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Effect of Porosity and Permeability on Rayleigh-Taylor Instability in Nanofluids

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A B S T R A C T

This study focuses on exploring the effects of porosity and permeability on the Rayleigh-Taylor instability in a configuration that is constituted with two horizontal layers of nanofluids. Normal mode technique is used to develop the dispersion relation for this problem. Both stable and unstable mode of the configuration is discussed in detail. The critical values of growth rate parameter for porosity are obtained and tabulated for quantitative analysis. For unstable mode, it is observed that porosity provides the stabilization effect to the configuration in the presence/absence of nanoparticles but the effect of stabilization is more dominant in the presence of nanoparticles. Permeability is delivering destabilization on the configuration but rate of pouring destabilization is slower with mix of nanoparticles.

Keywords: Nanofluids; Rayleigh-Taylor instability; Porous medium; Permeability.

Nomenclature

k_1	wave number	$\boldsymbol{u}$	velocity
n	growth rate parameter	ϕ	volume fraction of nanoparticles
ε	porosity	μ	viscosity of fluid
ρ_p	density of nanoparticles	ρ	density of nanofluid
K_m	permeability	ρ_f	density of basefluid

1.1Introdution

The Rayleigh-Taylor instability is a fingering instability that occurs at the interface of two fluids of different densities when the light fluid pushes the heavy fluid. Lord Rayleigh (1900) investigated the equilibrium characteristics of an inviscid incompressible fluid of variable density stratified in horizontal planes. Rayleigh demonstrated that $\frac{d\rho}{dz}$ must be negative everywhere in the flow domain in order for a stratified heterogeneous fluid to be stable. Chandrasekhar (1981) extended Rayleigh's treatment to include the effect of viscosity. If $\frac{d^2\mu}{dz^2}$ is positive everywhere inside the flow domain, then the oscillatory modes are stable. In addition, Chandrasekhar developed a variational principle for resolving the underlying characteristic value problem.

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Selig (1964) developed an alternative calculation scheme to address the same problem and established that the restriction on $\frac{d^2\mu}{dz^2}$ imposed by Chandrasekhar for the validity of the principle of exchange of stabilities is not required. There are diverse situations for occurrence of Rayleigh- Taylor instability. Some of them are mentioned below:

1. Natural phenomena:
    a) When the outer portion of the collapsed core of the massive star is overturned
    b) Bubble formation
    c) The formation of highly luminous twin exhaust jets in rotating gas clouds in an external gravitational potential.
2. Technological Example
    a) Laser implosion of deuterium-tritium fusion.
    b) Electromagnetic implosion of a metal.

In 1950, Taylor (1950) supported this experiment by stating the occurrence of instability in accelerated fluids. Bhatia (1974) was a pioneer in the development of RTI for viscous compressible plasma. Exact solutions for semi-infinite plasma in the presence of a magnetic field were obtained using the variational principle and interpreted to show that both viscosity and magnetic field have a stabilizing influence, whereas compressibility has a destabilizing influence. Different researchers are considering different types of fluids to investigate the effect of RTI in fluids. Chandrasekhar (1981) discussed RTI for two different densities of incompressible and viscous fluids. Following that, Sharma et al. (2001) investigated RTI of Rivlin-Ericksen elastico-viscous fluids in the presence of a magnetic field in a porous medium.

Some researchers, including Prajapati and Chhajlani (2010) and Sharma et al. (2016), demonstrated the analysis of various instabilities in porous media. Awasthi (2014) concentrated on two viscous, incompressible fluids in porous media that were enclosed between two horizontal cylindrical surfaces. For nonlinear RTI of cylindrical flow, he emphasised that flow through porous media is more stable than pure flow. He observed that heat and mass transfer increase system stability while porosity decreases it. Sharma et al. (2016) investigated two superimposed incompressible fluids with suspended dust particles in the presence of a magnetic field, and they discussed that porosity causes for stabilizing influence in magnetized and unmagnetized cases. Recently, Ahuja and Girotra (2021) investigated the Rayleigh-Taylor instability in nanofluids while accounting for surface tension, Atwood number, and nanoparticle volume fraction.

The current study is a reasonable attempt to analyze RTI in nanofluids via porous medium. This article contains all previously unexplored RTI insights. This research involves the development of a mathematical expression for RTI in nanofluids that incorporates various parameters such as surface tension, porosity, permeability, and nanofluid volume fraction. The modified dispersion relation is obtained using the normal mode technique and analyzed on three different platforms with different parameter characteristics.

2. Mathematical Formulation of Problem and Solution

The considered system has two infinite layers of different nanofluids superimposed on each other and separated by a plane $z = 0$. It is assumed that nanoparticles have uniform shape and size and having homogeneous distribution among these two layers when considered in base fluids. The region under investigation is supposed to have two layers say $z < 0$ as lower layer and $z > 0$ as upper layer having $\rho_i, \mu_i, \varepsilon_i, K_i$ and ϕ_i as density, viscosity, porosity, permeability and volume fraction of nanofluids where '*i*' stands for indicating upper and lower layer as *i*=1 for lower layered fluid and *i*=2 for upper layer fluid.

It is worth mentioning that nanoparticles are having higher density than those of base fluids. The system is working under the effect of acceleration due to gravity **g** (0, 0, -g). In the system ***v***, ***p***, $\rho, \mu, \phi, D_B, K_m, \varepsilon, T$ *and* $\delta(z - z_s)$ denote the velocity, pressure, density, viscosity, volume fraction, the Brownian diffusion coefficient, permeability, porosity, surface tension of interfacial surface, and Dirac δ function.

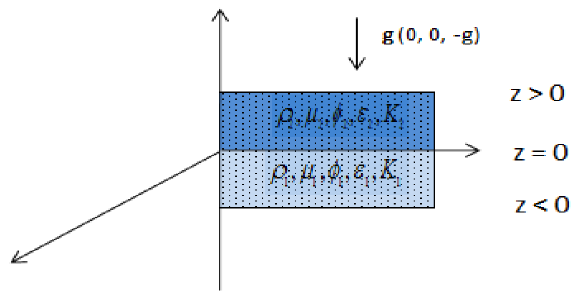

Figure 1. Physical Configuration

The basic equations like continuity, momentum, volume fraction and density are

$$\nabla.\boldsymbol{u} = 0, \tag{1}$$

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right] = -\nabla\boldsymbol{p} + \frac{\mu}{\varepsilon}\nabla^2\boldsymbol{u} + \rho\,\boldsymbol{g} - \frac{\mu}{K_m}\boldsymbol{u} + \sum\left[T\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)z_s\right]\delta(z\text{-}z_s), \tag{2}$$

$$\frac{\partial \rho}{\partial t} = -\frac{w}{\varepsilon}D\rho, \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \boldsymbol{u}\cdot\nabla\phi = \nabla\cdot\left[D_B\nabla\phi\right], \tag{4}$$

$$\rho_i = \phi_i\rho_{pi} + \left(1-\phi_i\right)\rho_{fi}, \tag{5}$$

where i=1 for lower layered fluid and i=2 for upper layer fluid.
And

$$\rho = \rho_1 + \rho_2, \tag{6}$$

with

$$\rho = \phi\,\rho_p + (1-\phi)\,\rho_f. \tag{7}$$

Now, by applying perturbation and normal mode techniques on Eqs. (1-6) the following set of equations is obtained

$$ik_x u + ik_y v + Dw = 0, \tag{8}$$

$$n\delta\phi = D_B(D^2 - k_1^{\,2})\phi, \tag{9}$$

$$n\delta\rho_f = -\frac{w}{\varepsilon}D\rho_f, \tag{10}$$

$$n\delta\rho_p = -\frac{w}{\varepsilon}D\rho_p, \tag{11}$$

$$\frac{\rho}{\varepsilon}nu = -ik_x(\delta p) + \frac{\mu}{\varepsilon}(D^2 - k_1^{\,2})u - \frac{\mu}{K_m}u, \tag{12}$$

$$\frac{\rho}{\varepsilon}nv = -ik_y(\delta p) + \frac{\mu}{\varepsilon}(D^2 - k_1^{\,2})v - \frac{\mu}{K_m}v, \tag{13}$$

$$\frac{\rho}{\varepsilon}nw = -D(\delta p) + \frac{\mu}{\varepsilon}\nabla^2(D^2 - k_1^{\,2})w - \mathbf{g}\left((\rho_p - \rho_f)\delta\phi + (1-\phi)\delta\rho_f + \phi(\delta\rho_p)\right) - \frac{\mu}{K_m}w + \sum\left[T(-k_1^{\,2})z_s\right]\delta(z - z_s). \tag{14}$$

Eq. (14) is further illustrated as

$$\frac{\rho}{\varepsilon}nw = -D(\delta p) + \frac{\mu}{\varepsilon}\nabla^2(D^2-k_1^{\,2})w - g\left[(\rho_p-\rho_f)D_B(D^2-k_1^{\,2})\phi + (1-\phi)\left(\frac{-w}{n\varepsilon}\right)D\rho_f + \phi\left(\frac{-w}{n\varepsilon}\right)D\rho_p\right]$$
$$-\frac{\mu}{K_m}w + \sum\left[T(-k_1^{\,2})\left(\frac{-w}{n\varepsilon}\right)\right]\delta(z-z_s). \tag{15}$$

By applying some algebraic operation to eliminate the pressure term, the resulting equation is

$$\frac{n}{\varepsilon}\left[D(\rho Dw)-\rho k_1^{\,2}w\right] = \frac{\mu}{\varepsilon}(D^2-k_1^{\,2})(D^2-k_1^{\,2})w - \frac{\mu}{K_m}(D^2-k_1^{\,2})w + gk_1^{\,2}\left[(\rho_p-\rho_f)(D^2-k_1^{\,2})\phi\right.$$
$$\left.-\frac{w}{n\varepsilon}(1-\phi)\mathrm{D}\rho_f - \frac{w}{n\varepsilon}\phi D\rho_p\right] + Tk_1^{\,4}\frac{w}{n\varepsilon}\delta(z-z_s). \tag{16}$$

For $\phi = 0$, Eq. (16) may be written as

$$D\left(\frac{\rho}{\varepsilon}+\frac{\mu}{K_m n}\right)Dw - \frac{\mu}{n\varepsilon}(D^2-k_1^{\,2})\mathrm{D}\ w = k_1^{\,2}\left[\left(\frac{\rho}{\varepsilon}+\frac{\mu}{K_m n}\right)w + Tk_1^2\frac{w}{n\varepsilon}\delta(z-z_s) - \frac{gw}{n^2\varepsilon}\mathrm{D}\rho\right]w. \tag{17}$$

Equation (17) represents the general relation of RTI of two superimposed fluids and coincides with the dispersion relation of Sharma and Bhardwaj (1994) in the absence of viscous term. In the absence of porosity, permeability, viscosity and nanoparticles, Eq. (17) coincides with the result of Sharma *et. al* (2010) without considering rotation. Further, Eq. (16) can be rewritten in simplified form as

$$\left[D(\rho Dw)-\rho k_1^{\,2}w\right] = \frac{\mu}{n}(D(D^2-k_1^{\,2}))Dw - \frac{\mu}{n}(D^2-k_1^{\,2})k_1^{\,2}w - \frac{\mu\varepsilon}{nK_m}D^2w + \frac{\mu}{K_m n}k_1^{\,2}w$$
$$+gk_1^{\,2}\left[\frac{\varepsilon}{n^2}(\rho_p-\rho_f)\phi_0 - \frac{w}{n^2}(1-\phi)\mathrm{D}\rho_f - \frac{w}{n^2}\phi D\rho_p\right] + Tk_1^{\,4}\frac{w}{n^2}\delta(z-z_s). \tag{18}$$

Integrating Eq. (18) across the surface gives

$$(\rho_2 Dw_2 - \rho_1 Dw_1) = \left[\frac{1}{n}\left\{\mu_2(D^2-k_1^{\,2})Dw_2 - \mu_1(D^2-k_1^{\,2})\ Dw_1\right\}\right] - \left[\frac{\mu_2\varepsilon_2}{nK_2}Dw_2 - \frac{\mu_1\varepsilon_1}{nK_1}Dw_1\right] \tag{19}$$
$$+\frac{g\,k_1^{\,2}}{n^2}(\rho_p-\rho_f)\phi_0\varepsilon_0 - \frac{1-\phi}{n^2}g\,k_1^{\,2}(\rho_{f2}-\rho_{f1})w_0 - \frac{\phi}{n^2}g\,k_1^{\,2}(\rho_{p2}-\rho_{p1})w_0 - T\frac{w_s k_1^{\,4}}{n^2}.$$

'w' must satisfy boundary conditions like w, Dw and $\mu\left(D^2+k_1^{\,2}\right)w$ must be continuous (Chandrasekhar (1981)) which gives the following equation

$$\left(\frac{-\mu_1\mu_2\rho^4}{4\mu^4 k_1^{\,3}}\right)n^5-\frac{\rho^3}{4\mu^4 k_1}\left[\frac{4\mu_1\mu_2\varepsilon}{\mu k_1^2 K_m}+\frac{1}{\mu}\left\{(\mu_2+\mu_1)^2-6\mu_1\mu_2\right\}-(\mu_1-\mu_2)\mathrm{A}\right]n^4-\left[\frac{3}{2}\frac{\mu_1\mu_2\rho^2}{k_1^3\mu^2K_m^2}\varepsilon^2\right.$$
$$+\varepsilon\left\{\frac{-\rho^2}{2\mu k_1 K_m}(\mu_1-\mu_2)A+\frac{3\rho^2}{4\mu^2 k_1 K_m}\left((\mu_2+\mu_1)^2+6\mu_1\mu_2\right)\right\}+\frac{3\rho^2}{4\mu^4 k_1}\left\{(\mu_2+\mu_1)^2-\frac{4}{3}\mu_1\mu_2\right\}$$
$$\left.+\frac{\rho^2}{4\mu^2 k_1}(\mu_1-\mu_2)\left(\frac{\mu_1\varepsilon_1}{K_1}-\frac{\mu_2\varepsilon_2}{K_2}\right)\right]n^3-\left[\frac{\mu_1\mu_2\rho}{k_1^3\mu K_m^3}\varepsilon^3+\varepsilon^2\left\{\frac{-1}{4k_1K_m^2}(\mu_1-\mu_2)\rho A+\frac{3\rho}{4\mu k_1 K_m^2}\right.\right.$$
$$\left.\left((\mu_2+\mu_1)^2+6\mu_1\mu_2\right)\right\}+\frac{\varepsilon}{\mu K_m}\left\{\frac{(\mu_1-\mu_2)}{2k_1}\left(\frac{\mu_1\varepsilon_1}{K_1}-\frac{\mu_2\varepsilon_2}{K_2}\right)+3k_1\rho\left((\mu_2+\mu_1)^2+\frac{4}{3}\mu_1\mu_2\right)\right\}$$
$$\left.-\frac{(\mu_2+\mu_1)\rho}{\mu}\left\{2k_1^{\,3}(\mu_2+\mu_1)+\frac{1}{2\mu}\rho^2\left\{\frac{\mathrm{T}\,\mathrm{k}_1^{\,2}}{\rho}-gA\right\}\right]n^2-\left[\frac{\mu_1\mu_2\rho}{4k_1^3K_m^4}\varepsilon^4+\frac{\varepsilon^3}{4k_1K_m^3}\left((\mu_2+\mu_1)^2\right.\right.$$
$$\left.+8\mu_1\mu_2\right)+\frac{\varepsilon^2}{K_m^2}\left\{\frac{1}{2}\left((\mu_2+\mu_1)^2+8\mu_1\mu_2\right)+\frac{(\mu_1-\mu_2)}{4k_1}\left(\frac{\mu_1\varepsilon_1}{K_1}-\frac{\mu_2\varepsilon_2}{K_2}\right)\right\}+\frac{\varepsilon}{K_m}\left\{(\mu_2+\mu_1)\left(2k_1^3\right.\right.$$
$$\left.\left.(\mu_2+\mu_1)+\frac{\rho^2}{\mu}\left\{\frac{\mathrm{T}\,\mathrm{k}_1^{\,2}}{\rho}-\mathrm{g}\,A\right\}\right)+\frac{\rho^2 k_1}{4\mu^3}(\mu_2+\mu_1)\left\{\frac{\mathrm{T}\,\mathrm{k}_1^{\,2}}{\rho}-\mathrm{g}\,A\right\}\right]n-\frac{\varepsilon^2\rho}{2K_m^2}(\mu_2+\mu_1)$$
$$\left\{\frac{\mathrm{T}\,\mathrm{k}_1^{\,2}}{\rho}-\mathrm{g}\,A\right\}-\frac{2\varepsilon k_1^2}{K_m}(\mu_2+\mu_1)\left\{\frac{\mathrm{T}\,\mathrm{k}_1^{\,2}}{\rho}-\mathrm{g}\,A\right\}-g\,k_1^{\,4}\phi_0(\rho_f-\rho_p)=0, \tag{20}$$

where A is defined as Atwood number as $\dfrac{\rho_2-\rho_1}{\rho_2+\rho_1}$.

3. Discussion

The following graphs are interpreating the effects of porosity and permeability on Rayleigh-Taylor instability in nanofluids. Figure 2 and Figure 3 are showing the stabilizing behaviour of porosity without nanoparticles and with nanoparticles. Figure 4 and Figure 5 are displaying the behaviour of permeability which clearly depict that permeability is destabilizing the configuration where as the critical values of growth rate is reduced with the addition of nanoparticles.

Figure 6 and Figure 7 are showing the effect of porosity on nanoparticles' volume fraction as the destabilization rate is reduced.

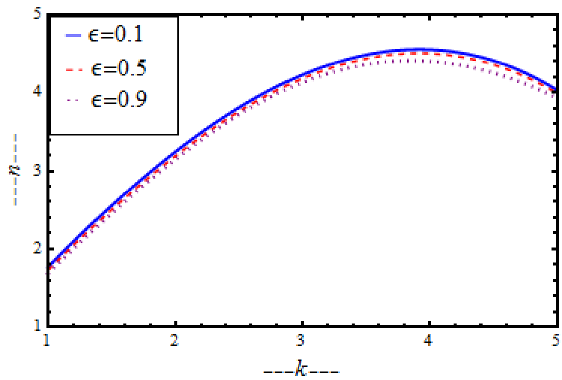

Figure 2: **Porosity without nanoparticals in RTI**

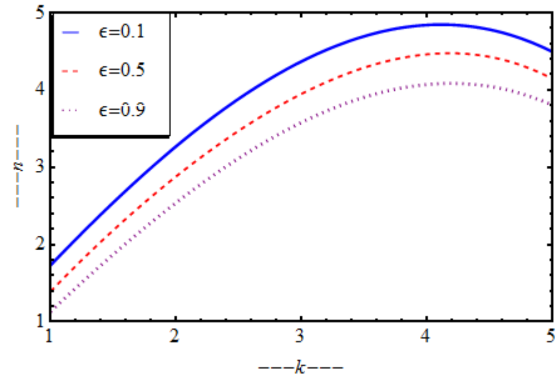

Figure 3: Porosity with nanoparticles in RTI

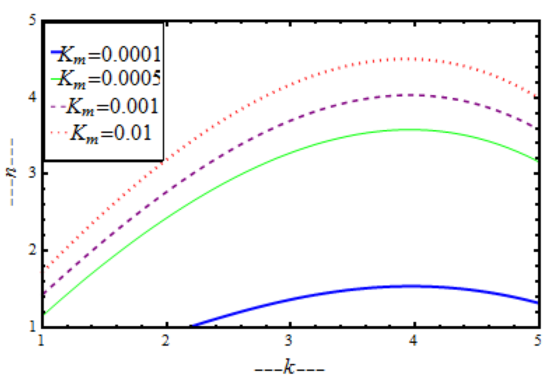

Figure 4: **Permeability without nanoparticals in RTI**

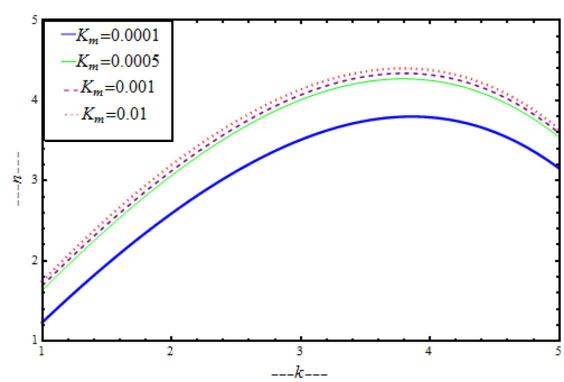

Figure 5: Permeability withnanoparticals in RTI

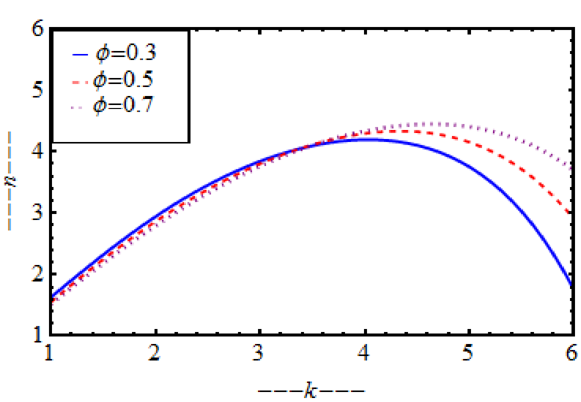

Figure 6: Volume fraction of nanoparticles with same nanoparticles

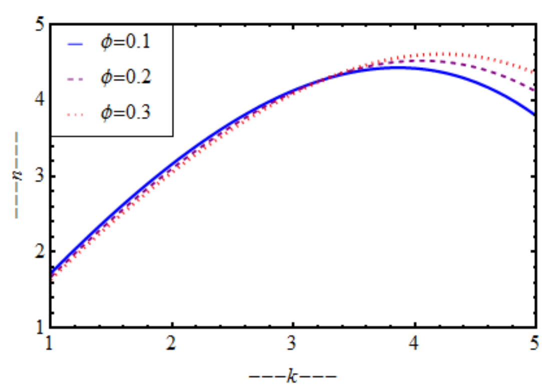

Figure 7: : Volume fraction of nanoparticles with different nanoparticles

4 Conclusion

The following facts are stated as conclusive remarks

i) Porosity factor is also interpreting the success in controlling the onset of Rayleigh Taylor instability in all three structures of absence and presence of same/ different nanoparticles. It is strongly confirmed through tables and figures of porosity parameter that presence of nanoparticles (same/different) proves to be much more focused on ensuring the stability.

ii) Permeability plays a significant role in destabilizing the system in all cases till certain value of permeability but after this value there is the destability effect is found to be negligible.

iii) Now as study continues to unveil the facts about the volume fraction, the very clear fact is disclosed that it helps Atwood number to destabilize the system more rapidly in compare to its absence but it also helps surface tension to stabilize at good rate. When nanoparticles are suspended at higher concentration, initially it takes the credit to stabilize the system (at slower rate) till at certain limit (critical value), then later it is adding its efforts to destabilize the unstable mode of configuration in both of the cases (same/different nanofluids are suspended). Therefore, this is relevant to say that presence of nanoparticles has a great impact on the considered configuration of RT instability.

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