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ON THE NON-HOMOGENEOUS TERNARY BI-QUADRATIC EQUATION

$$xz\,(x+z) = 2y^4$$

S.Vidhyalakshmi[1], M.A.Gopalan[2]

[1]Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University,Trichy-620 002,Tamil Nadu, India.
Email: vidhyasigc@gmail.com

[2]Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.
Email: mayilgopalan@gmail.com

ABSTRACT

This paper focuses on finding non-zero distinct integer solutions to the non-homogeneous ternary bi-quadratic diophantine equation given by $xz(x+z) = 2y^4$. Different sets of integer solutions are presented.

Key words: non-homogeneous bi-quadratic, ternary bi-quadratic ,integer solutions

Notations:

$$P_\alpha^3 = \frac{\alpha(\alpha+1)(\alpha+2)}{6},$$

$$P_\alpha^5 = \frac{\alpha^2(\alpha+1)}{2},$$

$$CP_\alpha^3 = \frac{\alpha^3+\alpha}{2},$$

$$CP_\alpha^4 = \frac{2\alpha^3+\alpha}{3},$$

$$CP_\alpha^{12} = 2\alpha^3 - \alpha$$

1. INTRODUCTION

The Diophantine equations are rich in variety and offer an unlimited field for research [1-4]. In particular refer [5-25] for a few problems on Biquadratic equation with 3 unknowns. This paper concerns with yet another interesting Biquadratic Diophantine equation with three variables given by $xz(x+z)=2y^4$ for determining its infinitely many non-zero distinct integral solutions.

Method of analysis:

The non-homogeneous ternary bi-quadratic equation under consideration is

$$xz(x+z)=2y^4 \tag{1}$$

Introduction of the linear transformations

$$x=u+v,\ z=u-v,\ y=u\,,\ u\neq v\neq 0 \tag{2}$$

in (1) leads to

$$u^2-u^3=v^2$$

which is satisfied by

$$u=(1-\alpha^2),\ v=\alpha(1-\alpha^2)$$

In view of (2), the corresponding integer solutions to (1) are given by

$$x=(1-\alpha)(1+\alpha)^2\,,\ z=(1+\alpha)(1-\alpha)^2\,,\ y=(1-\alpha^2) \tag{3}$$

A few interesting relations among the solutions are exhibited below:

(i). $x^3+z^3+6xyz=8y^3$

(ii). $x^2+4y^2+z^2=4y(x+z)-2xz$

(iii). $x^6=z^6+64\alpha^3y^6+12\alpha x^2y^2z^2$

(iv). $x^4+z^4+16\alpha^2y^4=2x^2z^2+8\alpha y^2(x^2-z^2)$

(v). $x^3=z^3+8\alpha^3y^3+6\alpha xyz$

(vi). $x^2+4\alpha^2y^2+z^2=4\alpha y(x-z)+2xz$

(vii). $9-3(x+y+z)$ is a perfect square

(viii). $6-2(x+y+z)$ is a square multiple of 6

(ix). $\begin{cases} \dfrac{x^3-z^3}{y^3}-4\alpha=4CP_\alpha^3, \\ \dfrac{x^3-z^3}{y^3}-5\alpha=3CP_\alpha^4, \\ \dfrac{x^3-z^3}{y^3}-7\alpha=CP_\alpha^{12} \end{cases}$

(x). $z - x = 12\,P^{3}_{\alpha-1}$

(xi). $z - x = 4\,P^{5}_{\alpha} - 4t_{3,\alpha}$

(xii). $\dfrac{x^4 - z^4}{y^4} = 16\,CP^{3}_{\alpha}$

(xiii). $\dfrac{x^4 + z^4 + 16\,y^4}{y^4}$ is a square multiple of 2

It is worth mentioning that, apart from (2), one may employ other choices of linear transformations leading to different sets of solutions . For simplicity and brevity , we present below a few sets of solutions to (1) :

Set 1:

$$x = 8k^2(1-k),\ y = 4k(1-k),\ z = 8k(1-k)^2$$

Set 2:

$$x = 2(2-k)(2+k)^2,\ y = 2(4-k^2),\ z = 2(2+k)(2-k)^2$$

Set 3:

$$x = 9k^2(2-3k),\ y = 3k(2-3k),\ z = 3k(2-3k)^2$$

Set 4:

$$x = (16-k)(16+k)^2,\ y = 2(16^2-k^2),\ z = (16+k)(16-k)^2$$

Set 5:

$$x = (32^2-k^2)(64+2k),\ y = 4(32^2-k^2),\ z = (32^2-k^2)(64-2k)$$

2. CONCLUSION

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous bi-quadratic Diophantine equation with three unknowns given by $xz(x+z) = 2\,y^4$ .One may search for other sets of integer solutions to the considered equation as well as other choices of the fourth degree diophantine equations with multi-variables

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