



Brief Considerations About the Theory of General Relativity

*Celso Luis Levada¹, Osvaldo Missiato¹, Rosangela Oliveira Colabone¹, Antonio Luiz Ferrari¹
Huemerson Maceti², Ivan José Lautenschleguer²*

¹Air Force Academy of Pirassununga-Brazil

²Herminio Ometto Foundations-Uniararas-Brazil

ABSTRACT

The General Theory of Relativity (GRT) is a theory capable of describing the universe in the presence of gravitational fields, amplifying and generalizing the concepts involved in the Special Theory of Relativity. It allows us to interpret gravitational fields as a deformation in Minkowski space-time caused by distributions of matter. Said theory was also able to predict and describe phenomena that Newton's gravitation was unable to explain, such as the deviation in the trajectory of starlight due to the presence of the sun (gravitational lensing) and the more accurate calculation of Mercury's perihelion. The prediction of gravitational waves, already detected! The GRT also plays a fundamental role in cosmology and with it was possible to describe our expanding universe. This text intends to present and discuss the main aspects for understanding the GRT.

INTRODUCTION

In the General Theory of Relativity, EINSTEIN⁽¹⁾ redefines gravity, examining the influence of space and time on the gravitational attraction between bodies. The basis of the general theory of relativity is the Equivalence Principle, which can be stated as follows: Let us consider two frames of reference: 1st an unaccelerated inertial frame of reference in which there is a uniform gravitational field and 2nd a uniformly accelerated frame of reference but in which there is no gravitational field. Accelerated motion in the absence of a gravitational field is indistinguishable from unaccelerated motion in the presence of a gravitational field. The local effects of gravity are the same as being in an accelerated frame of reference. These two frames of reference are physically equivalent. This principle also has another formula that equates acceleration with gravity. It is not possible to distinguish, in a small region of space-time, the difference between the acceleration of an object and the existence of traditionally postulated gravitational force. In this context, EINSTEIN⁽¹⁾ abandons the Newtonian notion of force and introduces the notion of curved space. Bodies produce a curvature of space around them, and the greater the mass of the body, the greater the curvature. In other words, for the general theory of relativity, gravity is simply a warping of spacetime caused by a very massive object. The EINSTEIN⁽¹⁾ Theory predicts that light is also attracted to bodies, but this effect would be small and, thus, could only be observed when light passed close to massive bodies, such as the Sun. Therefore, large concentrations of matter lead to large deformations of space-time, from which not even light can escape, an example of this is the existence of Black Holes. Combining special relativity with quantum physics, quantum electrodynamics (EDQ or QED) was formulated, which is a relativistic quantum field theory. To complete the studies, it is necessary to reconcile Quantum Mechanics with the General Theory of Relativity, but the calculations that take these two great theories into account produce results that are incompatible with physical reality.

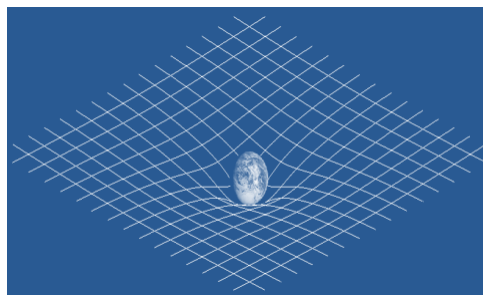


Fig1- Illustration of the curvature of space-time. Source: <https://pt.wikipedia.org/wiki/Espaco-tempo>

In general, there is a substitution of the four-dimensional mathematical space from the Minkowski geometry of Special Relativity to the Riemann geometry of General Relativity. Minkowski geometry ¹⁾ adds a fourth axis to the space-time continuum, Riemann geometry curves all four axes. To explore the issues inherent to the theory of relativity, one must study the Schwarzschild metric.

METRIC

The relationships that provide the distance between two points in space, like the expressions for “ds” that we obtained in this text, are called “metrics”, thus allowing us to obtain distances in a given geometry. Using the metric one can define notions such as distances, volume, angles, past, future and curvature. As can be deduced from the text by BERGMANN²⁾, in general relativity the metric tensor, or simply metric, transmits all information about the causal and geometric structure of space-time, being considered a second-order tensor field that describes the curvature of space-time. General Relativity especially is focused on finding the metric for a given configuration of matter and then calculating the properties of that metric and how other issues will behave when under the influence of that metric. As gravitation is seen in the General Theory of Relativity as a result of curvature, the metric tensor is closely related to the gravitational field. It is common to think of metric as something that comes along with spacetime itself; integral part of its description, no less fundamental than space or time. The metric tensor is a feature of a coordinate system that, among other things, describes how distances are measured in that coordinate system. The proposition of a metric for special and general relativity makes it possible to determine the movement of particles. We can introduce the concept of metric as follows: in the usual Euclidean space, that is, the three-dimensional space that, in a Cartesian coordinate system x, y, z) gives the distance Δs between two points separated by Δx, Δy, Δz) the following value

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

As we read in CAMENZIND³⁾, in the case of infinitesimal distances we have

$$ds^2 = dx^2 + dy^2 + dz^2$$

If we are working in another coordinate system, such as, for example, the cylindrical z, r, φ) or spherical r, θ, φ) system, this would be written, respectively,

$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2$$

$$ds^2 = dr^2 + r^2 [d\theta^2 + \text{sen}^2 \theta d\phi^2]$$

If we were in the context of special relativity, in which time is also considered as a coordinate, we would have the Minkowski metric, given by

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$$

Let us see, then, how we can write the metric for the specific case of a black hole. Let us assume, for simplicity, that the generated spacetime is static, so that the metric cannot depend on time, as well as spherically symmetric, following the treatment given by VISSER⁴⁾, we can, in this way, write the metric as

$$ds^2 = -e^{\phi(r)} dt^2 + \frac{1}{1 - \frac{br}{r}} dr^2 + r^2 d\theta^2 + \sin^2 \theta d\phi^2$$

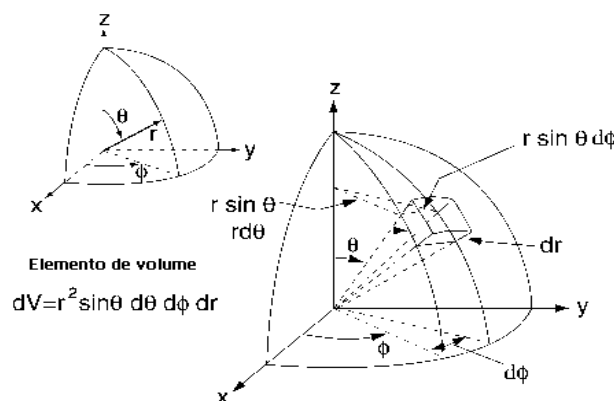


Figure 2 represents the variables r, θ and φ used in the previous equation

Source: disciplinas.ist.utl.pt/qgeral/biomedica/eq_schro.doc

The general theory of relativity also predicts the existence of gravitational waves, which are ripples in the general geometry of space and time produced by moving masses. Fluctuation of spacetime curvature that is propagated as a wave.

EINSTEIN FIELD EQUATIONS

In 1915, TONG 5) had developed the so-called Einstein Field Equations. The Einstein field equations are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Essentially, the equation tells us how a given amount of mass and energy warps spacetime. The left side of the equation,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu},$$

describes the curvature of spacetime whose effect we perceive as the gravitational force. It is the analogue of the F term on the left side of Newton's equation $F = G \frac{m_1 m_2}{r^2}$). On the other hand, the term $T_{\mu\nu}$ on the right side of the equation describes everything there is to know about the way mass, energy, momentum, and pressure are distributed throughout the Universe. It is like the term in Newton's equation, though much more complex than that. All these things are needed to figure out how space and time bend. The technical term $T_{\mu\nu}$ is the energy-momentum tensor. The constant G that appears on the right side of the equation is Newton's constant and c is the speed of light. And as for the Greek letters μ, ν that appear as subscripted? To understand⁵⁾ what they mean, first note that spacetime has four dimensions. There are three dimensions of space and one dimension of time, the said Greek letters indicating each of these four dimensions and their various combinations. The Greek letters μ and ν are indices, which can take the values 0, 1, 2 or 3. So, in fact, the above equation hides an entire collection of equations corresponding to the possible combinations of values that such indices can receive. The value of 0 corresponds to time and the values 1, 2 and 3 to the three dimensions of space.

For example,⁵⁾:

$$R_{00} - \frac{1}{2}Rg_{00} = \frac{8\pi G}{c^4} T_{00}, \quad R_{01} - \frac{1}{2}Rg_{01} = \frac{8\pi G}{c^4} T_{01}, \quad R_{11} - \frac{1}{2}Rg_{11} = \frac{8\pi G}{c^4} T_{11}$$

And so on.

Like each of μ, ν can take on four values, this gives a total of $4 \times 4 = 16$ equations. However, the equation $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ is the same as the equation $R_{\nu\mu} - \frac{1}{2}Rg_{\nu\mu} = \frac{8\pi G}{c^4} T_{\nu\mu}$ (This reduces the total number of equations to ten TONG5)). That is, what looks like an equation is actually a set of ten coupled nonlinear partial differential equations. They cannot be resolved separately. It is very difficult to solve Einstein's equations in any kind of generality, and it is usually necessary to make some assumptions and simplifications. To solve these equations, we can break them down into simpler equations. One of the simplifying assumptions assumes that the metric has a significant degree of symmetry, which considerably facilitates solving the equation. That is, the best simplification is to use a metric that has symmetric properties. With the introduction of the cosmological constant we have

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

In this equation we have that: $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, which specifies the geometry of spacetime, Λ is the cosmological constant, G is Newton's gravitational constant, c is the speed of light in vacuum and $T_{\mu\nu}$ is the voltage-energy tensor.

STATIC OR EXPANDING?

EINSTEIN¹⁾ assumed that the universe was static and unchanging and added the cosmological constant to his equations to contain gravity so that his equations had a solution that agreed with the static model. It was a way of counterbalancing the attractive effects of gravity on ordinary matter, which would otherwise cause a static and spatially finite universe to collapse or expand forever. Einstein's static model was the first relativistic cosmological model and, in addition to being static, it was finite with spherical spatial symmetry. This model, currently considered outdated, corresponded to a very convenient origin of a series of theoretical proposals that aimed to understand the general structure of the universe, from the point of view of space and time. However, according to DAMINELI⁶⁾, after some time, Hubble proved that the universe is indeed expanding, so it is not static, a fact that led Einstein to reject his own suggestion. But much has changed and currently, the cosmological constant can be interpreted as the "energy density of vacuum", a source of energy and momentum that is present even in the absence of matter fields. This interpretation is important because quantum field theory predicts that a vacuum must have some kind of energy and momentum. Therefore, the existence of a cosmological constant is therefore equivalent to the existence of a non-zero vacuum energy. I would never think about it. What a great idea!

WHAT'S WRONG WITH GENERAL RELATIVITY?

This text is based on the article by FREIBERGER⁷⁾, which begins by saying that general relativity correctly describes what we observe on the scale of the solar system and everything works wonderfully on this scale and has been tested. Problems arise when we look at the Universe on very small or

exaggeratedly large scales. The problem is that general relativity cannot be easily quantified; we cannot find a quantum counterpart in the same way that we find electromagnetism. In fact, the problem of finding a quantum theory of gravity is so challenging and so important that many consider it one of the biggest problems in modern physics. Another mystery arises when we look at the Universe as a whole. We know that the Universe is expanding, that is, stars and galaxies are moving away from each other, and observations of distant objects have also shown that this expansion is accelerating. General relativity cannot explain what causes this acceleration. So we must admit that there is a mysterious form of energy that drives acceleration, called dark energy. Quantum physics offers an explanation for dark energy (FREIBERGER⁷⁾). According to this theory, a vacuum does not really exist in the sense that we generally understand it as empty space. Instead, particles constantly pop in and out of existence, resulting in a vacuum energy, an energy of space itself, that may be driving the accelerated expansion. There is a number that measures this vacuum energy, called the cosmological constant, whose value the study of particle physics, can estimate. The problem is that this estimated value is higher than what the observations suggest. The values, both observed and theoretical, show a huge difference, so remarkably large that something is wrong. These two problems, dark energy and the need to quantify gravity, provide some of the motivations for us to continue to study general relativity. In relation to black holes, we can understand that, theoretically, there is a singularity identified by the fact of having a point mass of zero volume and, therefore, infinite density. It is a physical impossibility and is, according to the theory, at the center point of a black hole. So it follows that black holes themselves are physical impossibilities. According to this solution, under certain conditions, the mass will undergo an irreversible gravitational collapse that leads to a singularity, creating an event horizon within which only Hawking radiation can escape. We believe that it is not the theory of relativity that is wrong, but what may be wrong is the physical interpretation of it in relation to certain phenomena, and many researchers think it is incomplete. It is a classical theory, so it ignores any quantum aspect, so phenomena such as dark matter are, in the view of other scientists, points that indicate deficiencies in general relativity (REDD⁸⁾; FERREIRA⁹⁾).

FINAL CONSIDERATIONS

According to SCHUTZ¹⁰⁾, Einstein's equations are very difficult to solve, so supercomputers are needed to find solutions and propose new solutions. One of the great challenges today is figuring out what happens to space-time when two very heavy objects, such as black holes, collide. How do we know that Einstein's theory is correct? In the hundred years since its publication, the theory has passed every test it has been subjected to. Despite its slightly esoteric nature, it is crucial in things most of us rely on a daily basis, like the GPS features on our smartphones and the sat nav devices in our cars. The theory opens up some new questions, which is why some physicists feel it needs to be modified. But whether or not this is really necessary, there is no doubt that general relativity is one of the most amazing achievements in the history of science. Einstein's general theory of relativity is the foundation for our understanding of black holes and the Universe at its largest scales. In general relativity, the Newtonian concept of gravitational force is abolished, being replaced by a new notion, that of the curvature of space-time. This in turn leads to well-tested predictions of phenomena such as the bending of light and gravitational time dilation, and others, such as gravitational waves, which are only now entering the direct detection regime. Einstein's equivalence principle is the basis for arriving at the GR field equations, which allows the application of general relativity to stellar collapse, neutron stars, black holes, gravitational waves, and cosmology. Regarding the black hole, we can say that it is a large body of matter that is so dense that almost nothing can escape its gravitational pull, at a certain distance, known as the Schwarzschild radius. We said next to nothing because ELERT¹¹⁾ mentions that the Hawking effect is the first combination of quantum theory with general relativity, he says that when considering quantum effects, a black hole radiates like a blackbody at temperature :

$$T = \frac{\hbar c^3}{8\pi k G M}$$

In the above formula, T is the temperature, h is the Planck constant, c is the speed of light, G is the gravitational constant, k is the Boltzmann constant, and M is the mass of the black hole. Probably, in a next article, we will discuss in more depth, the relationship of Hawking radiation with the black hole. In our view, there must be a theory between M.Q and R.G that can answer certain questions as a limiting case of both. That is, they are not wrong, but they are valid in your domain. See that in fluids turbulence is not yet adequately answered. In general relativity gravity is not a force as we are used to thinking. And what then? Gravity represents a distortion of space caused by the distribution of mass via curvature, that is, the greater the mass of a system, the greater the curvature and, therefore, the manifestation of acceleration.

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