIFgα-normal space there exist BIFgα-open sets U1 and V1 such that Uc ⊆ U1 and V c ⊆ V1.Let A = U c, B = V c.Then A and B are BIFgα-closed sets such that A⊆U, B⊆V and A∪B = 1∼.

(ii)⇒(iii)Let H be BIF-closed set and K be an BIF-open set containing H.Then Hc and K are BIF-open sets such that Hc∪ K = 1∼. Then by (ii) there exist BIFgα- closed sets M1 and M2 such that M1 ⊆ Hc and M2 ⊆ K and M1∪M2 = 1 ∼.Thus ,we obtain H subseteqMc, Kc ⊆ Mc and Mc ∩Mc = 0 ∼.Let U = Mc and V = Mc.Then U and V are BIFgα-open sets such that H ⊆ U ⊆ V c ⊆ K. As V c an BIFgα-closed set, we have H ⊆ U ⊆ BIFgαcl(U) ⊆ K.

(iii)⇒(iv)Let H and K be disjoint BIFgα -closed sets of X. Then H ⊆ K c where K c is BIFgα-open. By the part (iii), there exists a BIF-open subset U of X such that H ⊆ U ⊆BIFgαcl(U)⊆ Kc. Thus, BIFgαcl(U) ∩ K = 0∼.

(iv)⇒(v)Let H and K be any disjoint,BIFgα -closed set of X. Then by the part (iv),there exists a BIFgα -open set U containing H such that BIFgα c1(U) ∩ K = 0∼. Since BIFgαc1(U) is an BIFgα- closed. Thus BIFgα c1(U) and K are disjoint BIFgα -closed sets of X. Again by the part (iv), there exists a BIFgα -open set V in X such that K ⊆ V and BIFgα c1(U) ∩ BIFgα cl(V) = 0∼.

Theorem 3.7. BIF-normality is a topological property.

Proof. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a BIF-homeomorphism and let (X,τ) be a BIF- normal space..Let A and B be two disjoint,BIF-closed sets in Y.Since f is BIF- onto,then $f^{-1}(A\cap B)=f^{-1}(0\sim)\Rightarrow f^{-1}(A)\cap f^{-1}(B)=0\sim$.Since f is BIF- continuous,$f^{-1}(A)$ and $f^{-1}(B)$ are BIF-closed in X and $f^{-1}(A)\cap f^{-1}(B)=0\sim$ in X.Since (X,τ) is BIF-normal,there exists BIF,disjoint open sets U and V such that $f^{-1}(A)\subseteq U$ and $f^{-1}(B)\subseteq V$ and $U\cap V=0\sim$.Now we have,$f[f^{-1}(A)]\subseteq f(U)$ and $f[f^{-1}(B)]\subseteq f(V)$(i.e,)A⊆f(U) and B⊆f(V) with $f(U)\cap f(V)=f(U\cap V)=0\sim\Rightarrow f(0\sim)=0\sim$.Thus there exists G=f(U) and H=f(V) where A⊆G and B⊆H,G∩H=0 ∼.Hence (Y,σ) is a BIF-normal space.Thus,every homeomorphic image of a BIF-normal space is a BIF-normal space.Hence BIF-normality is a topological property.

Definition 3.8. A function $f : (X, \tau) \to (Y, \sigma)$ is strongly BIFgα-open map if f(M) is BIFgα-open set in Y for each BIFgα-open set M in X.

Theorem 3.9. If $f : (X, \tau) \to (Y, \sigma)$ is continuous, bijective, strongly BIFgα-open function from a BIFgα-normal space X onto a space Y, then Y is BIFgα-normal

Proof. Let E and F be disjoint BIF-closed sets in Y. Since f is continuous bijective $f^{-1}(E)$and $f^{-1}(F)$ are disjoint closed sets in X. Now X is BIFgα-normal, there exist disjoint BIFgα-open sets U and V such that $f^{-1}(E) \subseteq U$ and $f^{-1}(F) \subseteq V$. That is, E ⊆ f(U) and F ⊆ f(V). Since f is strongly BIFgα-open function f(U) and f(V) are BIFgα-opensets in Y and f is injective f(U) ∩f(V) = f(U ∩V) = 0∼. Thus, Y is BIFgα-normal.

Theorem 3.10. If $f : X \to Y$ is BIF-bijective, BIF-open,BIF -irresolute function from BIFgα-normal space X into a bipolar intuitionistic fuzzy topological space Y , then Y is BIFgα-normal.

Proof. Let A and B disjoint BIF-closed sets in Y.Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint BIF-closed sets in X as f is BIF-irresolute. Since X is BIFgα-normal, there exist disjoint BIFgα-open sets G and H in X such that $f^{-1}(A) \subseteq G$ and $f^{-1}(B) \subseteq H$. Again since f is BIF-bijective and BIF-open , f(G) and f(H) are disjoint BIFgα-open sets in Y such that A⊆f(G) and B⊆f(H). Hence Y is BIFgα- normal.

Theorem 3.11. If $f : X \to Y$ is BIF strongly gα-continuous injective, BIFgα- closed function and Y is BIFgα-normal space, then X is BIF-normal.

Proof. Let E and F be BIF-disjoint closed sets in X. Since f is BIFgα-closed injective, f(E) and f(F) are disjoint,BIFgα- closed sets in Y. By the definition of BIgα- normality of Y, there exist disjoint,BIFgα-open sets U and V such that f(E)⊆ U and f(F)*subseteq* V. This implies E⊆ $f^{-1}(U)$ and F⊆ $f^{-1}(U)$. Since f is BIF strongly gα-continuous $f^{-1}(U)$ and $f^{-1}(V)$ are BIF-disjoint open sets in X. Therefore, X is BIF-normal.

Definition 3.12. A mapping $f : X \to Y$ is said to be perfectly BIFgα- continuou if ,$f^{-1}(V)$ is clopen in X for every BIFgα-open set V in Y.

Definition 3.13. A space (X, τ) is said to be a BIF ultra- normal space if for any pair of disjoint,BIF-closed sets A and B there exists disjoint,BIF-clopen sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 3.14. A space (X, τ) is said to be a BIFgα ultra- normal space if for any pair of disjoint,BIF-closed sets A and B there exists disjoint,BIFgα-clopen sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Theorem 3.15. If $f : X \to Y$ is BIF perfectly gα-continuous, BIF-injective,closed function and f is BIFgα-normal space, then X is BIF-ultra normal.

Proof. Let E and F be disjoint BIF-closed sets in X. Since f is BIF-closed injective, f(E) and f(F) are disjoint BIF-closed sets in Y. By the definition of BIFgα-normality of f, there exist disjoint BIFgα-open sets U and V such that f(E)⊆ U and f(F)⊆ V. This implies, E⊆ $f^{-1}(U)$ and F ⊆ $f^{-1}(V)$. Since f is BIF perfectly gα-continuous and $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint BIF-clopen sets in X. Therefore, X is BIF-ultra normal.

Definition 3.16. A mapping $f : X \to Y$ is said to be contra BIFgα- continuous ,if $f^{-1}(V)$ is BIFgα-open in X for every BIFgα-closed set V in Y.

Theorem 3.17. If $f : X \to Y$ is a contra BIFgα- continuous,closed injective mapping and Y is an BIFgα ultra- normal space,then X is an BIFgα-normal space.

Proof. Let A and B be disjoint,BIF-closed subsets of X. Since f is a BIF-closed and injective mapping, f(A) and f(B) are disjoint,BIF-closed subsets of Y.Since Y is an BIFgα ultra-normal space,f(A) and f(B) are separated by disjoint,BIFgα-clopen sets V_1 and V_2 respectively.Since f is BIFgα-contra continuous,$f^{-1}(V_1)$ and $f^{-1}(V_2)$ are BIFgα-closed sets in X. Hence A⊆ $f^{-1}(V_1)$ and B⊆ $f^{-1}(V_2)$ and$f^{-1}(V_1) \cap f^{-1}(V_1)$=0∼. Thus X is a BIFg$\alpha$-normal space.

Definition 3.18. A function $f : (X, \tau) \to (Y, \sigma)$ is called almost BIFgα-irresolute if for each BIF point x($\alpha, \beta, \gamma, \delta$) in X and each BIFg$\alpha$ -neighbourhood V of f(x), BIFg$\alpha cl(f^{-1}(V))$ is an BIFgα-neighbourhood of x($\alpha, \beta, \gamma, \delta$).

Theorem 3.19. If $f : (X, \tau) \to (Y, \sigma)$ is a strongly BIFgα-open,continuous,almost BIFgα-irresolute function from a BIFgα-normal space X onto a space Y,then Y is BIFgα-normal.

Proof. Let A be an BIF- closed set of Y and B an BIF-open set of Y contain ing A .Then since f is continuous $f^{-1}(A)$ is BIF-closed and$f^{-1}(B)$ BIF-open set in X such that $f^{-1}(A) \subseteq f^{-1}(B)$. Since X is an BIFg$\alpha$- normal there exits an BIFgα-open set U in X such that $f^{-1}(A)$ ⊆ U ⊆ BIFgαcl(U) ⊆ $f^{-1}(B)$, $f[f^{-1}(A)]$ ⊆ f(U) ⊆ f[BIFgαcl(U)] ⊆ $f[f^{-1}(B)]$.Since f is an strongly BIFgα -open,almost BIFgα irresolute- surjection function,we obtain $A \subseteq$ f(U) ⊆ BIFgαcl[f(U)] ⊆ B .Then again by Theorem 6.2.6,the space(Y, σ) is BIFgα-normal.

Theorem 3.20. Let $f : (X, \tau) \to (Y, \sigma)$ be a BIF-bijective map.If f is a BIFgα- open map,BIF-continuous and (X,τ) be BIF- normal then (Y,σ) is BIFgα-normal.

Proof. Let (X,τ) be a BIF-normal.Let A and B be two disjoint,BIF-closed sets in Y.Since f is BIF-continuous,$f^{-1}(A)$ and $f^{-1}(B)$ are BIF-closed in X.Since (X,τ) is BIF-normal,there exists disjoint,BIF-open sets U and V such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$ .That is ,$f[f^{-1}(A)] \subseteq f(U)$ and $f[f^{-1}(B)] \subseteq f(V)$,that is $A \subseteq f(U) and B \subseteq f(V)$.Since f is BIFg$\alpha$-open map,f(U) and f(V) are BIFgα-open sets in Y such that $A \subseteq f(U)$ and $B \subseteq f(V)$.Since f is injective,$f(U) \cap f(V) = f(U \cap V) = f(0\sim) = 0\sim$.Thus (Y,$\sigma$) is BIFg$\alpha$-normal

(v)⇒(i)Let H and K be any disjoint BIFgα -closed sets of X. Then by the part (v), there exist BIFgα-open sets U and V such that H ⊆ U,K ⊆ V and BIFgαc1(U) ∩ BIFgαc1(V) = 0∼. Therefore, we obtain that U ∩ V = 0∼.Hence X is BIFgα-normal.

Theorem 3.21. Let $f : (X, \tau) \to (Y, \sigma)$ be a BIF-bijective map.If f is a BIF-closed map,BIFgα-continuous and (Y,σ) be normal then (X,τ) is BIFgα-normal.

Proof. Let E and F be two disjoint,BIF-closed sets in X.Since f is a BIF-closed map,f(E) and f(F) are BIF-closed sets in Y.Since,(Y,σ) is BIF-normal,there exists disjoint,BIF-open sets G and H such that f(E)⊆ G and f(F)⊆ H.This implies,E⊆ $f^{-1}(G)$ and F⊆ $f^{-1}(H)$.Since f is BIFgα-

continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are BIFgα- open sets in X.Further, $f(G) \cap f(H) = f(G \cap H) = f(0\sim) = 0\sim$.Thus (X,$\tau$) is BIFgα-normal.

Theorem 3.22. Let $f : (X, \tau) \to (Y, \sigma)$ be a BIF-bijective map.If f is a BIFgα- irresolute,closed map and (Y,σ) be BIFgα-normal then (X,τ) is BIFgα-normal.

Proof. Let E and F be two disjoint,BIF-closed sets in Y.Since f is a BIF-closed,injection map,f(E) and f(F) are BIF-closed sets in Y.Since,(Y,σ) is BIFgα-normal,there exists disjoint,BIFgα-open sets G and H such that f(E)⊆ G and f(F)⊆ H.This implies,E⊆ $f^{-1}(G)$ and F⊆ $f^{-1}(H)$.Since f is BIFgα-irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are BIFgα- open sets in X.Further, $f(G) \cap f(H) = f(G \cap H) = f(0\sim) = 0\sim$.Thus (X,$\tau$) is BIFgα-normal.

Theorem 3.23. The image of a BIFgα-normal space under an BIF-open,continuous injective function is BIFgα-normal.

Proof. Let X be a BIFgα-normal space and let $f : (X, \tau) \to (Y, \sigma)$ be a BIF- open,continuous injective function.We need to prove that f(X) is BIFgα-normal.Let A and B be two disjoint BIF-closed sets in f(X).Since f is BIF-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are BIF-closed sets in (X,τ).Since (X,τ) is BIFgα-normal,there exists disjoint BIFgα-open sets M and N such that $f^{-1}(A) \subseteq M$ and $f^{-1}(B) \subseteq N$ and M∩N =0∼.Since f is BIF-open,injective function,we have $A \subseteq f(M)$, $B \subseteq f(N)$ and $f(M) \cap f(N) = f(M \cap N) = f(0\sim) = 0\sim$.Hence f(U) is BIFg$\alpha$-normal.

Theorem 3.24. A BIFgα-closed subspace of a BIFgα-normal space is BIFgα normal.

Proof. Let X be a BIFgα-normal space. Let Y be a BIF-closed subspace of X. Let A and B be a pair of disjoint,BIF-closed sets in Y. Then A and B are disjoint,BIF-closed sets in X. Since X is BIFgα-normal, there exist disjoint,BIFgα-open sets G and H in X such that A⊆G and B⊆H. Since G and H are BIFgα-open in X, then Y ∩ G and Y∩ H are BIFgα-open in Y. Also we have A ⊆ G, B ⊆ H which implies Y∩A⊆Y∩G,
Y∩B⊆Y∩H. So,A⊆Y∩G, B⊆Y∩H and (Y∩G)∩(Y∩H) = Y∩(G∩H) =0∼. Thus for each pair of disjoint,BIFgα-closed sets A, B in Y, there exist disjoint,BIFgα- open sets Y ∩ G and Y ∩ H such that A⊆Y∩G and B⊆Y∩H. Hence Y is BIFgα- normal

4.Bipolar intuitionistic fuzzy gα- almost normal spaces and Bipolar intuitionistic fuzzy gα- mildly normal spaces

Definition4.1. A space (X, τ) is said to be a BIF gα- almost normal space if for each BIF-closed set,A and BIF regular closed set,B such that A∩B=0 ∼ there exists disjoint, BIFgα-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 4.2. A space (X, τ) is said to be a BIF gα-mildly normal space if for any pair of disjoint BIF regular-closed sets A and B there exists disjoint,BIFgα-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Theorem 4.3. Every BIF-normal space is BIFgα almost-normal space.

Proof. Let A be a BIF-closed set and B be a BIF-regular closed set such that A∩B=0∼.Every BIF-regular closed set is BIF-closed set,therefore B is a BIF-closed set.Since (X,τ) is BIF-normal space,for the pair of disjoint,BIF-closed sets there exists BIF-open sets U and V suvh that A⊆U and B⊆V and U∩V=0∼.Every BIF- open set is BIFgα-open set.Thus,for the BIF-closed set A and BIF-regular closed set B such that A∩B=0∼ there exists a pair of disjoint,BIFgα-open sets U and V in X,such that A⊆U and B⊆V and U∩V=0∼.Therefore (X,τ) is BIF gα-almost normal space

The converse of the above theorem is not true as shown in the example below.

Example 4.4. Let X={a,b} and let τ={0∼,1∼,A,G} be a BIFT on X where

A=$\left\{x, \frac{a}{<0,0,1,-1>}, \frac{b}{<0.4,-0.4,0.4,-0.4>}\right\}$, B=$\left\{x, \frac{u}{<1,-1,0,0>}, \frac{v}{<0.4,-0.4,0.4,-0.4>}\right\}$.

Theorem 4.5. Every BIF-normal space is BIFgα-mildly normal space.

Proof. Let A and B be a BIF-regular closed sets such that A∩B=0∼.Every BIF- regular closed set is BIF-closed set,therefore A and B are BIF-closed sets.Since (X,τ) is BIF-normal space,for the pair of disjoint,BIF-closed sets there exists BIF- open sets U and V suvh that A⊆U and B⊆V and U∩V=0∼.Every BIF-open set is BIFgα-open set.Thus,for the pair of BIF-regular closed sets A and B such that A∩B=0∼ there exists a pair of disjoint,BIFgα-open sets U and V in X,such that A⊆U and B⊆V and U∩V=0∼.Therefore (X,τ) is BIF gα-mildly normal space.

The converse of the above theorem is not true as shown in the example below.

Example 4.6. Let X={a,b} and let τ={0∼,1∼,G,A} be a BIFT on X where X where

A=$\left\{x, \frac{a}{<0,0,1,-1>}, \frac{b}{<0.3,-0.3,0.3,-0.3>}\right\}$, B=$\left\{x, \frac{u}{<1,-1,0,0>}, \frac{v}{<0.3,-0.3,0.3,-0.3>}\right\}$.For the pair of disjoint BIF-regular closed sets 0∼ and G^c,such that $0\sim \cap G^c = 0\sim$ there exists a pair of disjoint,BIFgα-open sets 0∼ and A such that $0\sim \subseteq 0\sim$ and $G^c \subseteq A$.Therefore (X,τ) is BIF mildly gα-normal space.But for the BIF-closed sets 0∼ and A^c there does not exist disjoint BIF-open sets,satisfying the condition of BIF-normal space.Therefore BIF gα-mildly normal space need not be BIF-normal space.

Theorem 4.7. Let (X,τ) be a bipolar intuitionistic fuzzy topological space.

(i) Every BIF gα-normal space is BIF gα-almost normal space.

(ii) Every BIF gα-normal space is BIF gα-mildly normal space.

(iii) Every BIF gα-almost normal space is BIF gα-mildly normal space.

Proof.

*(i)*Let (X,τ) be a BIF gα-normal space.Let A and B be two BIF-closed sets out of which one is a BIF-regular closed set such that A∩B=0∼ in X.Every BIF- regular closed set is BIF-closed and since (X,τ) is BIF gα-normal space,for each pair of disjoint,BIF-closed sets there exists a pair of disjoint,BIF gα-closed sets U and V such that $A \subseteq U$ and $B \subseteq V$ .Therefore (X,τ)is BIF gα-almost normal space.

(ii)Let (X,τ) be a BIF gα-normal space.Let A and B be two disjoint, BIF- regular closed sets such that A∩B=0∼ in X.Every BIF-regular closed set is BIF-closed and since (X,τ) is BIF gα-normal space,for each pair of disjoint,BIF-closed sets there exists a pair of disjoint,BIF gα-closed sets U and V such that $A \subseteq U$ and $B \subseteq V$ .Therefore (X,τ)is BIF gα-mildly normal space.

(iii)Let (X,τ) be a BIF gα-almost normal space.Let A and B be two BIF-closed sets out of which one is a BIF-regular closed set such that A∩B=0∼ in X.Every BIF-regular closed set is BIF-closed and since (X,τ) is BIF almost gα-normal space,for each BIF-closed set,A and BIF regular closed set,B such that A∩B=0 ∼,there exists BIF gα-closed sets U and V such that $A \subseteq U$ and $B \subseteq V$ .Therefore (X,τ)is BIF gα-almost normal space. The converse of the above theorem is not true as shown in the examples below.

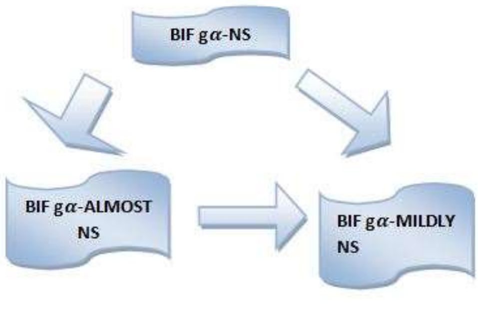
Example 4.8. Let X={a, b} and let τ = {0 ∼, 1 ∼, G, A, B, C, D} be a BIFT on X where A=$\left\{x, \frac{a}{<0,-1,1,0>}, \frac{b}{<1,0,0,-1>}\right\}$, B=$\left\{x, \frac{u}{<1,0,0,-1>}, \frac{v}{<0,-1,1,0>}\right\}$, C=$\left\{x, \frac{a}{<0,-1,1,0>}, \frac{b}{<1,-1,0,0>}\right\}$, D=$\left\{x, \frac{a}{<0,0,1,-1>}, \frac{b}{<1,0,0,-1>}\right\}$, G=$\left\{x, \frac{a}{<0,0,1,-1>}, \frac{b}{<1,-1,0,0>}\right\}$.For the disjoint,bipolar intuitionistic fuzzy closed set 0∼ and BIF-regular closed set B^c,such that 0∼ ∩ B^c = 0 ∼ there exists a pair of disjoint,BIFgα-open sets 0∼ and B such that 0∼ ⊆ 0∼ and $B^c \subseteq C$.Therefore (X,τ) is BIF almost gα-normal space.But for the pair of BIF-closed sets,0∼ and D^c there does not exist disjoint BIFgα-open sets,satisfying the condition of BIFgα-normal space.Therefore BIF gα-almost normal space need not be BIFgα-normal space.

Example 4.9. Let X={a, b} and let τ = {0 ∼, 1 ∼, A, B, C} be a BIFT on X where A=$\left\{x, \frac{a}{<0,0,1,-1>}, \frac{b}{<0.5,-0.5,0.5,-0.5>}\right\}$, B=$\left\{x, \frac{u}{<1,-1,0,0>}, \frac{v}{<0,0,1,-1>}\right\}$, C=$\left\{x, \frac{a}{<1,-1,0,0>}, \frac{b}{<0.5,-0.5,0.5,-0.5>}\right\}$.For the pair of disjoint bipolar intuitionistic fuzzy regular closed sets

BIFgα-open sets,satisfying the condition of BIFgα-normal space.Therefore BIF gα-mildly normalspace need not be BIF gα-normal space.

Example 4.10. Let X={a, b} and let τ = {0 ∼, 1 ∼, G, A} be a BIFT on X where A=$\left\{x, \frac{a}{<1,-1,0,0>}, \frac{b}{<0.5,-0.5,0.5,-0.5>}\right\}$, G=$\left\{x, \frac{u}{<0,0,1,-1>}, \frac{v}{<0.5,-0.5,0.5,-0.5>}\right\}$..For the pair of disjoint BIRCS,0∼ and G^c,such that 0∼ ∩ G^c=0∼ there exists a pair of disjoint,BIF gαOS,0∼ and A such that 0∼ ⊆ 0∼ and G^c ⊆A.Therefore (X,τ) is BIFgαNS.But for the BIFCS, 0∼ and BIFCS A^c there does not exist disjoint BIFgαOS,satisfying the condition of BIFgα almost NS.Theefore BIFgα mildly NS need not be BIFgα almost NS.

Figure 4.1: Inter-relationship between BIFgα-normal space, BIFgα mildly normalspace,BIFgα almost normal space



Theorem 4.11. BIF almost-normality is a topological property.

Proof. Similar to theorem 3.7

Theorem 4.12. BIF mild-normality is a topological property.

Proof. Similar to theorem 3.7

Theorem 4.13. For a BIFTS(X,τ) the following are equivalent

(i)X is BIF gα-almost normal space.

(ii) For every pair of BIF sets M and N,one of which is BIF-open and the other is BIF-regular open whose union is 1∼,there exists BIF gα-closed sets A and B such that $A \subseteq M$ and $B \subseteq N$ and A∪B=1∼.

(iii) For every BIF-closed set A and every BIF-regular open set B containing A,there exists a BIF gα-open set N such that A⊆ $N \subseteq BIFg\alpha cl(N) \subseteq B$.

Proof. Proof is similar to theorem 3.6

Definition 4.14. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called BIF r-map,if $f^{-1}(V)$ is BIF-regular open in X for every BIF- regular open set V in Y.

Corollary 4.15. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a completely BIF- continuous,strongly BIFα-open and almost BIFgα-irresolute surjection from an BIFgα-almost normal space X onto the space Y,then Y is almost BIFgα-normal.

Theorem 4.16. For a BIFTS(X,τ) the following are equivalent

(i)X is BIF gα-mildly normal space.

(ii) For every pair of BIF regular open sets M and N, whose union is 1∼,there exists BIF gα-closed sets A and B such that $A \subseteq M$ and $B \subseteq N$ and A∪B=1∼.

(iii) For any BIF regular -closed set A and every BIF-regular open set B containing A, there exists a BIF gα-open set N such that A⊆ $N \subseteq BIFg\alpha cl(N) \subseteq B$.

Proof. Proof is similar to theorem 3.6

Theorem 4.17. If $f : X \rightarrow Y$ is BIF-completely continuous injective,BIFgα- open function and X is BIF gα-mildly normal space, then F is BIFgα-normal.

Proof. Let E and F be disjoint BIF-closed sets in F. Since f is BIF-completely continuous bijective, $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint BIF-regular closed sets in X. By the definition of BIF gα-mildly normality of X, there exist disjoint BIFgα-open sets U and V such that $f^{-1}(E) \subseteq$ U and $f^{-1}(F) \subseteq$ V. This implies, E ⊆ f(U) and F ⊆ f(V). Since f is BIFgα-open,injective f(U) and f(V) are disjoint BIFgα-open sets in F.Therefore, F is BIFgα-normal.

Theorem 4.18 If $f : X \rightarrow Y$ is BIFgα-continuous,almost closed,surjection and Y is BIFgα-normal space then X is BIFgα-mildly normal space.

Proof. Let A and B be disjoint BIF-regular closed sets in X.Since f is almost closed map,f(A) and f(B) are BIF-closed in Y.Since Y is BIFgα-normal space,there exists disjoint, BIFgα-open sets U and V in Y such that f(A)⊆U and f(B)⊆V.Since f is BIFgα-continuous,$f^{-1}(U)$ and $f^{-1}(V)$ are BIFgα-open sets containing A and B respectively.Therefore (X,τ) is BIFgα-mildly normal space.

5. Bipolar intuitionistic fuzzy gα-strongly normal spaces

Definition 5.1. A space (X, τ) is said to be a BIF gα-strongly normal space if for any pair of disjoint BIFgα-closed sets A and B there exists disjoint, BIFgα-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Proposition 5.2. Let (X,τ) be a bipolar intuitionistic fuzzy topological space.

(i) Every BIF gα-strongly normal space is BIF-normal space.

(ii) Every BIF gα-strongly normal space is BIF gα-mildly normal space.

(iii) Every BIF gα- strongly normal space is BIF gα-almost normal space.

(iv) Every BIF gα- strongly normal space is BIF gα-normal space.

Proof:

(i) Let (X,τ) be a BIF gα- strongly normal space.Let A and B be two disjoint BIF- closed sets in X.Every BIF-closed set is BIFgα-closed set.Therefore A and B are BIFgα closed sets in X and they are disjoint.Since (X,τ) is BIF gα- strongly normal,for each disjoint pair of BIFgα closed set A and B, there exist disjoint BIFgα-open sets U and V such that A⊆U and B⊆V. Thus (X,τ) is BIF-normal space.

(ii) Let (X,τ) be a BIF gα- strongly normal space.Let A and B be two disjoint,BIFgα- closed sets in X.Since (X,τ) is gα- strongly normal,for each disjoint pair of BIFgα closed set A and B, there exist disjoint BIFgα-open sets U and V such that A⊆U and B⊆V. Thus (X,τ) is BIF gα-mildly normal space.

(iii) Let (X,τ) be a BIF gα-strongly normal space.Let A and B be two disjoint,BIF- closed sets out of which one is BIF-regular closed in

X.Every BIF-regular closed is BIFgα-closed set and every BIF-closed set is BIFgα closed set.Since (X,τ) is gα-strongly normal,for each disjoint pair of BIFgα closed set A and B, there exist disjoint BIFgα-open sets U and V such that A⊆U and B⊆V. Thus (X,τ)is BIF gα-mildly normal space.

(iv) Let (X,τ) be a BIF gα-strongly normal space.Let A and B be two disjoint,BIF- closed sets in X.Every BIF-closed set is BIFgα closed set in X.Since (X,τ) is BIFgα-strongly normal,for each disjoint pair of BIFgα-closed sets A and B, there exist disjoint BIFgα-open sets U and V such that A⊆U and B⊆V. Thus (X,τ)is BIF gα-normal space.

The converse of the above theorem is not true as shown in the examples below.

Example 5.3. Let X={a, b} and let τ = {0 ∼, 1 ∼, A, B, C} be a BIFT on X where A=$\left\{x, \frac{a}{<0,0,1,-1>}, \frac{b}{<0.5,-0.5,0.5,-0.5>}\right\}$,

B=$\left\{x, \frac{u}{<1,-1,0,0>}, \frac{v}{<0,0,1,-1>}\right\}$, C=$\left\{x, \frac{a}{<1,-1,0,0>}, \frac{b}{<0.5,-0.5,0.5,-0.5>}\right\}$.For the pair of disjoint,BIF-closed sets 0∼ and C^c there exists a pair of disjoint,BIFgα-open sets 0∼ and A such that 0∼ ⊆ 0∼ and $C^c \subseteq A$.Therefore (X,τ) is BIF-normal space.But for the pair of BIFgα-closed sets,0∼ and B^c there does not exist disjoint BIFgα-open sets, satisfying the condition of BIF gα- strongly normal space.Therefore BIF-normal space need not be BIF gα- strongly normal space.

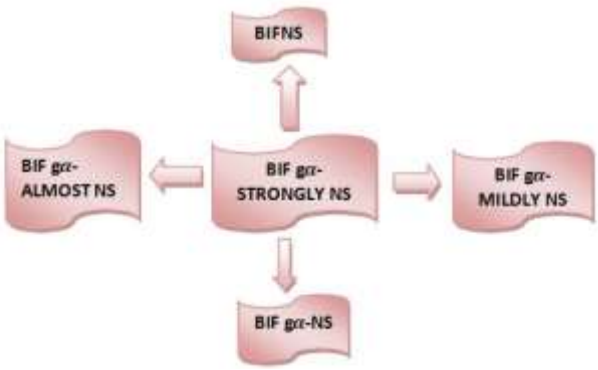
Example 5.4. Let X={a, b} and let τ = {0 ∼, 1 ∼, A,G} be a BIFT on X where A=$\left\{x, \frac{a}{<10,0,-1>}, \frac{b}{<0,-1,1,0>}\right\}$, G=$\left\{x, \frac{u}{<0,-1,1,0>}, \frac{v}{<1,0,0,-1>}\right\}$, For the pair of disjoint BIF-regular closed sets 0∼ and G^c,such that 0∼ ∩ G^c = 0 ∼ there exists a pair of disjoint,BIFgα-open sets 0∼ and A such that 0∼ ⊆ 0∼ and $G^c \subseteq A$. Therefore (X,τ) is BIF gα-mildly normal space.But for the pair of disjoint,BIFgα-closed set 0∼ and A^c there does not exist disjoint BIFgα-open sets,satisfying the condition of BIF gα- strongly normal space.Therefore BIF gα-mildly normal space need not be BIF gα- strongly normal space.

Example 5.5. Let X={a, b} and let τ = {0 ∼, 1 ∼, G, A, B, C, D} be a BIFT on X where A=$\left\{x, \frac{a}{<0,-1,1,0>}, \frac{b}{<1,0,0,-1>}\right\}$, B=$\left\{x, \frac{u}{<1,0,0,-1>}, \frac{v}{<0,-1,1,0>}\right\}$,

C=$\left\{x, \frac{a}{<0,-1,1,0>}, \frac{b}{<1,-1,0,0>}\right\}$, D=$\left\{x, \frac{a}{<0,0,1,-1>}, \frac{b}{<1,0,0,-1>}\right\}$, G=$\left\{x, \frac{a}{<0,0,1,-1>}, \frac{b}{<1,-1,0,0>}\right\}$. For the disjoint,BIF closed set 0∼ and BIF-regular closed set A^c,such that 0∼ ∩ A^c = 0 ∼ there exists a pair of disjoint,BIFgα-open sets 0∼ and B such that 0∼ ⊆ 0∼ and $A^c \subseteq B$.Therefore (X,τ) is BIF gα-almost normal space.But for the pair of BIF-closed sets,0∼ and G^c there does not exist disjoint BIFgα-open sets,satisfying the condition of BIF gα- strongly normal space.Therefore BIF gα-almost normal space need not be BIF gα-strongly normal space.

Example 5.6. Let X={a, b} and let τ = {0 ∼, 1 ∼, A, B} be a BIFT on X where A=$\left\{x, \frac{a}{<0,-1,1,0>}, \frac{b}{<0,-1,1,0>}\right\}$B=$\left\{x, \frac{u}{<1,0,0,-1>}, \frac{v}{<1,0,0,-1>}\right\}$ .For the pair of disjoint,BIF- closed sets 0 and B^c there exists a pair of disjoint,BIFgα-open sets 0 and A such that 0 ⊆ 0 and $B^c \subseteq A$.Therefore (X,τ) is BIFgα-normal space.But for the pair of BIF-closed sets,0 and A^c there does not exist disjoint BIFgα-open sets,satisfying the condition of BIF gα-normal space.Therefore BIFgα-normal space need not be BIF strongly gα-normal space.

Figure 5.1: Inter-relationship between strongly BIFgα-normal space,BIF-normal space,BIFgα-normal space,mildly BIFgα-normal space,almost BIFgα-normal space



Theorem 5.7. BIF strong-normality is a topological property.

Proof. Similar to theorem.

Definition 5.8. A space (X, τ) is said to be a BIF α-normal space if for any pair of disjoint BIF-closed sets A and B there exists disjoint,BIFα-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Theorem 5.9. Let (X,τ) be a bipolar intuitionistic fuzzy topological space. If (X,τ) is

(i) $T_{g\alpha}$-space and BIFgα-normal, then it is BIFgα-strongly normal.

(ii) T_r-space and BIFgα-mildly normal, then it is BIFgα-strongly normal.

(iii) T_r-space,$T_{g\alpha}$-space and BIFgα-almost normal, then it is BIFgα-strongly normal.

(iv) $T_{g\alpha}$-space and BIF-normal, then it is BIFgα-strongly normal.

Proof.

(i) Let A and B be disjoint,BIFgα-closed sets in X.Since (X,τ) is a $T_{g\alpha}$-space,every BIFgα-closed set is BIF-closed in X.Therefore A and B are disjoint,BIF-closed sets in X.Since (X,τ) is BIFgα-normal space,for each pair of disjoint,BIF-closed sets A and B,there exists a pair of disjoint,BIFgα-open sets U and V such that A⊆U and B⊆V.Therefore (X,τ) is BIFgα-strongly normal space.

(ii) Let A and B be disjoint,BIFgα-closed sets in X.Since (X,τ) is a T_r-space,every BIFgα-closed set is BIF regular-closed in X.Therefore A and B are disjoint,BIF regular-closed sets in X.Since (X,τ) is mildly BIFgα-normal space,for each pair of disjoint,BIF regular-closed sets A and B,there exists a pair of disjoint,BIFgα- open sets U and V such that A⊆U and B⊆V.Therefore (X,τ) is BIFgα- strongly normal space.

(iii) Let A and B be disjoint,BIFgα-closed sets in X.Since (X,τ) is a T_r-space,every BIFgα-closed set is BIF regular-closed in X and.Therefore A BIF regular-closed set in X.Since X is a $T_{g\alpha}$-space,BIFgα-closed set is BIF-closed in X.Since (X,τ) is almost BIFgα-normal space,for each pair of disjoint,BIF regular-closed set A and BIF-closed set B,there exists a pair of disjoint,BIFgα-open sets U and V such that A⊆U and B⊆V.Therefore (X,τ) is BIFgα- strongly normal space.

(iv) Let A and B be disjoint,BIFgα-closed sets in X.Since X is a $T_{g\alpha}$-space,BIFgα- closed set is BIF-closed in X.Since (X,τ) is BIF-normal space,for each pair of disjoint,BIF -closed set A and B,there exists a pair of disjoint,BIF-open sets U and V such that A⊆U and B⊆V.Every BIF-open set is BIFgα-open set.Therefore (X,τ) is BIFgα- strongly normal space.

Theorem 5.10. Let $f : (X, \tau) \to (Y, \sigma)$ be a BIF-bijective map.If f is a BIFgα- open map,BIF-continuous and (X,τ) be BIFgα- strongly normal then (Y,σ) is BIFgα-normal.

Proof. Let (X,τ) be a BIF-normal.Let A and B be two disjoint,BIF-closed sets in Y.Since f is BIF-continuous,$f^{-1}(A)$ and $f^{-1}(B)$ are BIF-closed in X.Every closed set is BIFgα closed ,therefore $f^{-1}(A)$ and $f^{-1}(B)$ are BIFgα-closed in X.Since (X,τ) is BIFgα- strongly normal,there exists disjoint,BIFgα-open sets U and V such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$ .That is,$f\,[f^{-1}(A)] \subseteq f\,(U\,)$ and $f\,[f^{-1}(B)] \subseteq f\,(V\,)$,that is $A \subseteq f\,(U\,)$ *and* $B \subseteq f\,(V\,)$.Since f is BIFgα-open map,f(U) and f(V) are BIFgα-open sets in Y such that $A \subseteq f\,(U\,)$ and $B \subseteq f\,(V\,)$.Since f is injective,$f\,(U\,) \cap f\,(V\,) = f\,(U \cap V\,) = f\,(0\,\sim) = 0\,\sim$.Thus (Y,$\sigma$) is BIFg$\alpha$-normal.

Theorem 5.11. Let $f : (X, \tau) \to (Y, \sigma)$ be a BIF-injective map.If f is a BIFgα- closed map,BIF-continuous and (Y,σ) be BIFgα-normal, then (X,τ) is BIFgα- strongly normal.

Proof. Let E and F be two disjoint,BIF-closed sets in X.Since f is a BIFgα-closed map,f(E) and f(F) are BIFgα-closed sets in Y.Since,(Y,σ) is BIFgα-normal,there exists disjoint,BIFgα-open sets G and H such that f(E)⊆ G and f(F)⊆ H.This implies,E⊆ $f^{-1}(G)$ and F⊆ $f^{-1}(H)$.Since f is BIFgα-continuous,$f^{-1}(G)$ and $f^{-1}(H)$ are BIFgα-open sets in X.Further,$f\,(G) \cap f\,(H) = f\,(G \cap H) = f\,(0\,\sim) = 0\,\sim$.Thus (X,$\tau$) is BIFgα-strongly normal.

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