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ON FINDING THE INTEGER SOLUTIONS OF TERNARY QUADRATIC DIOPHANTINE EQUATION

$$w^2 - 2z^2 + 2wx + 20zx = 56x^2$$

S.Vidhyalakshmi[1], M.A.Gopalan[2]

[1]*Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University,Trichy-620 002,Tamil Nadu, India. Email: vidhyasigc@gmail.com*

[2]*Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India. Email: mayilgopalan@gmail.com*

ABSTRACT

The cone represented by the ternary quadratic Diophantine equation $w^2 - 2z^2 + 2wx + 20zx = 56x^2$ is analyzed for its patterns of non-zero distinct integral solutions.

Keywords: *Ternary quadratic, Homogeneous quadratic, Integral solutions*

1. INTRODUCTION

The Diophantine equation offers an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-13] for quadratic equations with three unknowns. This communication concerns with yet another interesting equation $\mathrm{w}^2 - 2\mathrm{z}^2 + 2\mathrm{wx} + 20\mathrm{zx} = 56\mathrm{x}^2$ representing homogeneous quadratic with three unknowns for determining its infinitely many non-zero integral points.

2. METHOD OF ANALYSIS

The given ternary quadratic Diophantine equation is

$$\mathrm{w}^2 - 2\mathrm{z}^2 + 2\mathrm{wx} + 20\mathrm{zx} = 56\mathrm{x}^2 \quad (1)$$

On completing the squares,(1) is written as

$$\mathrm{P}^2 - 2\mathrm{Q}^2 = 7\mathrm{x}^2 \quad (2)$$

Where,

$$\mathrm{P} = \mathrm{w} + \mathrm{x}, \mathrm{Q} = \mathrm{z} - 5\mathrm{x} \quad (3)$$

Let us see the different patterns of solving the above equation (3) and thus,

Obtain the different choices of x, w and z satisfying (1).

Choice I:

Let us assume

$$\mathrm{x} = \mathrm{a}^2 - 2\mathrm{b}^2 \quad (4)$$

We can write 7 as

$$7 = (3 + \sqrt{2})(3 - \sqrt{2}) \tag{5}$$

Using (4) and (5) in (2) and employing the method of factorization ,consider

$$P + \sqrt{2}Q = (3 + \sqrt{2})(a + \sqrt{2}b)^2 \tag{6}$$

Equating the rational and irrational parts,the values of P and Q are obtained.

In view of (3),one has

$$w = 2a^2 + 8b^2 + 4ab, z = 6a^2 - 8b^2 + 6ab \tag{7}$$

Thus,(4) and (7) represent the integer solutions to (1).

Note 1:

In addition to (5),one may have

$$7 = (5 + 3\sqrt{2})(5 - 3\sqrt{2}),$$
$$7 = (13 + 9\sqrt{2})(13 - 9\sqrt{2})$$

Following the above procedure,we have two more integer solutions to (1).

Choice II:

Write (2) as

$$P^2 - 2Q^2 = 7x^2 * 1 \tag{8}$$

Take 1 on the R.H.S. of (8) as

$$1 = (3 + 2\sqrt{2})(3 - 2\sqrt{2}) \tag{9}$$

Using (4),(5) & (9) in (8) and employing the method of factorization ,consider

$$P + \sqrt{2}Q = (13 + 9\sqrt{2})(a + \sqrt{2}b)^2 \tag{10}$$

Equating the rational and irrational parts,the values of P and Q are obtained.

In view of (3), one has

$$w = 12a^2 + 28b^2 + 36ab, z = 14a^2 + 8b^2 + 26ab \tag{11}$$

Thus,(4) and (11) represent the integer solutions to (1).

Note 2:

The integer 1 on the R.H.S. of (8) is also taken as

$$1=\frac{(2r^2+s^2+\sqrt{2}2rs)(2r^2+s^2-\sqrt{2}2rs)}{(2r^2-s^2)^2}$$

Following the above procedure ,a different solution to (1) is obtained.

Choice III:

Rewrite (2) as

$$P^2-7x^2=2Q^2 \tag{12}$$

Let us assume

$$Q=a^2-7b^2 \tag{13}$$

We can write 2 as

$$2=(3+\sqrt{7})(3-\sqrt{7}) \tag{14}$$

Using (13) and (14) in (12) and employing the method of factorization ,consider

$$P+\sqrt{7}x=(3+\sqrt{7})(a+\sqrt{7}b)^2 \tag{15}$$

Equating the rational and irrational parts,the values of P and x are obtained.

In view of (13) and (3),one has

$$w=2a^2+14b^2+8ab, z=6a^2+28b^2+30ab, x=a^2+7b^2+6ab \tag{16}$$

Thus, (16) represents the integer solutions to (1).

Note 3:

Apart from (14), 2 on the R.H.S. of (12) is also taken as

$$2=\frac{(5+\sqrt{7})(5-\sqrt{7})}{9},$$
$$2=\frac{(13+\sqrt{7})(13-\sqrt{7})}{81}$$

which lead to different set of solutions to (1).

Choice IV:

Rewrite (12) as

$$P^2 - 7x^2 = 2Q^2 * 1 \tag{17}$$

We can write 1 on the R.H.S. of (17) as

$$1 = (8 + 3\sqrt{7})(8 - 3\sqrt{7}) \tag{18}$$

Using (13) , (14) &(18) in (17) and employing the method of factorization ,consider

$$P + \sqrt{7}x = (45 + 17\sqrt{7})(a + \sqrt{7}b)^2 \tag{19}$$

Equating the rational and irrational parts,the values of P and x are obtained.

In view of (13) and (3),one has

$$w = 28(a^2 + 7b^2) + 148ab, z = 86a^2 + 588b^2 + 450ab,$$
$$x = 17(a^2 + 7b^2) + 90ab$$

(20)

Thus, (20) represents the integer solutions to (1).

Note 4:

Apart from (18), 1 on the R.H.S. of (17) is also taken as

$$1 = \frac{(11 + 4\sqrt{7})(11 - 4\sqrt{7})}{9},$$
$$1 = \frac{(4 + \sqrt{7})(4 - \sqrt{7})}{9},$$
$$1 = \frac{(7r^2 + s^2 + \sqrt{7}2rs)(7r^2 + s^2 - \sqrt{7}2rs)}{(7r^2 - s^2)^2}$$

which lead to a different set of solutions to (1).

Choice V:

Rewrite (2) as

$$P^2 = 2Q^2 + 7x^2 \tag{21}$$

Consider

$$Q = u + 7v, x = u - 2v, P = 3R \tag{22}$$

Now (21) implies

$$R^2 = u^2 + 14v^2 \tag{23}$$

satisfied by

$$R = 14r^2 + s^2, u = 14r^2 - s^2, v = 2rs$$

In view of (22),we have

$$Q = 14r^2 - s^2 + 14rs, P = 3(14r^2 + s^2) \qquad (24)$$

and

$$x = 14r^2 - s^2 - 4rs \qquad (25)$$

Using (24) and (25) in (3),one has

$$w = 28r^2 + 4s^2 + 4rs, z = 84r^2 - 6s^2 - 6rs$$

which ,along with (25), represent the integer solution of (1).

Note :

Note that (23) may be represented as the system of double equations as shown in Table 1

Below:

Table 1:System of double equations

SYSTEM	I	II	III	IV
R+u	v^2	$7v^2$	14v	7v
R-u	14	2	v	2v

Solving each of the above system of double equations, the values of R,u ,v are obtained. From (22) and (3) the integer solutions of (1) are found. For brevity,the integer solutions thus obtained are exhibited below:

Solutions from system I:

$$x = 2k^2 - 4k - 7, w = 4k^2 + 4k + 28, z = 12k^2 - 6k - 42$$

Solutions from system II:

$$x = 14k^2 - 4k - 1, w = 28k^2 + 4k + 4, z = 84k^2 - 6k - 6$$

Solutions from system III:

$$x = 9k, w = 36k, z = 72k$$

Solutions from system IV:

$$x = k, w = 26k, z = 24k$$

3. CONCLUSION

In this paper, we have made an attempt to obtain all integer solutions to (1). To conclude, one may search for integer solutions to other choices of ternary quadratic diophantine equations.

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